

Neural Network Aided Computation of Generalized Spatial Modulation Capacity

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Abstract—Generalized Spatial Modulation (GSM) is being considered for future high-capacity and energy efficient terrestrial networks. A variant such as Polarized Modulation (PMod) has also a role in Dual Polarization Mobile Satellite Systems. The implementation of adaptive GSM systems requires fast methods to evaluate the channel dependent GSM capacity, which amounts to solve multi-dimensional integrals without closed-form solutions. For this purpose, we propose the use of a Multilayer Feedforward Neural Network and an associated feature selection algorithm. The resulting method is highly accurate and with much lower complexity than alternative numerical methods.

Index Terms—Index Modulations, Generalized Spatial Modulation, Polarized Modulation, Machine Learning, Multilayer Feedforward Neural Network.

I. INTRODUCTION

The family of Index Modulations (IM) schemes [1] is gaining traction for next generation terrestrial and satellite networks. Among others, we can cite Generalized Spatial Modulation (GSM), its more simpler variant Spatial Modulation (SM) and Polarized Modulation (PMod). In all of them, part of the information is encoded in the selection of the building blocks, antennas in the case of SM and GSM, or polarizations in the case of PMod.

SM and GSM have been proposed for future 5G networks [1], since they increase the spectral efficiency compared with single antenna systems with simpler hardware requirements as compared with other multi-antenna techniques, reducing the power consumption. On the other hand, PMod allows to increase the spectral efficiency of the scarce spectrum mobile satellite systems, through the use of Dual Polarization and Multiple-Input-Multiple-Output (MIMO) signal processing techniques [2]. Moreover, some works [3] highlight PMod as a means to improve satellite coverage in remote areas to serve the vast number of Machine-to-Machine (M2M) devices.

The capacity calculation of any modulation scheme is not only interesting from a theoretical point of view, but it also has a

practical interest. For example, the application of Adaptive Coding and Modulation (ACM) requires the evaluation of the instantaneous capacity to select the proper Modulation and Coding Scheme (MCS). In [4] the authors provide some analytical approximations to the Mutual Information (MI), i.e., the capacity constrained to specific constellations, of SM systems. However, similar approximations for GSM are not found in the literature to the best of our knowledge; the expression to compute the true GSM capacity is presented in [5], a multi-dimensional integral which does not admit a closed-form solution.

In this work we evaluate the capacity of a GSM link by using a very simple neural network. Namely, we use a Multilayer Feedforward Neural Network (MFNN) with some input features properly selected by using an algorithm which preprocess the channel matrix and the Signal to Noise Ratio (SNR). Although in the last years several works applying Machine Learning to Communications have appeared, see [6], the application of neural networks to obtain the capacity of a non-conventional modulation scheme is something new.

Simulation results show that neural networks can compute successfully the capacity of SM/PMod and GSM with a very low error and moderate complexity, significantly lower than that of methods such as Monte Carlo. Thus, link adaptation methods in both terrestrial and satellite systems can track more accurately the channel capacity on the fly.

This paper is structured as follows. Section II explains our system model and introduces GSM briefly. Then, in Section III the integral expressions to compute the true capacity of GSM are given. Later, Section IV provides an overview of MFNNs and it also details the algorithm to select the neural network input features. Lastly, Section V contains the main simulation results before the presentation of the main conclusions.

II. SYSTEM MODEL

Generalized Spatial Modulation (GSM) is a family of multi-antenna modulation schemes where information is transmitted not only by modulating the amplitude, phase and/or frequency of a sinusoidal carrier, but also by selecting the group of

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antennas employed to transmit the modulated symbol(s) [1]. In general, we consider a $N_t \times N_r$ MIMO system, with N_t transmit antennas, N_r receive antennas and R RF (Radio Frequency) chains for conveying one modulated symbol taken from a modulation alphabet \mathcal{S} with cardinality M .

As particular cases, **Spatial Modulation (SM)** activates only one antenna at a time, and requires just one RF chain [1]. Note that SM 2×2 is equivalent to PMod. On the other hand, **Single Symbol GSM (SS-GSM)** sends the same symbol through all the ($R > 1$) active antennas during a channel use [7]. The total number of bits conveyed in these two systems is expressed as

$$\eta = \lfloor \log_2 \binom{N_t}{R} \rfloor + \log_2 M. \quad (1)$$

In the most general case, known as **Multi Symbol GSM (MS-GSM)**, the achievable spectral efficiency increases by sending R different symbols through the R active antennas [8]. Note that if $R = N_t$ in MS-GSM, then we have a conventional MIMO system. In this paper we will restrict ourselves to the SM and SS-GSM schemes, without excluding some hints on the evaluation of the capacity for the MS-GSM case.

The base-band samples for a given discrete-time instant of an SM or SS-GSM link can be modeled as

$$\mathbf{y} = \sqrt{\gamma/R} \cdot \mathbf{H} \cdot \mathbf{x} + \mathbf{w}. \quad (2)$$

$\mathbf{y} \in \mathbb{C}^{N_r}$ is the received vector, γ is the average SNR, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix, $\mathbf{x} \in \mathbb{C}^{N_t}$ is the transmitted signal and $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_r})$ is the Additive White Gaussian Noise (AWGN). Following the notation of [5], the transmit signal, $\mathbf{x} = \mathbf{A}\mathbf{s} = \mathbf{A}\mathbf{1}s$, is constructed with the vector of transmitted symbols $\mathbf{s} \in \mathbb{C}^R$ and an antenna activation pattern matrix $\mathbf{A} \in \mathcal{A}$. Here $\mathbf{1}$ denotes an $R \times 1$ all ones vector and s the modulation symbol. The set \mathcal{A} contains $N_t \times R$ sparse matrices, with at most one non-zero entry per row. Each column of \mathbf{A} contains a single 1 entry, in the row corresponding to the number of the active antenna. For example, to activate the antennas 1, 2 and 5 in a 6×6 system with 3 RF chains, the corresponding antenna activation pattern matrix would read as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}^T. \quad (3)$$

In general, the number of possible antenna activation choices is not a power of two and the transmitter and receiver have to agree on the use of a set of

$$L = |\mathcal{A}| = 2^{\lfloor \log_2 \binom{N_t}{R} \rfloor} \quad (4)$$

antenna activation pattern matrices. The set of all possible transmit signal vectors is then

$$\{\mathbf{x}: \mathbf{x} = \mathbf{A}_i \mathbf{1} s_k, \mathbf{A}_i \in \mathcal{A}, s_k \in \mathcal{S}\}. \quad (5)$$

We assume that all \mathbf{A}_i and s_k are equiprobable due to the lack of CSIT (Channel State Information at the Transmitter)

and that two independent sequences of information bits are used to select \mathbf{A}_i and s_k .

III. GSM CAPACITY

In this section we review the expression of the capacity of a GSM link with Gaussian signaling. Then, this is an upper bound for the use of a finite alphabet. The GSM channel capacity, for a fixed channel matrix, can be expressed as

$$C_{GSM} = I(\mathbf{x}; \mathbf{y}) = h(\mathbf{y}) - h(\mathbf{y}|\mathbf{x}) = h(\mathbf{y}) - h(\mathbf{w}), \quad (6)$$

with $I(\cdot; \cdot)$ denoting the mutual information (MI) among two random variables and $h(\cdot)$ the differential entropy. The entropy of the noise is simply $h(\mathbf{w}) = \log_2 \det(\pi e \mathbf{I}_{N_r})$ and \mathbf{y} follows a Gaussian mixture distribution [5]

$$p(\mathbf{y}) = \sum_{i=1}^L p(\mathbf{y}|\mathbf{A}_i) p(\mathbf{A}_i) = \frac{1}{L} \sum_{i=1}^L \mathcal{CN}(\boldsymbol{\mu}_i, \boldsymbol{\Phi}_i) \quad (7)$$

with parameters:

$$\boldsymbol{\mu}_i = \mathbb{E}\{\mathbf{y}|\mathbf{A}_i\} = \mathbb{E}\left\{\sqrt{\frac{\gamma}{R}} \mathbf{H} \mathbf{A}_i \mathbf{s} + \mathbf{w}\right\} = \mathbf{0} \quad (8)$$

$$\boldsymbol{\Phi}_i = \mathbb{E}\{\mathbf{y}\mathbf{y}^H|\mathbf{A}_i\} = \frac{\gamma}{R} \mathbf{H} \mathbf{A}_i \mathbb{E}\{\mathbf{s}\mathbf{s}^H\} \mathbf{A}_i^H \mathbf{H}^H + \mathbf{I}_{N_r}. \quad (9)$$

We will assume normalized unit-power symbols, so that the covariance matrix of the symbols vector \mathbf{s} is given for each case by one of the following values:

- **SM:** $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = 1$,
- **SS-GSM:** $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{1}_{R \times R}$, i.e., a matrix with all ones,
- **MS-GSM:** $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_R$, i.e., the identity matrix.

The differential entropy of \mathbf{y} requires the evaluation of

$$h(\mathbf{y}) = -\frac{1}{L} \sum_{i=1}^L \int_{\mathbf{y}} \mathcal{CN}(\mathbf{0}, \boldsymbol{\Phi}_i) \log_2 \left(\frac{1}{L} \sum_{j=1}^L \mathcal{CN}(\mathbf{0}, \boldsymbol{\Phi}_j) \right) d\mathbf{y}, \quad (10)$$

to obtain the GSM capacity from (6). Numerical integration or Monte Carlo simulations are in general required to compute (10), which is not practical when trying to estimate the channel capacity on-the-fly for adaptation purposes.

IV. MFNN AND FEATURES SELECTION

The evaluation of the GSM capacity (6) can be interpreted as a non-linear mapping from the tuple (γ, \mathbf{H}) to the value of the capacity C_{GSM} . Multilayer Feedforward Neural Networks (MFNNs), well-known for their fitting capabilities of non-linear functions [9], will be used here for the estimation of C_{GSM} . In particular, the MFNN to be employed, a one hidden layer network with N neurons is shown in Fig. 1. The extraction of the neural network input features from (γ, \mathbf{H}) has a paramount importance and therefore it is explained thoroughly in the following subsection. Alternatively, a deep neural network could be considered, by using directly the channel matrix entries -scaled by $\sqrt{\gamma}$ - as inputs rather than a carefully chosen set of input features obtained from our

knowledge of the problem. However, we have obtained much better results with single layer networks, requiring much shorter training periods; in addition, smoother training can be expected, in the sense that the parameters of the net converge easily to those values offering a good performance.

As it can be seen in Fig. 1, each of the F neural network inputs goes through a linear pre-processing block to adjust the neurons input to the range $[-1, +1]$. Then, there is a hidden layer of N neurons, each applying a weighted linear combination of its inputs, a bias and a non-linear activation function, in our case the hyperbolic tangent. Then it comes the output layer, with one linear neuron, and a final stage to accommodate the range of the network outcome to the capacity range. The different parameters of the network will be extracted from supervised learning with the Levenberg-Marquardt (LM) backpropagation algorithm [10] using the Mean Squared Error (MSE) as performance metric.

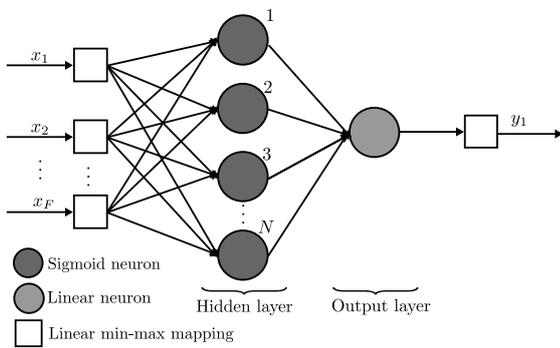


Fig. 1: Diagram of the neural network.

The true capacity values are obtained by solving the integral of (10) with Monte Carlo simulations. The network training is performed off-line, so that the receivers simply have to use the trained net. This entails a much lower complexity than obtaining (10), which requires the computation of L integrals of $2N_r$ real variables. The accurate calculation of high-dimensional integrals is a complex problem, since the number of function evaluations required for a given accuracy increases exponentially with the number of dimensions. Then, the use of a slow method such as Monte Carlo scheme is even faster than traditional numerical integration [11].

A. Input Variables Selection

Our proposal for the selection of the NN inputs is based on the expressions of the pairwise error probability (PEP). With Maximum Likelihood decoding, and using s and s' to denote two different modulation symbols and l and l' for two spatial symbols, i.e., two different selection of matrices from \mathcal{A} , the expression of the PEP between (s, l) and (s', l') is

$$P_e(l, s, l', s') = Q \left(\sqrt{\frac{\gamma}{2R}} \|\mathbf{H}\mathbf{A}_l \mathbf{1}s - \mathbf{H}\mathbf{A}_{l'} \mathbf{1}s'\| \right). \quad (11)$$

This expression, given by [12] for SM, shows the dependence of the PEP with the distance among the received symbols in

the absence of noise and with the SNR, which weights them with the noise power. Not surprisingly, the channel capacity will be also affected by the involved distances [13].

The square of the distance between two arbitrary noise-free received spatial-modulated symbols is written as

$$d = \sqrt{\gamma/R} \cdot \|\mathbf{H}\mathbf{A}_l \mathbf{1}s - \mathbf{H}\mathbf{A}_{l'} \mathbf{1}s'\|^2 = \|\mathbf{c}_l s - \mathbf{c}_{l'} s'\|^2, \quad (12)$$

where we have defined the column vector $\mathbf{c}_l = \sqrt{\gamma/R} \mathbf{H}\mathbf{A}_l \mathbf{1}$. With simple manipulations we obtain

$$d = \|\mathbf{c}_l\|^2 |s|^2 + \|\mathbf{c}_{l'}\|^2 |s'|^2 - 2\Re\{\mathbf{c}_l^H \mathbf{c}_{l'} s^* s'\}. \quad (13)$$

This shows that, apart from the properties of the modulated symbols, the capacity in a SM system also depends on the norms of the columns of the channel matrix and the scalar product between the columns. In the particular case of SS-GSM, it depends on the norms and scalar product of the sums of columns given by the antenna activation pattern matrices of the set \mathcal{A} . The scalar product between two complex column vectors can be expressed as

$$\mathbf{c}_1^H \mathbf{c}_2 = \|\mathbf{c}_1\| \cdot \|\mathbf{c}_2\| \cdot \cos \Theta_H \cdot e^{i\varphi}, \quad (14)$$

where, two angles between the complex vectors are used, namely Θ_H and φ . The so-called Hermitian angle Θ_H belongs to the interval $[0, \pi/2]$ whereas φ , named Kasner's pseudo-angle, takes values between $-\pi$ and π [14].

Based on all of the above, we know how to proceed to pre-process \mathbf{H} and γ to extract the relevant features to feed the neural network. Thus, Algorithm 1 details all the steps needed to feed the network. Firstly, the vectors \mathbf{c}_l are calculated with the channel matrix \mathbf{H} and the set of antenna activation pattern matrices \mathcal{A} . Then, these vectors are employed to obtain the three types of inputs of the neural network: the norms of the vectors \mathcal{N} , the Hermitian angles \mathcal{H} , and Kasner's pseudo-angles \mathcal{K} between all the pairs of these vectors \mathbf{c}_l .

The number of norms, and specially the angles, increase rapidly with the number of transmit antennas and RF chains. For example, with $N_t = 8$ and $R = 2$ there are $L = 16$ vectors \mathbf{c}_l according to (4) and $\binom{L}{R} = 120$ angles. As a consequence, to keep the number of NN inputs within reasonable bounds, we use a few values characterizing the distribution of both the norms and the angles, rather than feeding the entire set of values. When L or $\binom{L}{R}$ becomes higher than 8 (a value selected empirically), the inputs of the MFNN become the quantiles of norms and angles for some fixed probabilities. We have observed experimentally that the characterization of the Cumulative Distribution Function (CDF) using just $Q = 9$ probabilities (which include, among others, the minimum, the median and the maximum) for obtaining the quantiles, allows to make a good estimation of the GSM capacity while maintaining a reduced number of neural network inputs.

V. SIMULATION RESULTS

Seven scenarios were simulated to assess the merits of using MFNNs to compute the GSM channel capacity. The number

Algorithm 1 Pre-processing of γ and \mathbf{H} to obtain the neural network inputs

Input: γ , $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$, $\mathcal{A} = \{\mathbf{A}_l, l = 1, 2, \dots, L\}$, Q .

Output: \mathcal{N} , \mathcal{H} , \mathcal{K} .

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1: for  $l = 1$  to  $L$  do
2:   Calculate the column vector  $\mathbf{c}_l = \sqrt{\gamma/R} \mathbf{H} \mathbf{A}_l \mathbf{1}$ 
3:   Include its norm  $\|\mathbf{c}_l\|^2$  in the set  $\mathcal{N}$ .
4: end for
5: for  $k = 1$  to  $\binom{L}{2}$  do
6:   Calculate Hermitian angle  $\theta_H$  and Kasner's pseudoangle  $\varphi$  between a pair of column vectors  $\mathbf{c}_l$  and  $\mathbf{c}_{l'}$ .
7:   Include  $\theta_H$  in the set  $\mathcal{H}$  and  $\varphi$  in the set  $\mathcal{K}$ .
8: end for
9:  $\text{quants} \leftarrow \text{linspace}(0, 1, Q)$  {Equally spaced  $Q$  values among 0 and 1}
10: if  $L \leq 8$  then
11:    $\mathcal{N} \leftarrow \text{sort}(\mathcal{N})$  {Sort in ascending order}
12: else
13:    $\mathcal{N} \leftarrow \text{quantile}(\mathcal{N})$  {Calculate the quantiles of the values of  $\mathcal{N}$  for the cumulative probabilities quants}
14: end if
15: if  $\binom{L}{2} > 8$  then
16:    $\mathcal{H} \leftarrow \text{quantile}(\mathcal{H}, \text{quants})$ 
17:    $\mathcal{K} \leftarrow \text{quantile}(\mathcal{K}, \text{quants})$ 
18: end if

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of transmit and receive antennas was identical, ranging from 2 to 8, whereas the number of RF chains R was 1, 2 and 3. The seven scenarios are:

- 1) SM 2×2 (PMod), SM 4×4 and SM 8×8
- 2) SS-GSM 6×6 with $R = 2$ and $R = 3$
- 3) SS-GSM 8×8 with $R = 2$ and $R = 3$

For each particular scenario a dataset of 50,000 realizations of \mathbf{H} was generated, with matrices following a unit-variance Rayleigh distribution, i.e., $h_{ij} \sim \mathcal{CN}(0, 1)$. The SNR is drawn from a uniform random variable between -20 and 20 dB. The true capacity of each tuple (γ, \mathbf{H}) was calculated with (6) and (10), by using a Monte Carlo simulation with $5,000 \cdot L$ realizations of \mathbf{y} , where L denotes the number of spatial symbols.

For each scenario the dataset was divided into two independent parts. 15% of the samples were reserved for the final test of the performance of the MFNN. The remaining 70% and 15% were employed for training and validation of the neural network, respectively. Each network, one per scenario, was trained 10 times using different random initial values for the weights and biases. Finally, the parameters which provide the lowest MSE of the 10 independent trainings were retained. 10 and 20 neurons in the hidden layer of the MFNN were tested. As a reference of the computing time with Matlab[®], each training typically lasted less than 5 minutes, whereas the generation of the entire dataset with Monte Carlo simulations

required about 50 hours by using several cores of a processor in parallel.

Table I shows the results obtained with a one-hidden layer 20-neurons MFNN. In each case, apart from the MSE, we also include the typical error (3 times the standard deviation of the error) and the maximum error, all of them measured on the samples reserved for testing. The number of neurons and inputs of the net, the number spatial symbols L and the number of angles involved in each scenario $\binom{L}{2}$ are also included.

As it can be seen in Table I, the MSE is always in the order of 10^{-4} for all the scenarios. The error, seemingly following a Gaussian distribution, shows a typical and maximum value almost always below 0.07 and 0.10, respectively. This good estimation of the GSM capacity would make it possible for adaptive transmitters the selection of the Modulation and Coding Scheme (MCS) according to the capacity calculated and fed back by the receiver.

In order to grasp the relative magnitude of the error, Fig. 2 shows the ergodic capacity as a function of the average SNR for the different SM and SS-GSM cases, computed with the same datasets, i.e., for Rayleigh distributed channel matrices. Note that the transmit combined symbols $\mathbf{h}_l s$ are not Gaussian, which would be required to achieve the channel capacity of GSM [15]. This is why the capacity that we are computing and displaying in the figure is constrained to the specific selection mechanism of the antennas described in the paper. Thus, the capacity curves are not necessarily convex as usual for Gaussian symbols.

The proposed MFNN is a very efficient way of calculating the capacity of SM and GSM systems that, otherwise, would require resorting to long Monte Carlo simulations. For example, obtaining a value of capacity for a SS-GSM 8×8 system with $R = 3$ RF chains requires 5.58 ms (98% for the pre-processing) with the MFNN based capacity estimation. However, the Monte Carlo simulation lasts between 100 and 10,000 times more, depending on the required level of accuracy. All these times are based on computation time in Matlab[®] run in a computer equipped with an i7-4510U 2 GHz processor.

As to MS-GSM, with independent symbols transmitted per each RF chain, the norms and angles between the column vectors \mathbf{c}_l are not rich enough to yield a good capacity estimate. Although not shown here, we have used the eigenvalues of the submatrices of \mathbf{H} given by the antenna selection. Thus, if the quantiles of the eigenvalues are used as inputs to the NN, the MSE is around 10^{-3} , which grows in one order of magnitude if the inputs employed for SS-GSM are used instead.

VI. CONCLUSIONS

GSM can play an important role in future 5G terrestrial and satellite networks to increase the spectral efficiency while maintaining a reduced number of RF chains and, consequently,

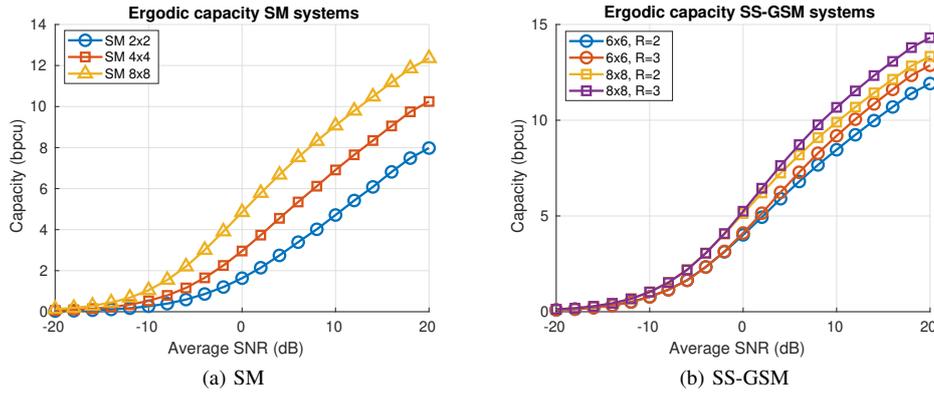


Fig. 2: Ergodic capacity for Rayleigh channel, $h_{ij} \sim \mathcal{CN}(0, 1)$, of several SM and SS-GSM systems.

Scenario	MSE	3σ	Max. error	Num. Neurons	NN inputs	L	$\binom{L}{2}$
SM 2×2 (PMod)	$5.27 \cdot 10^{-4}$	0.069	0.092	20	$4 = 2 + 2$	2	1
SM 4×4	$6.53 \cdot 10^{-4}$	0.077	0.142	20	$16 = 4 + 2 \times 6$	4	6
SM 8×8	$4.85 \cdot 10^{-4}$	0.066	0.097	20	$26 = 8 + 2 \times 9$	8	28
SS-GSM 6×6 , R=2	$4.00 \cdot 10^{-4}$	0.060	0.082	20	$26 = 8 + 2 \times 9$	8	28
SS-GSM 6×6 , R=3	$2.82 \cdot 10^{-4}$	0.050	0.109	20	$27 = 9 + 2 \times 9$	16	120
SS-GSM 8×8 , R=2	$2.75 \cdot 10^{-4}$	0.050	0.080	20	$27 = 9 + 2 \times 9$	16	120
SS-GSM 8×8 , R=3	$3.08 \cdot 10^{-4}$	0.053	0.090	20	$27 = 9 + 2 \times 9$	32	496

TABLE I: Performance of a 20-neurons MFNN for calculating the GSM capacity in several scenarios.

a low power consumption. The computation of the GSM capacity has a practical impact for link adaptation purposes; to this end, a Multilayer Feedforward Neural Network has been proposed. The usefulness of the network is highly dependent on the feature extraction, so that a simple algorithm was devised to obtain the network inputs from the channel matrix and the SNR. The accuracy of this new method was shown by using Rayleigh fading matrices in a wide range of SNR conditions and for several SM and SS-GSM schemes. Future terrestrial and mobile satellite systems can benefit from tools as those presented in this paper for the practical use of spatial modulation schemes in multi-antenna and multi-polarization links.

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