

LOW COMPLEXITY PRIMARY USER PROTECTION FOR COGNITIVE OFDM

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ABSTRACT

A novel active interference cancellation (AIC) scheme for primary user (PU) protection is presented for application to cognitive OFDM systems, in which out-of-band radiation spilling over the PU protected band is to be minimized. A set of cancellation subcarriers are modulated by appropriate linear combinations of the remaining data subcarriers. The combination coefficients are fixed and need not be changed on a symbol-by-symbol basis, in contrast with previous AIC approaches. Thus, the optimization can be carried out offline, drastically reducing the online implementation cost and power consumption. The proposed scheme is shown through simulations to outperform current AIC solutions at a lower computational cost.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely adopted as the modulation technique for many broadband wireless communication systems because of its high spectrum efficiency and robustness against multipath fading. Further, its natural bandwidth partitioning makes it a particularly well suited modulation scheme for cognitive systems, where the transmit signal needs to be adjusted according to the available transmission spectrum. Nevertheless, the high out-of-band radiation (OBR) characteristic of OFDM remains a limiting factor for its application to cognitive systems, since it results in high interference for primary users (PUs) lying within the secondary user (SU) OFDM band.

In recent years, considerable attention has been given to this problem, and several solutions have been reported. The use of multiple choice sequences and constellation expansion techniques were proposed in [1] and [2] respectively. Both techniques require the transmission of side information to the receiver and thus increase the system overhead. On the other hand, active interference cancellation (AIC) oriented schemes [3]-[7] and precoding techniques [8]-[12] do not require side

information at the receiver and are shown to have good OBR reduction performance. Precoding schemes naturally lead to low complexity implementations at the transmitter; however, the receiver needs to be aware of this fact and implement appropriate decoding of the received data. On the other hand, AIC schemes dedicate a subset of cancellation subcarriers in order to reduce OBR, without altering the data subcarriers. This operation is completely transparent to the receiver, which just needs to discard cancellation subcarriers, and thus a main advantage of AIC is its straightforward implementation in current systems.

In AIC schemes, the cancellation subcarriers are modulated by some function (usually a linear combination) of the symbols transmitted in the data subcarriers. Most solutions in the literature need to recompute the weights of the cancellation subcarriers at each OFDM symbol, making online computational cost a main concern [4]. This problem is exacerbated by the need to impose additional constraints in the optimization problem in order to keep the power allocated to cancellation subcarriers at bay, as in the constrained Least Squares (LS) approach of [3]. Reduced-complexity LS formulations were applied in [6] and [7], but the resulting power allocated to cancellation subcarriers is not kept under control. In contrast, the low-complexity implementation in [5] imposes individual power constraints on each cancellation subcarrier; however, PU protection performance is significantly reduced.

In this context, the main contribution of this paper is to derive a low complexity AIC scheme, without sacrificing PU protection performance. Different to reported AIC schemes, where optimization is performed over a discrete set of frequencies, the proposed approach is based on the direct minimization of the radiated power spilling over the PU protected band, computed as the integral of the power spectral density (PSD) over such band. This approach results in an AIC solution independent of the particular transmitted symbol, thus having a small online computational cost, since the cancellation weights can be computed offline. It is shown that the proposed formulation outperforms the schemes that use a set of discrete frequencies within the band, and does away with the problem of deciding which specific frequencies to include in the set.

The remainder of this paper is organized as follows. The

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signal model is presented in Sec. 2. In Sec. 3 the proposed AIC structure is defined, and the interference minimization over the PU band is derived. A performance evaluation to verify the effectiveness of the proposed approach is given in Sec. 4. Finally, conclusions are drawn in Sec. 5.

2. SIGNAL MODEL

A cognitive SU OFDM transmission with N subcarriers and power equally distributed among data subcarriers is considered. Focus is made on the case where a narrowband PU lies within the considered SU transmission bandwidth. It is assumed that the band \mathcal{B} corresponding to the PU can be fitted within N_P contiguous SU subcarriers. SU subcarriers are allocated as follows: N_P subcarriers (aligned with band \mathcal{B}) plus N_C subcarriers (usually taking $N_C/2$ at each side of band \mathcal{B}) are reserved for the OBR reduction task. The remaining $N_D = N - N_P - N_C$ subcarriers are unaffected and used for data transmission.

Based on this subcarrier allocation, an $N \times N_D$ matrix \mathbf{S} is defined, containing the N_D columns of of the $N \times N$ identity matrix \mathbf{I}_N corresponding to the data subcarriers. Analogously, we define the $N \times (N_P + N_C)$ matrix \mathbf{T} containing the columns on \mathbf{I}_N corresponding to the reserved subcarriers. Using these definitions, the $N \times 1$ vector modulating the SU subcarriers for a given OFDM symbol can be written as

$$\mathbf{x} = [x_0 \quad x_1 \quad \cdots \quad x_{N-1}]^T = \mathbf{S}\mathbf{d} + \mathbf{T}\mathbf{c}, \quad (1)$$

where \mathbf{d} is the $N_D \times 1$ data vector, and \mathbf{c} is a $(N_P + N_C) \times 1$ vector containing the cancellation coefficients to be modulated on the reserved subcarriers.

To keep the presentation simple, conventional OFDM is considered, in which a rectangular transmission window is employed. Let Δ_f be the subcarrier spacing, and $T = T_{\text{cp}} + T_u$ the OFDM symbol duration, with $T_u = 1/\Delta_f$ and T_{cp} the length of the cyclic prefix. The *normalized* cyclic prefix length is then $a = T_{\text{cp}}/T_u = T_{\text{cp}}\Delta_f$, and the spectrum corresponding to the k -th subcarrier, windowed over one OFDM symbol, is given for $k = 0, 1, \dots, N - 1$ by

$$\phi_k(f) = \text{sinc} \left((1 + a) \left(\frac{f}{\Delta_f} - k \right) \right). \quad (2)$$

Thus, the SU spectrum can be expressed as

$$X(f) = \sum_{k=0}^{N-1} x_k \phi_k(f) = \mathbf{x}^T \boldsymbol{\phi}(f), \quad (3)$$

where $\boldsymbol{\phi}(f) = [\phi_0(f) \quad \phi_1(f) \quad \cdots \quad \phi_{N-1}(f)]^T$. From (1) and (3), the PU protection problem amounts to choosing the cancellation coefficients \mathbf{c} (subject to appropriate design constraints) such that the resulting spectrum $X(f)$, measured over band \mathcal{B} , is 'small' in some sense. In the next section we address this problem by considering the radiated power over \mathcal{B} as objective function.

3. PSD BASED LOW COMPLEXITY AIC

3.1. Derivation

We consider generating the cancellation coefficients \mathbf{c} as linear combinations of the data symbols, i.e.,

$$\mathbf{c} = \boldsymbol{\Theta}\mathbf{d}, \quad (4)$$

where the $(N_P + N_C) \times N_D$ matrix $\boldsymbol{\Theta}$ is the parameter to be optimized. Note that $\boldsymbol{\Theta}$ is *fixed* and does not change from one OFDM symbol to the next (as long as the band to protect does not change). Therefore, it can be computed offline, and thus the online complexity of the AIC scheme boils down to the computation of (4) for each OFDM symbol.

Inserting (4) in (1) gives

$$\mathbf{x} = (\mathbf{S} + \mathbf{T}\boldsymbol{\Theta})\mathbf{d} = \mathbf{G}\mathbf{d}. \quad (5)$$

Since the operator $\mathbf{G} = \mathbf{S} + \mathbf{T}\boldsymbol{\Theta}$ is memoryless and static (time-invariant), the signal PSD can be approximated as

$$\begin{aligned} P_x(f) &\approx E \left\{ |X(f)|^2 \right\} = \boldsymbol{\phi}^H(f) E \{ \mathbf{x}\mathbf{x}^H \} \boldsymbol{\phi}(f) \\ &= \boldsymbol{\phi}^H(f) \mathbf{G} E \{ \mathbf{d}\mathbf{d}^H \} \mathbf{G}^H \boldsymbol{\phi}(f) \\ &= \text{tr} \{ \mathbf{G}^H \boldsymbol{\Phi}(f) \mathbf{G} \}, \end{aligned} \quad (6)$$

where we have assumed that the data are zero-mean i.i.d. with covariance $E \{ \mathbf{d}\mathbf{d}^H \} = \mathbf{I}_{N_D}$, and we have also introduced the matrix $\boldsymbol{\Phi}(f) = \boldsymbol{\phi}(f)\boldsymbol{\phi}^H(f)$, which is real-valued and symmetric.

The goal is to minimize the out-of-band radiation, under the constraint that the total transmitted power remains fixed, i.e.

$$\min_{\boldsymbol{\Theta}} \int_{\mathcal{B}} P_x(f) df \quad \text{s.t.} \quad \int_{-\infty}^{\infty} P_x(f) df = P_{\text{max}} \quad (7)$$

which can be rewritten as

$$\begin{aligned} \min_{\boldsymbol{\Theta}} \text{tr} \{ \mathbf{G}^H(\boldsymbol{\Theta}) \boldsymbol{\Phi}_{\mathcal{B}} \mathbf{G}(\boldsymbol{\Theta}) \} \\ \text{s.t.} \quad \text{tr} \{ \mathbf{G}^H(\boldsymbol{\Theta}) \boldsymbol{\Phi}_{\mathcal{T}} \mathbf{G}(\boldsymbol{\Theta}) \} = P_{\text{max}}, \end{aligned} \quad (8)$$

where the $N \times N$ matrices $\boldsymbol{\Phi}_{\mathcal{B}} = \int_{\mathcal{B}} \boldsymbol{\Phi}(f) df$ and $\boldsymbol{\Phi}_{\mathcal{T}} = \int_{-\infty}^{\infty} \boldsymbol{\Phi}(f) df$ are the integrals of the PSD over band \mathcal{B} and over the whole spectrum, respectively. Different to previously reported AIC schemes, the solution of (8), which concentrates most of the computational load, needs to be computed only once and that computation can be performed offline and stored.

By exploiting the fact that both $\boldsymbol{\Phi}_{\mathcal{B}}$ and $\boldsymbol{\Phi}_{\mathcal{T}}$ are real-valued and symmetric, the problem (8) can be solved via Lagrange multipliers and generalized singular value decomposition (gsvd) tools [13]. Let $\vec{\boldsymbol{\theta}} = \text{vec}(\boldsymbol{\Theta})$ and $\vec{\mathbf{s}} = \text{vec}(\mathbf{S})$ be the vectorization of matrices $\boldsymbol{\Theta}$ and \mathbf{S} respectively, and let

$$P_{d_{\mathcal{T}}} = \text{tr} \{ \mathbf{S}^T \boldsymbol{\Phi}_{\mathcal{T}} \mathbf{S} \}, \quad P_{d_{\mathcal{B}}} = \text{tr} \{ \mathbf{S}^T \boldsymbol{\Phi}_{\mathcal{B}} \mathbf{S} \} \quad (9)$$

be the contribution of the data subcarriers to the total power and to the power leaked over band \mathcal{B} , respectively. Then the problem (8) can be equivalently expressed as

$$\begin{aligned} \min_{\vec{\theta}} \quad & P_{dB} + \vec{\theta}^H \mathbf{A}^T \mathbf{A} \vec{\theta} + 2\Re \left\{ \vec{\theta}^H \mathbf{A}^T \mathbf{p} \right\} \\ \text{s.t.} \quad & \vec{\theta}^H \mathbf{B}^T \mathbf{B} \vec{\theta} + 2\Re \left\{ \vec{\theta}^H \mathbf{B}^T \mathbf{q} \right\} = \alpha P_{dT}, \end{aligned} \quad (10)$$

where $\alpha = (P_{\max} - P_{dT})/P_{dT} > 0$, whereas the matrices \mathbf{A} , \mathbf{B} and the vectors \mathbf{p} , \mathbf{q} are given by

$$\mathbf{A} = \mathbf{I}_{N_D} \otimes (\mathbf{P}_A^T \boldsymbol{\Lambda}_A^{1/2} \mathbf{P}_A), \quad (11)$$

$$\mathbf{B} = \mathbf{I}_{N_D} \otimes (\mathbf{P}_B^T \boldsymbol{\Lambda}_B^{1/2} \mathbf{P}_B), \quad (12)$$

$$\mathbf{p} = \mathbf{A}^{-1} (\mathbf{I}_{N_D} \otimes (\mathbf{T}^T \boldsymbol{\Phi}_B)) \vec{s}, \quad (13)$$

$$\mathbf{q} = \mathbf{B}^{-1} (\mathbf{I}_{N_D} \otimes (\mathbf{T}^T \boldsymbol{\Phi}_T)) \vec{s}, \quad (14)$$

with \mathbf{P}_A , $\boldsymbol{\Lambda}_A$, \mathbf{P}_B , $\boldsymbol{\Lambda}_B$ given by the eigendecompositions

$$\mathbf{T}^T \boldsymbol{\Phi}_B \mathbf{T} = \mathbf{P}_A^T \boldsymbol{\Lambda}_A \mathbf{P}_A, \quad \mathbf{T}^T \boldsymbol{\Phi}_T \mathbf{T} = \mathbf{P}_B^T \boldsymbol{\Lambda}_B \mathbf{P}_B. \quad (15)$$

From (10), and following [13, Ch.12], the optimal value of $\vec{\theta}$ is found to be

$$\vec{\theta} = -\mathbf{X} (\mathbf{D}_A^2 + \lambda \mathbf{D}_B^2)^{-1} (\mathbf{D}_A \mathbf{U}^T \mathbf{p} + \lambda \mathbf{D}_B \mathbf{V}^T \mathbf{q}), \quad (16)$$

where $\mathbf{U}^T \mathbf{A} \mathbf{X} = \mathbf{D}_A$ and $\mathbf{V}^T \mathbf{B} \mathbf{X} = \mathbf{D}_B$ are the gsvd of matrices \mathbf{A} and \mathbf{B} , and λ is the unique Lagrange multiplier that attains the power constraint. Finally, the linear combination matrix $\boldsymbol{\Theta} = \text{vec}^{-1}(\vec{\theta})$ is inserted in (4) to obtain the cancellation coefficients.

3.2. PSD-AIC computational cost

Disregarding matrices \mathbf{S} and \mathbf{T} that just map cancellation coefficients and data symbols to subcarriers, the online computational cost for PSD-AIC is given by the calculation of (4) which requires only $2 \times (N_C + N_P) \times N_D$ real operations (note that (16) is real-valued).

It is clear that most of the computational effort, i.e. the optimization of $\boldsymbol{\Theta}$, is performed offline. This is not the case in previous solutions [3],[5]. Further, the online computational cost of AIC schemes found in the literature depends directly on the frequency resolution, that is, the number of frequency points in band \mathcal{B} considered in the optimization. In our approach, although the integration step required in order to compute $\boldsymbol{\Phi}_B$ and $\boldsymbol{\Phi}_T$ has to be carried out numerically in general, which also involves some frequency resolution, this does not affect online complexity. As will be shown in Sec. 4, the proposed approach leads to improved PU protection performance thanks to the better spectral resolution available.

4. PERFORMANCE EVALUATION

The performance of the proposed scheme, termed PSD-AIC, is evaluated in this section. Comparison is made against the

	$N_C = 6$	$N_C = 8$	$N_C = 10$
Full load	-17.8	-17.8	-17.8
Null subcarriers	-20.6	-21.1	-21.6
PSD-AIC $\alpha = 0.01$	-28.1	-31.9	-36.8
PSD-AIC $\alpha = 0.02$	-30.2	-36.2	-40.3
PSD-AIC $\alpha = 0.03$	-32.0	-38.6	-41.8
PSD-AIC $\alpha = 0.04$	-33.6	-39.9	-42.5
PSD-AIC $\alpha = 0.05$	-34.9	-40.7	-43.1

Table 1. Mean notch depth over protected band \mathcal{B} (dB)

baseline scenarios of a fully loaded system (which only turns off the N_P subcarriers aligned with \mathcal{B}) and a system employing N_C null subcarriers to reduce OBR. Further, comparison against the cancellation subcarrier schemes of [3] and [5], which have comparable features is also provided.

To get a realistic evaluation, OFDM parameters are chosen based on current standards specifications [14, 15]. We consider an SU OFDM system consisting of $N = 1024$ subcarriers, together with a narrowband PU lying within the SU spectrum and with a bandwidth equivalent to $N_P = 20$ subcarriers. Data symbols to be modulated on the data subcarriers are i.i.d. and chosen from a 16-QAM constellation. A 5% CP is used, equivalent to 48 samples.

The power on the data subcarriers is kept fixed, while the parameter α in (10) is varied from 0.01 to 0.05 such that the extra power αP_{dT} spent on the cancellation subcarriers results a small fraction of the power of the data subcarriers, P_{dT} .

PU protection performance of the proposed PSD-AIC is considered in the results shown in Table 1. Specifically, it is shown that for $N_C = 8$ and $\alpha = 0.03$ the notch depth is increased 17.5 dB and more than 20 dB with respect to the null subcarriers and the fully loaded cases respectively, demonstrating the OBR reduction capability of the proposed scheme.

Figs. 1 and 2 show the behavior of the proposed scheme as the power on the cancellation subcarriers is increased while keeping N_C fixed, and as N_C is varied for a fixed cancellation power respectively. It can be noted how an increasing value of α improves the notch depth over the protected band, while it also increases the PSD peak values at the band edges due to the N_C cancellation subcarriers. It can also be noted that increasing N_C for a fixed α also improves the notch depth while reducing the PSD peak values as cancellation power is distributed among more subcarriers.

Figs. 3 and 4 compare the proposed active cancellation scheme with those from [3] and [5], referred to in the sequel as AIC and SR-AIC respectively. Both AIC and SR-AIC are based on the minimization of the SU spectrum over a discrete set of frequencies within \mathcal{B} . In particular, $M = 10$ samples per sidelobe are taken for the computations presented, as suggested by the authors to keep online computational load reasonable (the cancellation coefficients are computed *online* for

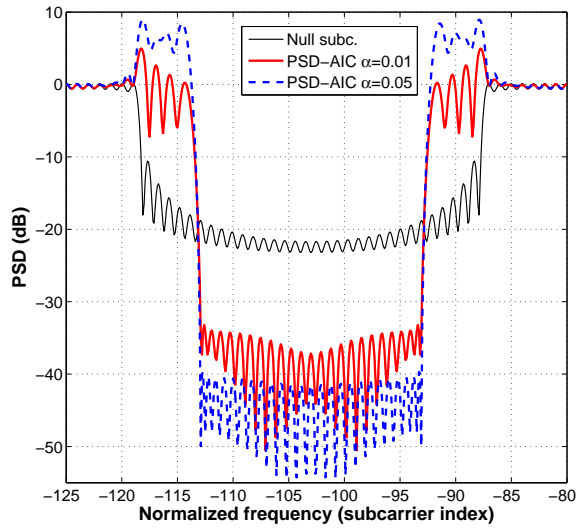


Fig. 1. PSDs of proposed PSD-AIC for $N_C = 10$ cancellation subcarriers and increasing values of α .

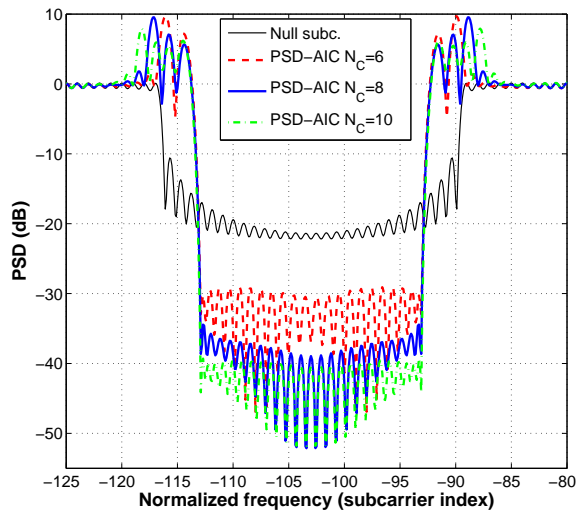


Fig. 2. PSDs of proposed PSD-AIC for $\alpha = 0.03$ and different amounts of cancellation subcarriers.

	Online complexity	Example
PSD-AIC	$2(N_C + N_P)N_D$	55, 776
AIC [3]	$\mathcal{O}(2NM + 1/2N_C^2M + 2/3N_C^3) + 2M(N_C + N_P)N_D$	> 578, 789
SR-AIC [5]	$2NM + 2M(N_C + N_P)N_D$	578, 240

Table 2. Online computational cost for compared AIC schemes. The last column indicates the approximate load for the parameters of Figs. 3 and 4.

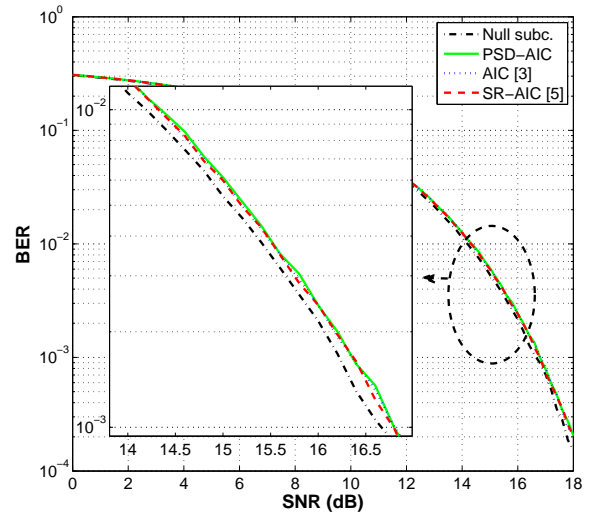


Fig. 3. BER performance for PSD-AIC, AIC [3] and SR-AIC [5] for $N_C = 8$ and $\alpha = 0.03$. Results are averaged over 500 OFDM symbols.

each OFDM symbol in both schemes). On the other hand, the matrices Φ_B and Φ_T featuring in the proposed method are evaluated numerically using a frequency resolution of 100 samples per sidelobe. Regarding the power constraint on the cancellation subcarriers, while AIC is designed under a power constraint equivalent to the one employed in this paper, this is not the case for SR-AIC. In SR-AIC an *individual* power constraint for each cancellation subcarrier is used. In the results presented here, these constraints are set all equal.

Fig. 3 shows the bit error rate (BER) performance vs. SNR by averaging over 500 OFDM symbols. Signals are scaled in order to have the same transmit power with all schemes. It can be seen that all AIC schemes included in this comparison exhibit a fixed SNR loss due to the power allocated to the cancellation subcarriers. As long as $N_C \ll N$, as will be the case in realistic situations, this SNR loss is not significant.

Although performance in terms of BER is similar for all compared AIC schemes, this is not the case for PU protection performance, as shown in Fig. 4. Normalized power spectra are plotted to this end using the same parameters as in Fig. 3. The main difference in performance between SR-AIC and both PSD-AIC and AIC arises for the different constraint used. The power constraints used in SR-AIC are more restrictive on the cancellation coefficients, leading to a significant performance loss in terms of OBR reduction. On the other hand, performance of AIC and proposed PSD-AIC is comparable, although favoring the proposed scheme. In this case the power constraints in the two optimization problems are equivalent; the better performance of PSD-AIC is obtained from the finer frequency resolution, which is obtained without compromising online computational cost.

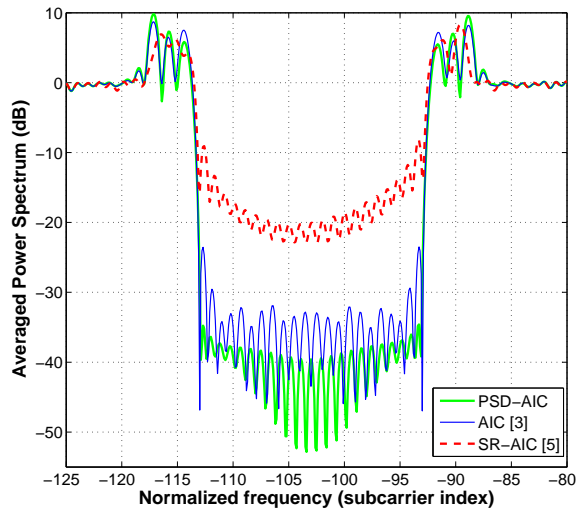


Fig. 4. Averaged power spectrum for PSD-AIC, AIC [3] and SR-AIC [5] for $N_C = 8$ and $\alpha = 0.03$. Results are averaged over 500 OFDM symbols.

Table 2 further emphasizes the complexity savings of the proposed structure. The online complexity for the three compared schemes is shown and the impact of M (the number of samples per sidelobe for the set of discrete frequencies) becomes evident. The large complexity savings of PSD-AIC come from the fact that its online computational cost is independent of the frequency resolution used. The computational cost for proposed PSD-AIC is less than 10% of that of AIC and SR-AIC resulting in computational savings of more than 90%.

5. CONCLUSIONS

A novel AIC structure was proposed for PU protection in cognitive OFDM systems, based on the definition of the cancellation subcarriers as linear combination of the data subcarriers. A low-complexity scheme for PU protection was derived from this structure, exploiting the fact that the structure definition enables most of the computational load to be performed offline. It was shown that the proposed scheme outperforms current AIC solutions in terms of PU protection at a much lower computational cost.

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