# Joint NDA Estimation of Carrier Frequency/Phase and SNR for Linearly Modulated Signals

Wilfried Gappmair, *Member, IEEE*, Roberto López-Valcarce, *Member, IEEE*, Carlos Mosquera, *Member, IEEE*

***Abstract*—We consider the problem of joint non-data-aided (NDA) estimation of carrier frequency and phase offsets, and the signal and noise powers, from the baud-rate samples of a linearly modulated signal in Gaussian noise. The Cramér-Rao lower bound (CRLB) is obtained, showing that, for quadrature-symmetric constellations, the carrier parameters are decoupled from signal and noise powers. A joint NDA maximum-likelihood (ML) estimator is developed, based on the application of the expectation-maximization (EM) algorithm. Its performance is close to the CRLB for a wide SNR range.**

***Index Terms*—Carrier offset estimation, SNR estimation, Cramér-Rao lower bound, maximum-likelihood estimation, expectation-maximization algorithm.**

## I. Introduction

Accurate estimation of carrier frequency and phase offsets before data detection is a mandatory task in high-speed digital receivers. Similarly, many advanced communication systems require knowledge of the signal-to-noise ratio (SNR) at the receiver side, e. g., in turbo decoding and link adaptation. Traditionally, these two estimation problems have been considered separately. In this respect, the Cramér-Rao lower bound (CRLB) for carrier frequency and phase estimation has been derived in [1] under the assumption that the signal and noise powers are known; in addition, many carrier recovery methods require that gain control be established first, which amounts to saying that at least the signal power is known. On the other hand, available results on SNR estimation either assume perfect carrier recovery [2], [3], or adopt an incoherent (i.e. envelope-based) approach [4] so as to achieve the necessary robustness to residual carrier offsets at the price of a degradation in estimation performance.

In a realistic scenario, however, all four parameters (frequency and phase offsets, signal and noise powers) must be estimated. In this context, the CRLB for non-data-aided (NDA) joint estimation of carrier parameters and SNR has been explored in [5] with respect to B(Q)PSK and MSK waveforms. We extend this approach to general quadrature-symmetric constellations, and develop a joint maximum-likelihood (ML) estimator for carrier frequency/phase and SNR, based on the expectation-maximization (EM) algorithm. This SNR estimator can cope with carrier offsets, unlike previous coherent estimators [3], and makes more efficient use of the data than envelope-based schemes.

W. Gappmair is with the Institute of Communication Networks and Satellite Communications, Graz University of Technology, Austria (e-mail: gappmair@tugraz.at).

R. López-Valcarce and C. Mosquera are with the Department of Signal Theory and Communications, University of Vigo, Spain (e-mail: valcarce@gts.tsc.uvigo.es, mosquera@gts.tsc.uvigo.es).

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## II. Signal Model

We assume that a linearly modulated signal is transmitted through a frequency-flat channel and that the symbol timing is perfectly recovered by the receiver. The data symbols $c_k$ are independently and equiprobably drawn from an $M$-ary constellation $\mathcal{C}$ with zero mean and unit variance. The $L$ received baud-spaced samples $r_k$ are affected by additive noise, magnitude scaling, and carrier offsets:

$$r_k = \sqrt{S}\, c_k\, \mathrm{e}^{\jmath(2\pi k\nu+\theta)} + \sqrt{N}\, w_k, \tag{1}$$

where $k \in \mathcal{K} = \{k_0, \dots, k_0 + L - 1\}$, $\nu$ is the frequency error (normalized to the symbol rate, and small enough so that intersymbol interference can be neglected), $\theta$ is the phase error, and $S$ and $N$ are respectively the signal and noise powers. The noise process $\{w_k\}$ is zero mean, white circular Gaussian with unit variance; hence, the SNR is given by $\rho \doteq S/N$.

## III. Cramér-Rao Lower Bound

Let $\mathbf{r}$ be the vector of observations and $\mathbf{u} = (u_1, u_2, u_3, u_4) = (\nu, \theta, \rho, N)$ the parameter vector to be estimated. The samples $r_k$ are conditionally Gaussian, and thus the related probability density function (pdf) is given by $f(r_k|\,\mathbf{u}, c) = \frac{1}{\pi N} \exp\{-\frac{1}{N}|r_k - \sqrt{\rho N}\, c\, \mathrm{e}^{\jmath(2\pi k\nu+\theta)}|^2\}$. Due to statistical independence, the joint pdf develops as $f(\mathbf{r}|\,\mathbf{u}) = \prod_{k\in\mathcal{K}} f(r_k|\,\mathbf{u})$, where

$$\begin{aligned} f(r_k|\,\mathbf{u}) &= \frac{1}{M}\sum_{c\in\mathcal{C}} f(r_k|\,\mathbf{u}, c) \\ &= \frac{1}{\pi N}\,\mathrm{e}^{-|r_k|^2/N}\,\frac{1}{M}\sum_{c\in\mathcal{C}} \mathrm{e}^{-\rho|c|^2} \\ &\quad \times \exp\left\{2\sqrt{\frac{\rho}{N}}\,\mathrm{Re}\left[r_k c^* \mathrm{e}^{-\jmath(2\pi k\nu+\theta)}\right]\right\}. \end{aligned} \tag{2}$$

The CRLB on the variance of any unbiased estimate of $u_i$ is given by the $i$-th diagonal element of the inverse of the Fisher information matrix (FIM) $\mathbf{I}(\mathbf{u})$, whose elements are defined as [6, Ch. 3]

$$\mathbf{I}_{ij}(\mathbf{u}) \doteq \mathrm{E}_{\mathbf{r}}\left[\frac{\partial\Lambda(\mathbf{r}|\mathbf{u})}{\partial u_i}\;\frac{\partial\Lambda(\mathbf{r}|\mathbf{u})}{\partial u_j}\right], \tag{3}$$

where $\Lambda(\mathbf{r}|\mathbf{u}) \doteq \log f(\mathbf{r}|\,\mathbf{u})$ is the log-likelihood function (LLF) of the parameters, and $\mathrm{E}_{\mathbf{r}}[\cdot]$ denotes expectation with respect to $f(\mathbf{r}|\,\mathbf{u})$. Then the following result holds (see the Appendix for the proof):

*Lemma 1:* Suppose that the constellation $\mathcal{C}$ has quadrature symmetry, i.e., if $c \in \mathcal{C}$, then $-c$, $c^*$ and $-c^*$ belong to $\mathcal{C}$ as well. Then the FIM is block diagonal as follows: $\mathbf{I}_{13}(\mathbf{u}) = \mathbf{I}_{14}(\mathbf{u}) = \mathbf{I}_{23}(\mathbf{u}) = \mathbf{I}_{24}(\mathbf{u}) = 0$. In addition, with the choice $k_0 = -(L-1)/2$, then $\mathbf{I}_{12}(\mathbf{u}) = 0$ as well.

Note that quadrature symmetry is satisfied for most practical constellations. This result implies that the parameter sets $\{\nu\}$, $\{\theta\}$ and $\{\rho, N\}$ are *decoupled*, i. e., the CRLB for any of these sets is the same, no matter if the other two are regarded as known or unknown. These bounds have been obtained in [1] for the carrier offset parameters (assuming $\rho$, $N$ known) and in [2] for the SNR (assuming $\nu$, $\theta$ known):

$$\mathrm{CRLB}_\nu = \frac{3F_M(\rho)}{2\pi^2 L(L^2-1)\rho}, \quad (4)$$

$$\mathrm{CRLB}_\theta = \frac{F_M(\rho)}{2L\rho}, \quad (5)$$

$$\mathrm{CRLB}_\rho = \frac{(2\rho+\rho^2)F_N(\rho)}{L}. \quad (6)$$

The NDA factors $F_M(\rho)$ and $F_N(\rho)$, derived in [1] and [2] respectively, are only functions of the constellation $\mathcal{C}$ and the true SNR; they quantify the loss with respect to the data-aided (DA) case. Both factors are larger than unity, approaching this limit asymptotically as $\rho \to \infty$. With the exception of MSK and B(Q)PSK schemes, for which analytical expressions exist [5], $F_M(\rho)$ and $F_N(\rho)$ must be numerically evaluated. For square QAM constellations, an efficient method (involving only one-dimensional integrals) for computing $F_N(\rho)$ is given in [7]. As a remark, the choice of a symmetric set $\mathcal{K}$ not only decouples the frequency and phase parameters, but also minimizes the CRLB for the phase [5].

Fig. 1 shows the CRLB (6) for SNR estimation (normalized to the true value $\rho^2$ and scaled by $L$) for 16-QAM and QPSK constellations, together with the corresponding bounds for envelope-based (EVB) estimators restricted to use only the magnitudes $\{|r_k|\}$ of the observations [4]. Asymptotically for high SNR, the CRLB for NDA EVB estimators is 3 dB above the corresponding CRLB for NDA I-Q based estimators, which in turn coincides with the DA CRLB in this region. These observations clearly motivate the search for schemes that jointly estimate the SNR and the carrier offset parameters.

## IV. ML Estimation based on the EM iteration

Lacking closed-form expressions, ML estimates have to be obtained by numerical methods. Among these, iterative gradient schemes are sensitive to stepsize tuning, whereas second-order schemes, like e.g. Newton's method, require costly evaluations of higher-order partial derivatives of the LLF[1]. On the other hand, NDA scenarios motivate the application of the EM algorithm [8], [9]. NDA SNR estimates based on the EM method have been previously reported in [3] (I-Q based, requiring previous carrier offset correction) and in [4] (envelope-based). Here we present an EM-based *joint* estimator of the SNR and the carrier offset parameters. Let $\mathbf{c}$ be the unknown vector of data symbols, so that we may consider $\mathbf{r}$ and $(\mathbf{r}, \mathbf{c})$ as incomplete and complete set of observables, respectively. For convenience, let $\mathbf{v} \doteq (\nu, \theta, S, N)$ be the unknown parameter vector, and $\hat{\mathbf{v}}^{(n)} = (\hat{\nu}^{(n)}, \hat{\theta}^{(n)}, \hat{S}^{(n)}, \hat{N}^{(n)})$ be the estimate on $\mathbf{v}$ at iteration $n$. In the expectation step of the EM scheme we must compute $\lambda(\mathbf{v}|\hat{\mathbf{v}}^{(n)}) \doteq \mathrm{E}_{\mathbf{c}}[\log f(\mathbf{r}|\,\mathbf{v},\mathbf{c})\,|\,\hat{\mathbf{v}}^{(n)}, \mathbf{r}]$, where $\mathrm{E}_{\mathbf{c}}[\cdot]$ denotes expectation with respect to the transmitted symbols. Observe that

[1]Another means to reduce the complexity of Newton's method is the method of scoring, which will not be pursued here.

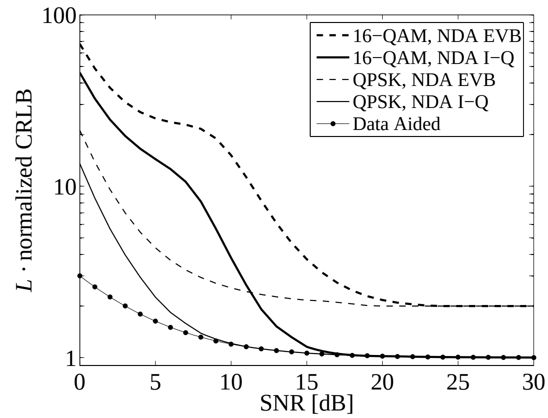


Fig. 1. Normalized Cramér-Rao lower bounds for SNR estimation (scaled by observation length $L$).

$$\log f(\mathbf{r}|\,\mathbf{v},\mathbf{c}) = -L\log\pi N - \frac{1}{N}\sum_{k\in\mathcal{K}}\Big\{|r_k|^2 + S|c_k|^2 - 2\sqrt{S}\,\mathrm{Re}\Big[r_k^* c_k \mathrm{e}^{\jmath(2\pi k\nu+\theta)}\Big]\Big\}. \quad (7)$$

Therefore, taking conditional expectations with respect to the $c_k$'s in (7), one obtains

$$\frac{\lambda(\mathbf{v}|\hat{\mathbf{v}}^{(n)})}{L} = -\log\pi N - \frac{1}{N}\left(M_2 - 2\sqrt{S}B^{(n)}(\nu,\theta) + SA^{(n)}\right) \quad (8)$$

where $M_p \doteq \frac{1}{L}\sum_{k\in\mathcal{K}}|r_k|^p$ denotes the $p$-th sample moment of the observations, and

$$A^{(n)} = \frac{1}{L}\sum_{k\in\mathcal{K}}\xi_k^{(n)}, \quad (9)$$

$$B^{(n)}(\nu,\theta) = \frac{1}{L}\sum_{k\in\mathcal{K}}\mathrm{Re}\left[r_k^*\eta_k^{(n)}\mathrm{e}^{\jmath(2\pi k\nu+\theta)}\right], \quad (10)$$

with $\eta_k^{(n)}$ and $\xi_k^{(n)}$ as *a posteriori* mean and mean-squared values of the $k$-th symbol:

$$\eta_k^{(n)} \doteq \sum_{c_i\in\mathcal{C}} P_{ik}^{(n)} c_i, \quad (11)$$

$$\xi_k^{(n)} \doteq \sum_{c_i\in\mathcal{C}} P_{ik}^{(n)} |c_i|^2. \quad (12)$$

$P_{ik}^{(n)}$ is the related *a posteriori* probability, which by Bayes' theorem is given as

$$P_{ik}^{(n)} \doteq \Pr(c_i|\,\hat{\mathbf{v}}^{(n)}, r_k) = \frac{f(r_k|\,\hat{\mathbf{v}}^{(n)}, c_i)\Pr(c_i)}{f(r_k|\,\hat{\mathbf{v}}^{(n)})}. \quad (13)$$

Assuming *a priori* equiprobable symbols, neither $\Pr(c_i) = \frac{1}{M}$ nor $f(r_k|\,\hat{\mathbf{v}}^{(n)})$ depend on index $i$; thus, they can be replaced by a normalization factor $\mu_k$ so that $P_{ik}^{(n)} = \mu_k f(r_k|\hat{\mathbf{v}}^{(n)}, c_i)$, with $\mu_k$ such that $\sum_{c_i\in\mathcal{C}} P_{ik}^{(n)} = 1$. Note that (11) corresponds to a *soft* decision on the $k$-th transmitted symbol.

In the maximization step, one must find $\hat{\mathbf{v}}^{(n+1)} = \arg\max_{\mathbf{v}} \lambda(\mathbf{v}|\,\hat{\mathbf{v}}^{(n)})$. This results in

$$\hat{\nu}^{(n+1)} = \arg\max_{\nu} \left| \sum_{k\in\mathcal{K}} r_k^* \eta_k^{(n)} \mathrm{e}^{\jmath 2\pi k\nu} \right|, \quad (14)$$

$$\hat{\theta}^{(n+1)} = -\arg \left[ \sum_{k\in\mathcal{K}} r_k^* \eta_k^{(n)} \mathrm{e}^{\jmath 2\pi k\hat{\nu}^{(n+1)}} \right], \quad (15)$$

$$\hat{S}^{(n+1)} = \frac{1}{[A^{(n)}]^2} \left| \frac{1}{L} \sum_{k\in\mathcal{K}} r_k^* \eta_k^{(n)} \mathrm{e}^{\jmath 2\pi k\hat{\nu}^{(n+1)}} \right|^2, \quad (16)$$

$$\hat{N}^{(n+1)} = M_2 - A^{(n)} \hat{S}^{(n+1)}. \quad (17)$$

A standard means to obtain $\hat{\nu}^{(n+1)}$ is to perform a $qL$-point FFT of the sequence $\{r_k^* \eta_k^{(n)},\ k \in \mathcal{K}\}$ and then apply parabolic interpolation around the maximum of this FFT (this is the approach used in Section V, with $q = 3$). The estimates (14) – (17) are used to compute the *a posteriori* probabilities $P_{ik}^{(n+1)}$ for the next cycle. The process is repeated until a pre-specified number of iterations $n_{\max}$ is reached; the final SNR estimate is then given by $\hat{\rho} = \hat{S}^{(n_{\max})}/\hat{N}^{(n_{\max})}$. In terms of computational cost, obtaining $\hat{nu}^{(n+1)}$ takes $\mathcal{O}(qL\log_2 qL)$ real multiplications, where the rest of steps require approximately $(4M+10)L$ multiplications per iteration. The number of iterations required for convergence depends on the modulation scheme and the SNR; noisy scenarios with dense constellations result in slower convergence.

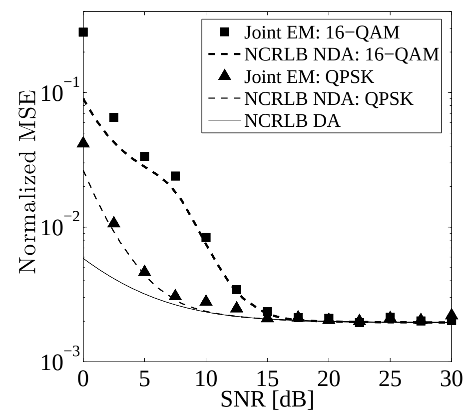


Fig. 2. SNR estimator performance and CRLBs ($L = 512$).

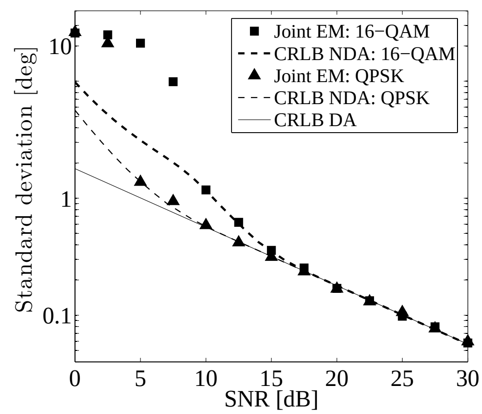


Fig. 3. Carrier phase estimator performance and CRLBs ($L = 512$).

## V. Numerical Results

We present results obtained with QPSK and 16-QAM via Monte Carlo simulations with $\nu = 0.03$, $\theta = 18^\circ$ and $L = 512$. The EM joint estimator is initialized as follows. The $M_2M_4$ estimates [10] of the signal and noise powers are taken as starting point: $\hat{S}^{(0)} = \sqrt{(2M_2^2 - M_4)/(2 - g_4)}$ (where $g_4 \doteq \mathrm{E}[|c_k|^4]$), and $\hat{N}^{(0)} = M_2 - \hat{S}^{(0)}$. The initial frequency estimate $\hat{\nu}^{(0)}$ is taken as the point at which the $L$-point FFT of the sequence $\{|r_k|^4,\ k \in \mathcal{K}\}$ attains its largest magnitude. Finally, the initial phase estimate is taken as $\hat{\theta}^{(0)} = -\frac{1}{4} \arg \left[ \sum_{k\in\mathcal{K}} \left( r_k \mathrm{e}^{-\jmath 2\pi k\hat{\nu}^{(0)}} \right)^4 \right]$.

Fig. 2 shows the mean square error of SNR estimates (normalized by $\rho^2$), as well as the normalized CRLBs for DA and NDA estimation [2], [10]. The performance of the estimator is fairly close to the theoretical limit, departing only at very low SNR values.

The performance of phase and frequency estimates is shown in Figs. 3 and 4 together with the corresponding CRLBs. Note that there exist ambiguities in the estimation of the carrier offsets due to the quadrature symmetry of the constellations and the lack of training symbols: only phases and frequencies respectively in the intervals $\theta \in [-45^\circ, 45^\circ]$ and $\nu \in [-\frac{1}{8}, \frac{1}{8}]$ are blindly identifiable.

As usually the case with NDA carrier estimates, a threshold effect is observed. The optimization problem from which the frequency estimate is obtained is prone to exhibiting local maxima far from the exact frequency value. When the SNR drops below some critical value, these ouliers may overtake the "true" maximum, resulting in a sharp increase of the estimation variance. Note that a "false" frequency estimate will result in a meaningless phase estimate as well. In fact, in the very low SNR region, $\hat{\theta}$ and $\hat{\nu}$ approximately follow uniform distributions in their respective ranges. The threshold SNR is higher for 16-QAM than for QPSK, as expected. From Fig. 2 it is seen, however, that the SNR estimator remains close to its CRLB even for SNR values below these thresholds. This robustness is likely to be a consequence of the parameter sets $\{\nu, \theta\}$ and $\{S, N\}$ being decoupled, as detailed in Section III.

## VI. Conclusions

Joint NDA estimation of carrier frequency and phase offsets, together with that of signal and noise powers, has been investigated for linearly modulated signals. The Cramér-Rao lower bound for this problem was derived, showing that the carrier parameters are decoupled from the power parameters for quadrature-symmetric constellations. In order to obtain the joint ML estimates, the EM algorithm, which computes soft decisions of the transmitted symbols as a byproduct, has been applied. By suitably modifying the corresponding *a priori* and *a posteriori* probabilities, the EM-based estimator can be straightforwardly adapted to the case in which training symbols are present in the received data packet.

$$g_\theta(z_k) = 2A_k(N)\sqrt{\frac{\rho}{N}}\sum_{c\in\mathcal{H}} \mathrm{e}^{-\rho|c|^2}\mathrm{Im}[c^* z_k]S_k(\rho,N), \quad (18)$$

$$g_\nu(z_k) = 4\pi k\, A_k(N)\sqrt{\frac{\rho}{N}}\sum_{c\in\mathcal{H}} \mathrm{e}^{-\rho|c|^2}\mathrm{Im}[c^* z_k]S_k(\rho,N) = 2\pi k\, g_\theta(z_k), \quad (19)$$

$$g_\rho(z_k) = A_k(N)\sum_{c\in\mathcal{H}} \mathrm{e}^{-\rho|c|^2}\left\{\frac{\mathrm{Re}[c^* z_k]}{\sqrt{\rho N}}S_k(\rho,N) - |c|^2 C_k(\rho,N)\right\}, \quad (20)$$

$$g_N(z_k) = A_k(N)\sum_{c\in\mathcal{H}} \mathrm{e}^{-\rho|c|^2}\left\{\left(-\frac{1}{N}+\frac{|z_k|^2}{N^2}\right)C_k(\rho,N) - \sqrt{\frac{\rho}{N^3}}\mathrm{Re}[c^* z_k]S_k(\rho,N)\right\}, \quad (21)$$

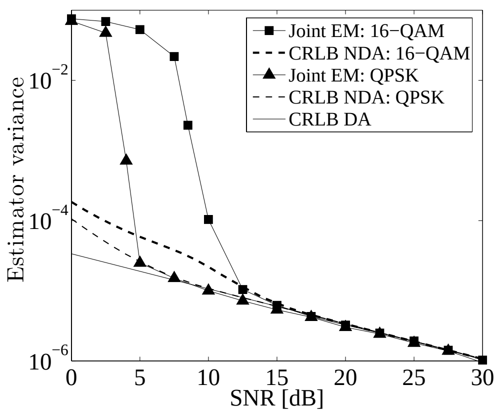


Fig. 4. Carrier frequency estimator performance and CRLBs ($L = 512$).

## Appendix
## Proof of Lemma 1

Conditioned on $\mathbf{u} = (u_1, u_2, u_3, u_4) = (\nu, \theta, \rho, N)$, the observations $r_k$ are independent, so that after some straightforward algebra the elements of the FIM can be expressed as

$$\mathbf{I}_{ij}(\mathbf{u}) = \sum_{k\in\mathcal{K}}\int_{\mathbb{C}} \frac{\partial f(r_k\,|\,\mathbf{u})}{\partial u_i}\frac{\partial f(r_k\,|\,\mathbf{u})}{\partial u_j}\frac{1}{f(r_k\,|\,\mathbf{u})}\,\mathrm{d}r_k, \quad (22)$$

where $\mathbb{C}$ denotes the complex plane. With $z_k \doteq r_k\,\mathrm{e}^{-\jmath(2\pi k\nu+\theta)}$, the pdf in (2) can be rewritten in a more convenient form as

$$g(z_k) \doteq f(r_k\,|\,\mathbf{u}) = A_k(N)\sum_{c\in\mathcal{H}} \mathrm{e}^{-\rho|c|^2}C_k(\rho,N), \quad (23)$$

where $A_k(N) \doteq \frac{2}{\pi MN}\,\mathrm{e}^{-|z_k|^2/N}$, $C_k(\rho,N) \doteq \cosh(2\sqrt{\rho/N}\,\mathrm{Re}[c^* z_k])$, and $\mathcal{H}$ is the subset of $\mathcal{C}$ comprising the symbols in the right (or left) complex semiplane. The partial derivatives $g_{u_i}(z_k) \doteq \partial g(z_k)/\partial u_i$ required in (22) are straightforwardly computed as (18) – (21) placed on top of this page, where $S_k(\rho,N) \doteq \sinh(2\sqrt{\rho/N}\,\mathrm{Re}[c^* z_k])$. From (23) – (21) and the quadrant symmetry of $\mathcal{C}$, it is easily verified that $g(z)$ and its partial derivatives satisfy the following properties:

$$g(z) = g(-z) = g(z^*) = g(-z^*), \quad (24)$$

$$g_{u_i}(z) = g_{u_i}(-z), \quad \text{for } i\in\{1,2,3,4\}, \quad (25)$$

$$g_{u_i}(z^*) = \begin{cases} -g_{u_i}(z), & \text{for } i\in\{1,2\}, \\ g_{u_i}(z), & \text{for } i\in\{3,4\}. \end{cases} \quad (26)$$

Now let $h_{ij}(z_k) \doteq g_{u_i}(z_k)g_{u_j}(z_k)/g(z_k)$. Then the symmetry properties (24) – (26) imply that $h_{ij}(-z) = h_{ij}(z)$ for all $i,j\in\{1,2,3,4\}$, whereas $h_{ij}(z^*) = -h_{ij}(z)$ if $i\in\{1,2\}$ and $j\in\{3,4\}$. Therefore,

$$\begin{aligned}\mathbf{I}_{ij}(\mathbf{u}) &= \sum_{k\in\mathcal{K}}\int_{\mathbb{C}} h_{ij}(z_k)\,\mathrm{d}z_k \\ &= \sum_{k\in\mathcal{K}}\int_{\mathbb{C}_1}\left[h_{ij}(z_k)+h_{ij}(-z_k)+h_{ij}(z_k^*)+h_{ij}(-z_k^*)\right]\mathrm{d}z_k \\ &= 0 \quad \text{if} \quad i\in\{1,2\} \quad \text{and} \quad j\in\{3,4\},\end{aligned} \quad (27)$$

where $\mathbb{C}_1$ is the first quadrant of the complex plane. Finally, the fact that $\nu$ and $\theta$ are uncoupled for $k_0 = -(L-1)/2$ follows immediately from

$$\begin{aligned}\mathbf{I}_{12}(\mathbf{u}) &= \sum_{k\in\mathcal{K}}\int_{\mathbb{C}_1} h_{12}(z_k)\,\mathrm{d}z_k \\ &= \sum_{k=-(L-1)/2}^{(L-1)/2} 2\pi k\int_{\mathbb{C}_1}\frac{g_\theta^2(z)}{g(z)}\,\mathrm{d}z = 0.\end{aligned} \quad (28)$$

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