Overlay Spectrum Reuse in a Broadcast Network: Covering the Whole Grayscale of Spaces

Alberto Rico-Alvariño*, Carlos Mosquera*†

*Signal Theory and Communications Department, Universidade de Vigo 36310 Vigo, Spain
email: {alberto,mosquera}@gts.uvigo.es
† Galician Research and Development Center in Advanced Telecommunications (GRADIANT), 36310 Vigo, Spain
email: mosquera@gradiant.org

***Abstract*—In this paper we address the problem of allowing secondary access to the spectrum assigned to a broadcast operator. We assume that the secondary transmitter knows of the primary message but is not able to apply interference cancellation techniques due to the lack of channel state information. Thus, interference cancellation can only be performed at the secondary receivers, provided they are able to decode the primary message.**

Unlike previous approaches, which study the particular cases of secondary access in a *white space* (far away from the influence of the primary transmitter) or a *black space* (inside the primary coverage zone), we present an analysis which is valid for any position of the secondary operation area. An interesting result arises from the study of *gray spaces* (a region which is near the primary coverage area), where the optimum power allocation for the secondary transmitter can increment the primary coverage area, thus benefiting the primary system.

***Index Terms*—Overlay, Cognitive Radio, Broadcast, Gray Spaces**

I. Introduction

The use of Cognitive Radio (CR) [1] to increase the efficiency of spectrum usage has been found to be a possible way to overcome the scarcity of radioelectric resources: by resorting to advanced communications techniques, one or more secondary users make use of a portion of the spectrum which is licensed to a primary operator. Obviously, this spectrum access has to be properly planned so the primary user Quality of Service (QoS) is not compromised by the presence of a secondary system.

Many different approaches to this secondary access have been investigated in the literature, but they can be roughly split into three different groups [2]:

- The *interweave* approach, where the secondary user senses the spectrum usage, detects the so-called *spectrum holes* (frequency bands which are not being used at the moment) and uses these resources to convey its own information. Although spectrum sensing is a very active research topic, the use of geolocation databases to determine the unused bands is emerging as a more practical mechanism [3].
- In the *underlay* spectrum access, the secondary user is allowed to transmit in the same resources as a primary user, but meeting a given interference constraint so the primary service is not compromised. In general, this approach implies the existence of interference between both primary and secondary networks, although the secondary receivers could opportunistically decode the primary message to remove part of this interference [4], [5]. The interference caused to the primary system can be reduced by *shaping* the spectrum [6]. Some works following this approach allow a minimum information exchange with the primary system [7] regarding the acceptable interference level.
- The *overlay* paradigm introduces the possibility of allocating a fraction of power to the transmission of the primary signal from the secondary transmitter. This interaction is possible if the secondary transmitter acts as a primary relay [8] or if the primary message is known a priori [9], and the secondary access can be *non-orthogonal* [10] (using the same time-frequency-space resources of the primary system) or *orthogonal* (exploiting the spatial dimension [11], or transmitting in free time slots [8] or frequency subbands [9]).

In this paper we will follow the overlay approach to study the insertion of a secondary transmitter in a broadcast primary network. In order to preserve the QoS of the primary user, we will constraint the secondary operation similarly to [12]: all the primary receivers have to be able to properly decode the primary signal without any modification in their hardware. As broadcast services are characterized by continuous transmission, this implies that the secondary access must be *non-orthogonal*.

Therefore, the existence of interference from the primary transmitter to the secondary receiver has to be managed somehow. In this paper we do not assume that the secondary transmitter is able to use Dirty Paper Coding [13], as it would require almost perfect channel state information at the transmitter [14]. However, interference cancellation can be performed at the secondary receiver, provided it is able

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to decode the primary message. This approach has been followed in [10] to study a secondary cellular system located inside the coverage area of a primary broadcast network, so interference cancellation is always possible. In this paper we study the case of secondary access in different positions with respect to the coverage zone of a broadcast network, so interference cancellation is not always possible unless the secondary transmitter *expanded* the coverage zone.

We will treat the previous idea by resorting to the concept of different *spaces*, similarly to [15]:

- **White Spaces** The secondary system is far from the primary coverage zone, so its performance is noise or power limited.
- **Black Spaces** The secondary system is inside the primary coverage zone, so it is able to decode the primary message and, therefore, cancel the interference.
- **Gray Spaces** The secondary system is outside the primary coverage zone, but it receives a considerably large amount of interference (that is not able to decode in principle) from the primary transmitter. Moreover, the interference caused to the primary system has to be properly controlled due to the proximity to the coverage zone.

The remaining of the paper is structured as follows: Section II introduces the system model; Section III presents the coding and decoding strategies for interference cancellation; Section IV presents the power allocation problem for a single secondary receiver; Section V extends the previous result to multiple secondary receivers; finally, Section VI concludes the paper.

II. System model

Let us assume a single Primary Transmitter (PT) transmitting at a rate R_p using P_p units of power, which is communicating with N_r Primary Receivers (PR).

A Secondary Transmitter (ST) with a total available power P_s is inserted in the network. We will assume that the ST has prior knowledge of the primary message, so the total power P_s has to be shared between the powers γ and ρ, allocated to the primary and secondary messages, respectively, so $\gamma + \rho \leq P_s$. The knowledge of the primary message can be obtained, for example, if the primary system operates as a Single Frequency Network (SFN), so the contents are delivered by a Distribution Network (DN), and the ST can obtain the primary waveform just by connecting to this DN [10]. The objective of the ST is to communicate with one or more Secondary Receivers (SR) at the highest possible rate while meeting a set of interference constraints regarding the correct reception of the primary message at the PR.

If we denote as $h_{p,i}$ the (complex) channel from the PT to the i-th PR, the capacity of the associated link, in absence of the ST, can be written as

$$C_{P,i}^{0} = C\left(\frac{|h_{p,i}|^2 P_p}{\sigma^2}\right) \tag{1}$$

with σ^2 the noise variance, assumed to be constant at all the receivers, and $C(x) \doteq \log_2(1+x)$. After the insertion of the ST, we will assume that the capacity can be expressed as

$$C_{P,i} = C\left(\frac{|h_{p,i}|^2 P_p + |h_{s,i}|^2 \gamma}{\sigma^2 + |h_{s,i}|^2 \rho}\right) \tag{2}$$

with $h_{s,i}$ the channel from the ST to the i-th PR. Note that we are assuming that the primary signal contributions coming from both primary and secondary transmitters are added, that would be the case of a cooperative transmission using an Alamouti Space Time Code (STC) [16], for example. Note that if the PT and ST transmit the same waveform (i.e., without STC) the channel gain will be of the form $|h_{p,i}P_p + \gamma h_{s,i}|^2$, so the interference will be constructive or destructive depending on the relative phase between channels [12]; in the case of a Orthogonal Frequency Division Multiplexing (OFDM) system, which has to be used in SFNs, this interference is going to be constructive on some carriers and destructive in others, so an alternative analysis like the one in [17] should be performed. In any case, we will restrict our analysis to the use of STC so the powers coming from both primary and secondary transmitter can be assumed to be *added*, like in (2).

As the PT is conveying data at a rate R_p, the i-th PR is able to decode the primary message provided that $C_{P,i} \geq R_p$ or, equivalently

$$\gamma \geq \frac{1}{|h_{s,i}|^2}\left(\Upsilon_0\sigma^2 - |h_{p,i}|^2 P_p\right) + \rho\Upsilon_0 = \rho\Upsilon_0 - \mathcal{M}_i,\ i = 1, \ldots, N_r \tag{3}$$

with $\Upsilon_0 \doteq 2^{R_p} - 1$ the required Signal to Interference plus Noise Ratio (SINR) for a correct reception of the primary message. We have denoted

$$\mathcal{M}_i \doteq \frac{1}{|h_{s,i}|^2}\left(|h_{p,i}|^2 P_p - \Upsilon_0\sigma^2\right),\ i = 1, \ldots, N_r \tag{4}$$

for the sake of clarity. Note that $|h_{s,i}|^2\mathcal{M}_i$ can be thought to be the *power margin* of the i-th PR before the insertion of the ST: $\mathcal{M}_i > 0$ means that the PR lies inside the primary coverage area, $\mathcal{M}_i < 0$ means that the PR lies outside the coverage area, and the receivers with $\mathcal{M}_i = 0$ are located at the coverage area edge. We consider that the primary service is not compromised if the N_r linear constraints in (3) are met or, equivalently, if the interference constraint is met in the worst PR case:

$$\gamma \geq \max_{i=1,\ldots,N_r} \rho\Upsilon_0 - \mathcal{M}_i, \tag{5}$$

so we must only take into account the receiver with a lower $\mathcal{M}_i$, $\mathcal{M} \doteq \min_{i=1,\ldots,N_r} \mathcal{M}_i$ or, equivalently, the one with a lower *power margin*. This idea is similar to that of a *critical TV receiver* in [10].

In Figure 1 there is a plot of the proposed system model.

III. Coding/decoding strategies

Now, we proceed to characterize the capacity of the secondary link. It is usually assumed in the literature that the ST uses Dirty Paper Coding (DPC) [13] to cancel the (a priori known) interference caused by the PT. However, the use of

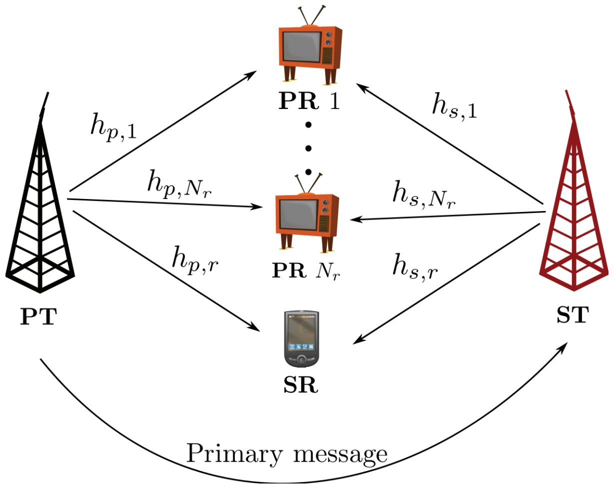

Fig. 1. System model: the primary and secondary systems interfere with each other, so the ST has to exploit the available side information (which can be obtained by connecting the ST to a Distribution Network [10], for example) to control the interference caused to the primary system.

DPC requires complete Channel State Information (CSI) at the transmitter, both in magnitude and phase [14]. Therefore, we will assume that the available CSI at the ST is similar to the Channel Quality Indicator (CQI) present in LTE [18], or obtained by means of field level measurements, so no phase information is available.

The channel seen from the SR is a Multiple Access Channel (MAC), so the capacity of a secondary link C_S is going to depend on the decodability of the primary message. In this section, we will fix the power allocation values (γ, ρ) and characterize the different decoding strategies a SR can use to recover the secondary message. Note that the power constraint $\gamma + \rho \leq P_s$ has to be met in all the cases and, as we will see next, sometimes this constraint is met with strict inequality at the optimum power allocation point. In the following, as depicted in Figure 1, we denote as $h_{s,r}$ and $h_{p,r}$ the channels from the ST and PT to the SR under study, respectively.

A. Treat as Noise

With this approach, the secondary receiver treats the primary interference as an additional noise source. Although this decoding strategy can be used in any interference regime, it is the only possible decoding procedure if the interference is weak even in absence of the ST transmitting the secondary message, i.e., the SNR of the primary signal at the SR is below the required SNR for decoding:

$$\frac{|h_{p,r}|^2 P_p + |h_{s,r}|^2 \gamma}{\sigma^2} < \Upsilon_0. \tag{6}$$

If this is the case, the primary rate R_p lies outside the capacity region of the MAC, so the SR can do nothing but treat the primary signal as noise. This strategy leads to the secondary link capacity

$$C_S = C\left(\frac{\rho |h_{s,r}|^2}{\sigma^2 + |h_{p,r}|^2 P_p + |h_{s,r}|^2 \gamma}\right). \tag{7}$$

B. Strong interference

If the primary interference is strong enough, even in presence of the secondary message

$$\frac{|h_{p,r}|^2 P_p + |h_{s,r}|^2 \gamma}{\sigma^2 + \rho |h_{s,r}|^2} \geq \Upsilon_0 \tag{8}$$

the SNR of the primary signal at the SR is above the threshold SNR, and, therefore, the SR is be able to decode the primary message and subtract it from the received signal, so the secondary link capacity is

$$C_S = C\left(\frac{\rho |h_{s,r}|^2}{\sigma^2}\right). \tag{9}$$

C. Medium interference

In this regime, the primary message is decodable prior to the transmission of the secondary message, but after its insertion is not:

$$\frac{|h_{p,r}|^2 P_p + |h_{s,r}|^2 \gamma}{\sigma^2 + \rho |h_{s,r}|^2} < \Upsilon_0 \leq \frac{|h_{p,r}|^2 P_p + |h_{s,r}|^2 \gamma}{\sigma^2}, \tag{10}$$

i.e., the SNR of the primary signal is above the threshold SNR if we do not take into account the interference caused by the insertion of the secondary message, and below the required SNR if we include this additional noise term.

This region corresponds to the classical *time-sharing* segment of the MAC. As *time-sharing* would imply the existence of synchronization between PT and ST, which is not desirable, the ST must resort to the use of Superposition Coding (SC) like in [4], which achieves the same capacity region, with the corresponding secondary link capacity given by

$$C_S = C\left(\frac{(\rho + \gamma) |h_{s,r}|^2 + |h_{p,r}|^2 P_p}{\sigma^2}\right) - R_p. \tag{11}$$

Note that this strategy requires the ST to split its secondary power ρ into ρ_1 and ρ_2 units of power, allocated to two different sources $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. In such a case, ρ_1 is chosen to meet

$$C\left(\frac{|h_{s,r}|^2 \gamma + |h_{p,r}|^2 P_p}{\sigma^2 + |h_{s,r}|^2 \rho_1}\right) \geq R_p \tag{12}$$

while $\rho_2 = \rho - \rho_1$. The encoding process is performed as follows: encode the message from source $\mathcal{S}_1$ into the codeword x_1 at a rate

$$R_1 = C\left(\frac{|h_{s,r}|^2 \rho_1}{\sigma^2}\right) \tag{13}$$

with ρ_1 the value obtained from (12) with equality; encode the message from source $\mathcal{S}_2$ into the codeword x_2 at a rate

$$R_2 = C\left(\frac{|h_{s,r}|^2 \rho_2}{\sigma^2 + |h_{s,r}|^2 \rho_1 + |h_{p,r}|^2 P_p}\right). \tag{14}$$

Decoding is performed as follows: decode x_2 treating x_1 and the primary message as noise, subtracting it from the received signal; decode the primary message treating x_1 as noise, subtracting it from the received signal; finally, decode x_1. It can be seen that the three rates (12), (13) and (14) lie inside the three dimensional MAC region, so the proposed decoding sequence is possible.

It can be easily seen that

$$R_1 + R_2 = C\left(\frac{(\rho+\gamma)|h_{s,r}|^2 + |h_{p,r}|^2 P_p}{\sigma^2}\right) - R_p. \quad (15)$$

IV. Optimum power allocation for a single secondary receiver

As the capacity function is defined in a piecewise way and, therefore, is not differentiable, we propose to solve three different optimization problems (one for each of the three decoding strategies) and afterwards select the one that leads to a higher capacity. The two last decoding strategies conform problems with a different associated *region constraint* (8) and (10), and the three of them share both a *power constraint* $\gamma + \rho \leq P_s$ and N_r *interference constraints* (or a worst case constraint (5)). In order to define in a compact way the *region constraints* we denote

$$\mathcal{M}^s \doteq \frac{1}{|h_{s,r}|^2}\left(|h_{p,r}|^2 P_p - \Upsilon_0 \sigma^2\right) \quad (16)$$

as the *power margin* for the primary signal at the SR under study. Note that, unlike $\mathcal{M}$, which is related to the interference constraints at the primary receivers, we will use this parameter to constrain the operation of the different decoding strategies at the SR to the corresponding interference regimes.

Now, we proceed to enunciate the three different problems:

A. Treat as noise

With this decoding strategy, we can enunciate the optimization problem as

$$\begin{aligned} \text{max.} \quad & f(\rho,\gamma) \doteq \frac{\rho|h_{s,r}|^2}{\sigma^2 + \gamma|h_{s,r}|^2 + |h_{p,r}|^2 P_p} \\ \text{s.t.} \quad & \gamma + \rho \leq P_s \\ & \gamma \geq \rho\Upsilon_0 - \mathcal{M} \\ & \gamma \geq 0 \\ & \rho \geq 0. \end{aligned} \quad (17)$$

This is a linear-fractional program, which can be easily converted to a Linear Program (LP) by the Charnes-Cooper transformation [19]. It is clear that if $\mathcal{M}^s > 0$, i.e., if the primary signal is decodable at the SR in absence of the ST, or if $\mathcal{M} \leq \mathcal{M}^s$, i.e., if the reinforcement of one of the primary receivers forces the SR to be able to decode the primary signal, then the SR could operate in the medium or strong interference regimes, thus leading to a higher capacity. Also, if some $\mathcal{M}_i < 0$ (so $\mathcal{M} < 0$) we are introducing a constraint on some PRs that are not able to decode the primary signal prior to the insertion of the ST - these receivers could be thought to be lying outside the *primary coverage area*, and the ST would be constrained to extend that coverage area.

Note that the objective function f is increasing with ρ and decreasing with γ, so the allocation of power to the primary mesasage decreases the capacity of the secondary user. Therefore, it would be desirable to allocate power only to the secondary message, but the presence of an active interference constraint forces to allocate $\Delta\gamma = \Upsilon_0 \Delta\rho$ units of power to the primary message for every $\Delta\rho$ units of power allocated to the secondary message or, equivalently, to move in the direction $\mathbf{x}$ given by

$$\mathbf{x} = \begin{bmatrix} \Delta\rho \\ \Delta\gamma \end{bmatrix} = k \begin{bmatrix} 1 \\ \Upsilon_0 \end{bmatrix}. \quad (18)$$

We can check if the objective function is increasing in the direction dictated by the constraint just by calculating the gradient ∇f of the objective function

$$\begin{bmatrix} \frac{\partial f}{\partial \rho} \\ \frac{\partial f}{\partial \gamma} \end{bmatrix} = \nabla f = \frac{|h_{s,r}|^2}{\sigma^2 + \gamma|h_{s,r}|^2 + |h_{p,r}|^2 P_p} \times \begin{bmatrix} 1 \\ -\frac{|h_{s,r}|^2}{\sigma^2 + \gamma|h_{s,r}|^2 + |h_{p,r}|^2 P_p} \end{bmatrix} \quad (19)$$

so if $\nabla f^T \mathbf{x} > 0$ the function is increasing in the constraint direction or, equivalently, if

$$\Upsilon(\gamma,\rho) < \frac{1}{\Upsilon_0} \quad (20)$$

with

$$\Upsilon(\gamma,\rho) = \frac{|h_{s,r}|^2 \rho}{\sigma^2 + \gamma|h_{s,r}|^2 + |h_{p,r}|^2 P_p} \quad (21)$$

the current SINR of the secondary link. Therefore, we conclude that allocating extra power $\mathbf{x}$ to both primary and secondary signal components in the direction dictated by the interference constraint is beneficial if the current SINR of the secondary link meets $\Upsilon(\gamma,\rho) < \frac{1}{\Upsilon_0}$. If this is not the case, then the optimum power allocation leaves some power without being used. A plot explaining the geometrical interpretation of this constraint is shown in Figure 2.

B. Strong interference

In this case, the optimization problem is stated as

$$\text{max.} \quad \frac{\rho|h_{s,r}|^2}{\sigma^2} \quad (22)$$

$$\text{s.t.} \quad \gamma + \rho \leq P_s \quad (23)$$

$$\gamma \geq \rho\Upsilon_0 - \mathcal{M} \quad (24)$$

$$\gamma \geq \rho\Upsilon_0 - \mathcal{M}^s \quad (25)$$

$$\gamma \geq 0$$

$$\rho \geq 0$$

which is a LP. Note that if $\mathcal{M}^s < \mathcal{M}$ then this region is forced, as the *worst case* PR has a smaller *power margin* than the SR; this could be the case of having a SR inside the primary coverage zone, for example. It is clear that in this region the ST is going to use all its available power, so by forcing an active power constraint (23) we can jointly write the interference (24) and region (25) constraints as

$$\rho \leq \frac{P_s + \min\{\mathcal{M}, \mathcal{M}^s\}}{1 + \Upsilon_0}, \quad (26)$$

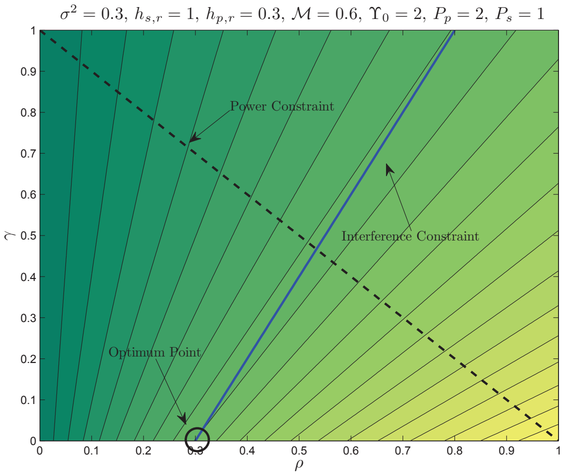

Fig. 2. Level curves for the objective function f (those zones with lighter colors ahcieve a larger capacity) and constraints of the optimization problem. If we are using an *Active Set* method [20], for example, starting at point $(\rho, \gamma) = (0, 0)$, on the first iteration we will get to the point $(0.3, 0)$. At this point, the interference constraint forces us to move along the direction in (18), which leads to lower levels of the objective function. Therefore, the optimum point is reached at $(\rho, \gamma) = (0.3, 0)$.

so the optimum value of ρ is

$$\rho = \min\left\{P_s, \frac{P_s + \min\{\mathcal{M}, \mathcal{M}^s\}}{1+\Upsilon_0}\right\}. \tag{27}$$

Note that if $P_s < -\min\{\mathcal{M}, \mathcal{M}^s\}$ (27) will be negative because the original problem is infeasible, as the available secondary power is not enough to allow the SR (or the worst case PR) to decode the primary message. If this is not the case, a rate

$$C_S = \min\left\{C\left(\frac{P_s|h_{s,r}|^2}{\sigma^2}\right), C\left(\frac{|h_{s,r}|^2}{\sigma^2}\frac{P_s + \max\{\mathcal{M}, \mathcal{M}^s\}}{1+\Upsilon_0}\right)\right\} \tag{28}$$

is achievable.

C. Medium interference

Finally, the third decoding strategy can be solved by the following optimization problem

$$\begin{aligned}
\text{max.} \quad & \frac{(\rho+\gamma)\,|h_{s,r}|^2}{\sigma^2} && (29)\\
\text{s.t.} \quad & \gamma + \rho \le P_s && (30)\\
& \gamma \ge \rho\Upsilon_0 - \mathcal{M} && (31)\\
& \gamma \le \rho\Upsilon_0 - \mathcal{M}^s && (32)\\
& \gamma \ge -\mathcal{M}^s && (33)\\
& \gamma \ge 0 \\
& \rho \ge 0.
\end{aligned}$$

Remark that the feasibility of this problem requires $\mathcal{M}^s \le \mathcal{M}$, $-\mathcal{M}^s \le P_s$ and $P_s\Upsilon_0 \ge \mathcal{M}^s$. If we assume an active power constraint (30), the region (32) and interference (31) constraints can be written as

$$\frac{\Upsilon_0 P_s - \mathcal{M}^s}{1+\Upsilon_0} \ge \gamma \ge \frac{\Upsilon_0 P_s - \mathcal{M}}{1+\Upsilon_0} \tag{34}$$

so after taking into account the other region constraint (33), if we set $\gamma = \max\left\{-\mathcal{M}^s, \frac{\Upsilon_0 P_s - \mathcal{M}}{1+\Upsilon_0}\right\}$ and the problem is feasible then the achievable capacity is

$$C_S = C\left(\frac{|h_{s,r}|^2 P_s + |h_{p,r}|^2 P_p}{\sigma^2}\right) - R_p. \tag{35}$$

Note that the medium and strong interference regimes lead to different power allocation policies, since in the former the primary message is not directly decodable, whereas in the latter it is. However, it can be seen that the optimum power allocation in both regimes leads to the same capacity for the secondary system. In order to prove it, we will assume that $P_s \ge -\mathcal{M}^s \ge -\mathcal{M}$ and $P_s\Upsilon_0 \ge \mathcal{M}^s$, which is the condition for joint feasibility of (29) and (22). Therefore, by substituting (16) in (28), the capacity in strong interference can be written as

$$\begin{aligned}
C_{S,Strong} &= C\left(\frac{|h_{s,r}|^2}{\sigma^2}\frac{P_s + \mathcal{M}^s}{1+\Upsilon_0}\right) && (36)\\
&= \log_2\left(1 + \frac{|h_{s,r}|^2 P_s - \Upsilon_0\sigma^2 + |h_{p,r}|^2 P_p}{\sigma^2(1+\Upsilon_0)}\right)\\
&= \log_2\left(\frac{|h_{s,r}|^2 P_s + \sigma^2 + |h_{p,r}|^2 P_p}{\sigma^2(1+\Upsilon_0)}\right)
\end{aligned}$$

Now, since $R_p = \log_2(1+\Upsilon_0)$, we can write (35) as

$$\begin{aligned}
C_{S,Medium} &= \log_2\left(1 + \frac{|h_{s,r}|^2 P_s + |h_{p,r}|^2 P_p}{\sigma^2}\right) && (37)\\
&\quad - \log_2(1+\Upsilon_0)\\
&= \log_2\left(\frac{\sigma^2 + |h_{s,r}|^2 P_s + |h_{p,r}|^2 P_p}{\sigma^2(1+\Upsilon_0)}\right)\\
&= C_{S,Strong}.
\end{aligned}$$

Note that the implications of this equality are quite significant, as a secondary transmission link working with this decoding strategy can be transformed into an equivalent system (i.e., with the same capacity) working with a strong interference and, therefore, the coverage area of the primary system could be extended.

D. Numerical results

The power allocation results were evaluated in different scenarios to obtain the resulting secondary user capacity. In order to get insight on the implications of the secondary system position, the results are shown as a function of this location. We assume that the PT is located at $\mathbf{x}_P = (0, 0)$, the ST at $\mathbf{x}_S = (r, 0)$ and the SR at $\mathbf{x}_R = (r + d, 0)$. We also assume a free space propagation loss model, so the attenuation between two different points separated x units of distance can be written as x^2/k, with $k = \left(\frac{\lambda}{4\pi}\right)^2$. In the numerical results, we have set $d = 2$ and $k = 2$.

In order to properly describe the interference constraint, we define

$$\Delta P_i \doteq |h_{p,i}|^2 P_p - \Upsilon_0\sigma^2, \tag{38}$$

so $\mathcal{M}_i = \frac{1}{|h_{s,i}|^2}\Delta P_i$, and we define $\Delta P \doteq \min\{\Delta P_i\}$.

If we restrict our secondary system to keep the same original coverage area, the worst case receiver would be located at the edge of the coverage zone, so $|h_{p,i}|^2 P_p/\sigma^2 = \Upsilon_0$ and, therefore, $\Delta P = 0$, i.e., the worst case receiver has a zero power margin. If this is the case, the existence of a white space is impossible, strictly speaking, as even an infinitesimal extra interference coming from the ST would cause this extreme *worst case* receiver to fail.

The capacity results are shown in Figure 3 for different positions of the ST. The coverage limit ≈ 4.3 is the position of the ST for which the SR crosses the coverage edge ≈ 6.3. It can be seen that the optimum power allocation forces to be in the medium/high interference regimes, while the *treat as noise* scenario tends to the limit capacity of $\log_2\left(1+\Upsilon_0^{-1}\right)$, as depicted in Section IV. Moreover, the system is quite far away from its performance in a true *white space*, with a capacity of $C_{noPU} = \log_2\left(1+\frac{|h_{s,r}|^2 P_s}{\sigma^2}\right)$.

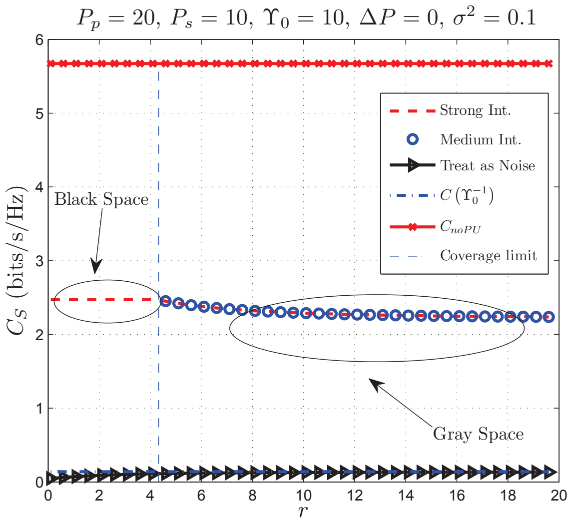

Fig. 3. Capacity for the different decoding strategies with no coverage margin ($\Delta P = 0$).

Now, we propose to include a *power margin* that allows the existence of a smooth transition from the *black space* to a true *white space*. This could be the case, for example, of knowing the absence of PR near the coverage edge. In this scenario, if the ST is inside the coverage zone we set $\mathcal{M} = 0$, as a PR could be located near the ST and, therefore, have a large $|h_{s,i}|^2$ value. When the ST is outside the coverage zone, the worst case receiver leads to a value of $\mathcal{M} = \frac{1}{|h_{s,i}|^2}\Delta P$, with $|h_{s,i}|^2$ calculated following the free space loss formula. Note that as the ST moves away from the coverage area $|h_{s,i}|^2 \to 0$, so the interference constraint (5) tends to $\gamma \geq -\infty$ for a positive ΔP, which is indeed a true *white space*, as there is no limit for the power allocated to the secondary message. In Figure 4 the achievable capacity results are shown for the three different decoding strategies. It can be seen that the behavior of the functions in the gray space is quite curious, as two clearly different zones exist: in the first one, near the coverage limit, the optimum power allocation forces to *expand* the coverage area and work in the medium-strong decoding region, thus *blackening* this gray space. However, as gray tends to white, the optimum decoding strategy consists on treating the primary interference as noise. Finally, when the ST is located very far from the coverage area, the secondary system capacity tends to the white space capacity.

In Figures 5 and 6 there are plots of the fraction of power allocated to the primary and secondary messages at the ST, respectively. It is remarkable the big difference between the medium and strong interference optimum power allocation, as they lead to quite different values, but to the same achievable rate for the ST. Another interesting result is the evolution of the power allocated to the primary and secondary message when using the *treat as noise* decoding: inside the coverage area this power is constant, and it can be seen that that no power remains unused. When the ST leaves the coverage area no power is allocated to the primary message ($\gamma = 0$), and ρ starts to grow rapidly. In this zone (gray space), the capacity increases substantially with the distance as a result of two different effects: as both the ST and SR are moving away from the coverage area, the interference coming from the PT is clearly reduced, and also the value of $\mathcal{M}$ increases, so more power can be allocated to the secondary message without breaking the interference constraint. When the interference constraint allows to allocate all the power to the secondary message (white space), it can be seen that the capacity continues to increase, but in this case much slower, as we have only one of the two effects in the gray space: the decrease of the interference coming from the PT.

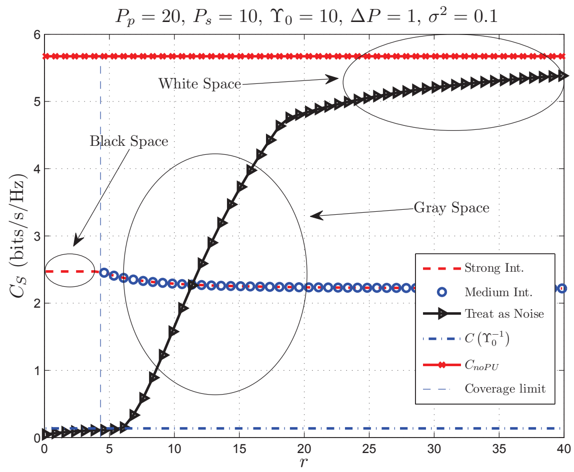

Fig. 4. Capacity for the different power decoding strategies with a coverage margin ($\Delta P = 1$).

V. Optimum Power Allocation for Multiple Receivers

In this section we will study the effect of having multiple SRs in the capacity of the system. Depending on the nature of

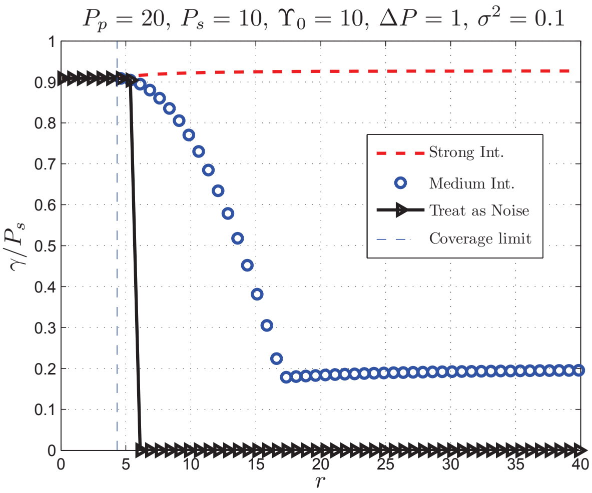

Fig. 5. Fraction of power allocated to the primary message as a function of the position of the ST.

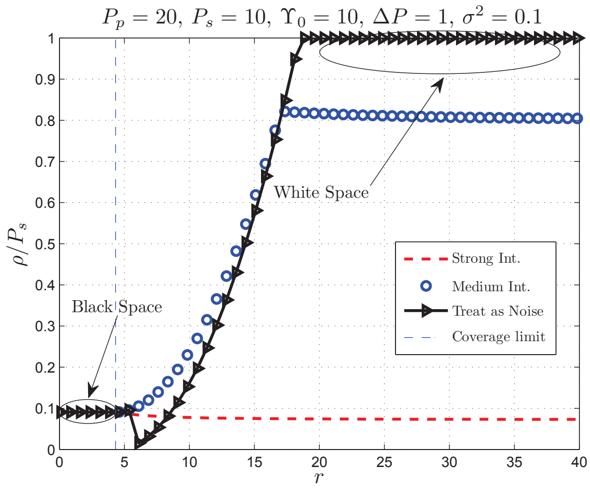

Fig. 6. Fraction of power allocated to the secondary message as a function of the position of the ST.

the secondary system, we can clearly distinguish two different scenarios:

- **Unicast** If this is the case, each SR is interested on a different message. An example of this kind of system could be a cellular network.
- **Multicast/Broadcast** Multiple SRs are interested in the same message. The main issue with this kind of systems is to adapt the transmission rate of the ST in order to deal with SRs in very different situations. An example of this type of system could be the insertion of a local TV transmitter *overlaid* on a global one, or the use of cellular-based multicast service like 3GPP Multimedia Broadcast Multicast Service (MBMS) in LTE [21].

Note that the unicast case can be solved just by considering one SR at a time (i.e., applying Time Division Multiplexing - TDM), and afterwards adopting the optimum power allocation and decoding strategy for that SR. The case of having multiple multicast groups can be reduced to the broadcast case just by applying TDM again. However, the broadcast case is quite involved.

Let us assume a broadcast scenario with N_s SRs. We will extend the notation from the previous section to denote as h_{s,r_k} and h_{p,r_k} the channels from the ST and PT to the k-th SR, for $k = 1 \ldots,\ N_s$. In this broadcast analysis, we will only take into account the *treat as noise* and *strong interference* decoding strategies, as the *medium interference* one achieves the same capacity as the latter, and it requires an optimum power splitting (into ρ_1 and ρ_2) which heavily depends on the channels seen by a given receiver, thus being unpractical for multiple receivers. We will denote as

$$\mathcal{D}_k \in \{\mathcal{N}, \mathcal{S}\} \tag{39}$$

the decoding strategy of the k-th SR, where $\mathcal{D}_k = \mathcal{N}$ denotes that the k-th SR is decoding the secondary message treating the primary one as noise, and $\mathcal{D}_k = \mathcal{S}$ denotes a strong interference operation. We denote the capacity under both operation modes as

$$C_k^{\mathcal{N}}(\rho, \gamma) = C\left(\frac{|h_{s,r_k}|^2 \rho}{\sigma^2 + |h_{p,r_k}|^2 P_p + |h_{s,r_k}|^2 \gamma}\right) \tag{40}$$

for the $\mathcal{N}$ operation and

$$C_k^{\mathcal{S}}(\rho, \gamma) = C\left(\frac{|h_{s,r_k}|^2 \rho}{\sigma^2}\right) \tag{41}$$

for the $\mathcal{S}$ operation. The latter requires the constraint

$$f_k^{\mathcal{S}}(\rho, \gamma) \doteq \Upsilon_0 \rho - \gamma - \mathcal{M}_{r_k}^s \leq 0 \tag{42}$$

to be met, where we have defined

$$\mathcal{M}_{r_k}^s \doteq \frac{1}{|h_{s,r_k}|^2}\left(|h_{p,r_k}|^2 P_p - \Upsilon_0 \sigma^2\right). \tag{43}$$

What we want to obtain is, given N_s receivers, the maximum rate R_s such that no PR is compromised and the N_s SRs are able to decode the secondary message. In the single receiver approach we had to solve a different optimization problem for each of the three decoding strategies. In this case, after dropping the *medium interference* strategy, we have that every receiver can operate with 2 different decoding procedures. Thus, our design variables are, in this case, the power allocation weights (ρ, γ) as well as the vector of decoding strategies $\boldsymbol{\mathcal{D}} = (\mathcal{D}_1, \ldots, \mathcal{D}_{N_s}) \in \{\mathcal{N}, \mathcal{S}\}^{N_s}$:

$$\begin{aligned} \text{max.} \quad & \min_{k=1,\ldots,N_s}\left\{C_k^{\mathcal{D}_k}(\rho, \gamma)\right\} \\ \text{s.t.} \quad & \gamma + \rho \leq P_s \\ & \gamma \geq \rho\Upsilon_0 - \mathcal{M} \\ & f_k^{\mathcal{D}_k}(\rho, \gamma) \leq 0,\ k = 1, \ldots, N_s \\ & \gamma \geq 0 \\ & \rho \geq 0 \end{aligned} \tag{44}$$

where we have introduced a *dummy constraint* $f_k^{\mathcal{N}}(\rho, \gamma) \doteq 0$ for clarity in the notation. Note that for a given $\boldsymbol{\mathcal{D}}$, the optimization problem (44) is a generalized linear fractional program [19], which is quasiconvex and, therefore, can be efficiently solved.

Thus, the main problem is to choose the optimum value of $\boldsymbol{\mathcal{D}}$ among the 2^{N_s} possible vectors, which could be computationally infeasible for a large number of receivers. However, note that:

- For a given power allocation (ρ, γ), it is clear that if a receiver is able to decode the primary message then treating it as noise is suboptimal.
- If the j-th user chooses to decode the primary message, i.e., $\mathcal{D}_j = \mathcal{S}$, then the constraint $f_j^{\mathcal{S}} \leq 0$ has to be met, and can be rewritten as $\gamma \geq \Upsilon_0 \rho - \mathcal{M}_{r_j}^s$, so if $f_j^{\mathcal{S}} \leq 0$ then $f_k^{\mathcal{S}} \leq 0$ for k such that $\mathcal{M}_{r_k}^s \geq \mathcal{M}_{r_j}^s$. This means that if a given user is able to decode the primary message, then all the receivers with a larger *power margin* will also be able to decode it.

These two facts have a very strong implication, as it suffices to try at most[1] $N_s + 1$ different decoding strategies, so the complexity of the problem is substantially reduced. If we assume that the receivers are ordered in decreasing order of $\mathcal{M}_{r_k}^s$, i.e., $\mathcal{M}_{r_k}^s \geq \mathcal{M}_{r_j}^s \, \forall j \geq k$, then only the following strategy vectors can be optimum

$$\begin{aligned} \boldsymbol{\mathcal{D}}^{(0} &= (\mathcal{N}, \mathcal{N}, ..., \mathcal{N}) \\ \boldsymbol{\mathcal{D}}^{(1} &= (\mathcal{S}, \mathcal{N}, ..., \mathcal{N}) \\ \boldsymbol{\mathcal{D}}^{(2} &= (\mathcal{S}, \mathcal{S}, ..., \mathcal{N}) \\ &\ldots \\ \boldsymbol{\mathcal{D}}^{(N_s} &= (\mathcal{S}, \mathcal{S}, ..., \mathcal{S}) \end{aligned} \quad (45)$$

so it suffices to solve $N_s + 1$ quasiconvex optimization problems to obtain the optimum power allocation and strategy vector.

A. Numerical results

We have numerically solved the optimization problem in (44) in different scenarios. In all the simulations we have set $P_p = 20$, $\sigma^2 = 0.1$, $\Upsilon_0 = 10$, and a free space propagation model was assumed, with $k = 2$. The PR positions were generated following an uniform distribution inside the coverage area, leaving a small portion as a protection area that allows the existence of a white space, similarly to the single receiver study. The SR positions were also generated following an uniform distribution in circles of different dimensions, depending on the scenario.

Figure 7 shows a secondary system positioned near the primary coverage edge, with the SR concentrated near the ST. In this case, the optimum decoding strategy is to decode the primary message for all the SR, so the primary coverage area is clearly extended. Moreover, due to the proximity of the SRs to the ST, a relatively high rate is achieved.

[1]This number can be further reduced if the interference constraint forces some of the SRs to be able to decode the primary message.

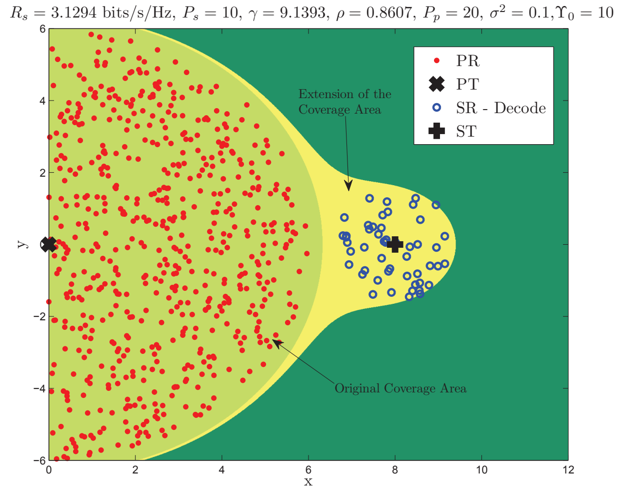

Fig. 7. Initial and final coverage area for a scenario with a ST with SRs concentrated near the transmitter. In this case, the optimum decoding strategy is to decode the primary transmitter in all the receivers, so the coverage area is expanded.

The picture changes dramatically if the SRs can be situated far away from the ST. In Figure 8 the optimum decoding strategy is $\mathcal{N}$ for the SRs situated far away from the primary coverage area, and $\mathcal{S}$ for those clearly inside it. In this case, there are some SRs that are situated far from the ST and near the coverage edge, so the signal coming from the ST is very attenuated and they receive a large amount of interference (which are not able to decode) from the PT. The achievable rate in this case is much smaller and, moreover, the primary coverage area is slightly diminished.

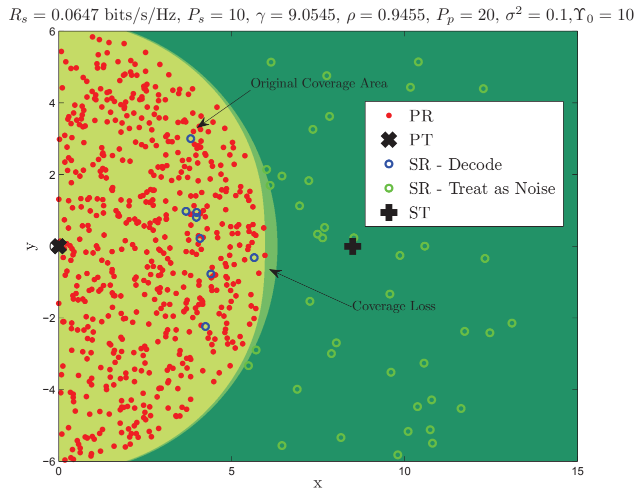

Fig. 8. Initial and final coverage area for a scenario with a ST with some SRs situated far away from the transmitter. In this case, the optimum decoding strategy is to decode the primary transmitter for the receivers inside the final coverage zone, and to treat it as noise for those outside it. The final coverage area is slightly smaller than the initial one.

If we move the secondary system far from the primary coverage edge, we end up in a white space scenario, which is shown in Figure 9. In this case all the power is allocated to the secondary message and the optimum decoding strategy is to treat as noise the very weak interference coming from the PT. The primary coverage area is also reduced in this case.

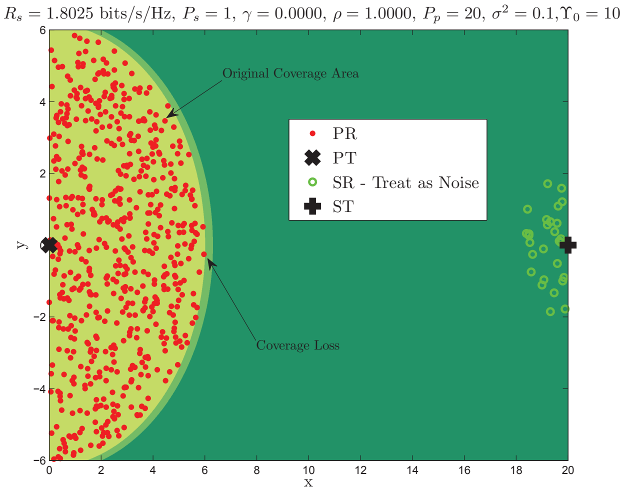

Fig. 9. Initial and final coverage area for a ST operating in a white space. In this case, all the power can be allocated to the secondary message without breaking the interference constraint. The transmit power of the secondary transmitter has been disminished to $P_s = 1$ to force the existence of the *white space* near the primary coverage zone.

A case of special interest can be the one where the insertion of the ST is clearly beneficial for the primary system: in many real-world cases a broadcaster (e.g. TV operators) needs to extend its coverage area in order to give its service to some new receivers. In this case, the primary operator could incur in a relatively high expense to cover maybe a few more receivers. Following the overlay paradigm, the primary operator could allow the existence of a secondary system with the condition of extending the primary coverage zone to a set of new PR. This scenario is shown in Figure 10, which is identical to that in Figure 7 except for the presence of PR outside the original coverage area. The resulting coverage area can be seen to be substantially larger than the one in Figure 7 so the new PRs lie inside it. This larger coverage area forces the ST to allocate more power to the primary message, so a smaller rate is achieved for the secondary system.

VI. Concluding remarks

In this paper we have studied the problem of overlay cognitive access to the spectrum licensed to a primary broadcast user. Surprisingly, in some cases the insertion of the secondary network is beneficial for the primary system, as it expands its coverage area. This kind of overlay operation could result of special interest in those cases where the primary service provider outsources the extension of his coverage area to a secondary operator.

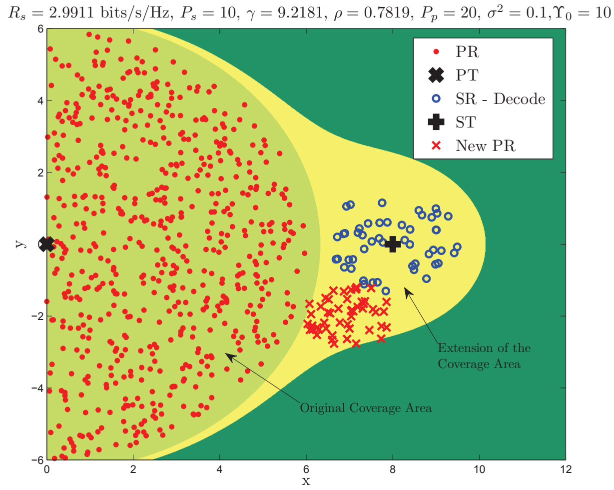

Fig. 10. Initial and final coverage zone for a ST forced to enlarge the primary coverage area by the insertion of *New PRs*.

Future lines of this work include the extension to fading channels, which are expected to lead to higher benefits due to the presence of diversity.

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