Dynamic Spectrum Leasing (DSL): A New Paradigm for Spectrum Sharing in Cognitive Radio Networks

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***Abstract*—Recently [1] proposed a dynamic spectrum leasing (DSL) paradigm for dynamic spectrum access in cognitive radio networks. In this paper, we formalize this concept by developing a more general game theoretic framework for the dynamic spectrum leasing and by carefully identifying requirements for the coexistence of primary and secondary systems via spectrum leasing. In contrast to hierarchical spectrum access, spectrum owners in proposed dynamic spectrum leasing networks, denoted as primary users, dynamically adjust the amount of secondary interference they are willing to tolerate in response to the demand from secondary transmitters. The secondary transmitters in turn attempt to achieve maximum possible throughput, or other suitably defined reward, opportunistically while not violating the interference limit set by the primary users. The new game-theoretic model, however, allows the secondary users to encourage the spectrum owners to push the interference cap upward based on demand. We have proposed a general structure for utility functions of primary users and the secondary users that allows the primary users to control the price and demand for spectrum access based on their required Quality-of-Service (QoS). We establish that with these utility functions the DSL game has a unique Nash equilibrium to which the best-response adaptation finally converges. Moreover, it is shown that the proposed coexistence and best-response adaptations can be achieved without having any significant interaction between the two systems. In fact, it is shown that the only requirement is that the primary system periodically broadcasts two parameter values. We use several examples to illustrate the system behavior at the equilibrium, and use the performance at the equilibrium to identify suitable system design parameters.**

***Index Terms*—Cognitive radios, DSL, dynamic spectrum access, dynamic spectrum leasing, dynamic spectrum sharing, game theory, power control.**

I. Introduction

In recent years, it has been observed that the scarcity of radio spectrum is mainly due to the inefficiency of traditional static spectrum allocation policies [2], [3]. This has prompted proposals for various *dynamic* spectrum access (DSA) approaches, that can be grouped primarily into three main classes: a) open-sharing, b) hierarchical-access, and c) dynamic exclusive use [2], [4]. The open-sharing approach advocates a model similar to the highly successful industrial, science and medicine (ISM) bands. The hierarchical spectrum access, on the other hand, attempts to improve the spectrum utilization in current allocations. The hierarchical access in which secondary users are allowed to opportunistically access the spectrum on the basis of no-interference to the primary (licensed) users, is arguably the method that has received the most attention in recent literature. Various spectrum underlay and overlay schemes have been proposed and investigated in recent years to achieve such hierarchical DSA in cognitive radio networks (see [5]–[10] and references therein). Cognitive radios have been chosen as an enabling platform in realizing such dynamic spectrum sharing due to their built-in cognition that can be used to observe, learn from and adjust to the RF interference environment [11]–[13].

In DSA, it is assumed that there is a primary system that owns the spectrum rights. The existing literature in underlay and overlay based secondary networks, however, impose the burden of interference management mainly on the secondary system. In particular, it is assumed that there is a maximum interference level that the primary system is willing to tolerate, and the secondary powers/activity are to be adjusted within this constraint. In [1], on the other hand, we proposed a new concept of *dynamic spectrum leasing* (DSL) as an approach for better spectrum utilization. Spectrum leasing is one of the solutions that has been suggested under the third option of dynamic exclusive-use model in which the spectrum licensees are granted the rights to sell or trade their spectrum to third parties [2], [4]. As opposed to passive spectrum sharing by the primary users as in hierarchical DSA, leasing means that the primary users have an incentive (e.g. monetary rewards as leasing payments) to allow secondary users to access their licensed spectrum. However, until [1], spectrum leasing has only been identified as a static, or off-line, sharing technique, with the possible exception of [14]. On the other hand, in [1] we proposed to achieve dynamic spectrum leasing by allowing the primary users to dynamically adjust the extent to which they are willing to lease their spectrum. Thus, the proposed DSL approach is well-suited for spectrum underlay systems in which both primary and secondary systems are expected to coexist simultaneously. However, unlike in hierarchical-access

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systems considered in existing literature, the primary users in a DSL network actively adapt the maximum secondary interference they are willing to tolerate, named the interference cap, according to the observed RF environment and their required Quality-of-Service (QoS). At this point it is also worth pointing out that the spectrum leasing considered in [14] differs from our DSL approach in several ways. Most importantly, it relies on cooperative communication involving primary and secondary systems whereas the proposed DSL scheme does not.

In this paper, we formalize our proposed DSL framework for cognitive radios [1]. Specifically, we first present a signal and system model for coexistence of primary and secondary systems under the dynamic spectrum leasing. Next, we develop a more general game theoretic formulation to model the interactions among primary and secondary systems that better captures the realities of such a dynamic spectrum leasing network. We propose a general structure for a suitable class of utility functions for both primary and secondary systems that reflect the demand for spectrum access by the secondary users, their payoffs in terms of a suitable performance measure and the primary user QoS requirements. We establish the conditions under which the proposed game-theoretic formulation has a unique Nash equilibrium to which both primary and secondary best-response adaptations would converge.

Naturally, any DSL system requires each system to know a certain amount of information about the other system. While in hierarchical-access systems it is usually assumed that only the secondary system needs to be aware of the primary operation, in a DSL network both systems will be aware of each other. However, it may arguably be desirable to minimize the awareness the primary system needs to have on the secondary operation. In this paper, we show that indeed successful dynamic spectrum leasing can be achieved still relegating most of the interference management burden to the secondary system and primary system having to periodically broadcast only two parameter values: Its tolerable interference cap and the total interference it is currently experiencing from the secondary transmissions. These are quantities that are readily available at the primary users (or can be easily estimated). Thus, we believe that the proposed DSL framework is indeed a viable solution for active spectrum sharing in cognitive radio networks.

The rest of this paper is organized as follows: In Section II we introduce a signal and system model for the proposed dynamic spectrum leasing network. Next, in Section III we develop a non-cooperative game for dynamic spectrum leasing. In this section we propose a general class of utility functions suitable for DSL and establish conditions under which the spectrum leasing game will converge to a Nash equilibrium. In Section IV we use several example DSL systems to illustrate the performance characteristics of the proposed game-theoretic dynamic spectrum leasing scheme. Specifically, we investigate the primary and secondary system coexistence within each other's required performance QoS constraints and based on that provide design guidelines for a DSL network. We also investigate the robustness of the best-response adaptations to time-varying channel fading conditions and the effect of this on the system equilibrium. Finally, Section V concludes the paper by summarizing our results and discussing possible future work.

II. System Model for Dynamic Spectrum Leasing

We assume that there is one primary wireless communication system that owns the license rights to the spectrum band of interest. The users in this primary system, however, may not be using its spectrum completely all the time, or may be able to tolerate a certain amount of additional co-channel interference without compromising required QoS constranits, leading to an inefficient utilization of radio spectrum. For simplicity of exposition, we focus on a particular channel in the primary system that is allocated to a single primary user (for example, as in FDMA). Thus there is only one primary transmitter of interest, and there are K secondary transmitters who are interested in accessing this spectrum band of interest to the maximum possible extent. The primary user is denoted as user 0, and the secondary users are labeled as users 1 through K. There are one primary receiver and one secondary receiver of interest[1]. The channel gain between the k-th transmitter (either primary or secondary) and the common secondary receiver is denoted by h_{sk}, and that between the k-th transmitter and the primary receiver is denoted by h_{pk}, for $k = 0, 1, \ldots, K$. Throughout the analysis in this paper we assume fading to be quasi-static, so that the coefficients stay fixed for a certain duration of time after which they change to new set of values. It should be mentioned that quasi-static fading model is frequently used in modeling many wireless communications environments [16]. Our model can also be complemented with a channel estimation and tracking algorithm to cope with slowly time-varying situations and as we will show later, the performance of the proposed DSL scheme is fairly robust against such time-varying fading.

The primary user is assumed to adapt its interference cap (IC), denoted by Q_0, which is the maximum total interference the primary user is willing to tolerate from secondary transmissions at any given time. By adjusting this interference cap Q_0, the primary user can control the total transmit power the secondary users impose on its licensed channel. The motivation for the primary user can be, for instance, the monetary reward obtained by allowing secondary users to access its licensed spectrum. In essence, then, the interference cap determines how much secondary user activity the primary user is ready to allow, and thus its reward should be an increasing function of the interference cap. However, we impose the realistic constraint that the primary user should always maintain a target signal-to-interference-plus-noise ratio (SINR) to ensure its required transmission QoS. Moreover, an unnecessarily large interference cap by the primary user could hinder both the secondary system and other primary transmitters' (though, for simplicity, not included in the current model) performance due to resulting high interference.

The goal of secondary transmitters is to capitalize on the allowed spectrum activity by the primary system by fully

[1] Generalization to more than one secondary receiver is straightforward, and is reported in [15].

utilizing the interference margin. Each secondary user may be assumed to act in its own interest to maximize its own utility. However, their transmission powers must be carefully regulated in order to ensure low interference to the primary user (within the IC) as well as to other secondary users. We use p_k to represent the transmission power of the k-th user, for $k = 0, 1, \ldots, K$.

Throughout this work, we will assume that both primary and secondary receivers are equipped with conventional matched-filter receivers[2]. The signal received at the primary and secondary receivers can be respectively written as

$$r_p(t) = A_0 b_0 s_0(t) + \sum_{k=1}^{K} \Theta_k A_k b_k s_k(t) + \sigma_p n_p(t) \quad (1)$$

$$r_s(t) = \sum_{k=1}^{K} B_k b_k s_k(t) + B_0 b_0 s_0(t) + \sigma_s n_s(t) \quad (2)$$

where $n_\cdot(t)$ are additive white Gaussian noise with unit spectral height, $\sigma_\cdot^2$ are the variance of the receiver noise, A_k and B_k, for $k = 0, 1, \ldots, K$, represent the received signal amplitude at the primary and secondary receiver respectively, and are defined as $A_k \doteq h_{pk}\sqrt{p_k}$ and $B_k \doteq h_{sk}\sqrt{p_k}$. Θ_k is a Bernoulli random variable such that $Prob\,(\Theta_k = 1) = q_k$ and $Prob\,(\Theta_k = 0) = 1 - q_k$, representing the randomness in secondary-user collisions with the primary transmission. In an overlay dynamic spectrum access system, the secondary users are prohibited from transmitting whenever primary users are using the spectrum. Thus, in an overlay system the secondary interference will be present at the primary receiver only when a secondary transmitter makes a mistake in its white-space detection procedure. Hence, q_k can be interpreted as the false-alarm probability of the white space detector at the k-th secondary transmitter in an overlay system. On the other hand, in an underlay dynamic spectrum sharing system, the secondary users are allowed to transmit always without regard to the timings of the primary transmissions albeit at a low power level. In this case, the secondary interference is always present at the primary receiver. We can use the above model to capture this situation simply by assuming that $q_k = 1$ (or $\Theta_k = 1$ with probability 1). Hence, the model (1)-(2) is general enough to be applicable for both underlay and overlay cognitive operations, although we envision for DSL to be more meaningful in spectrum underlay systems.

Assuming M discrete-time projections $r_m^{(p)} = < r_p(t), \psi_m^{(p)}(t) >$, for $m = 1, 2, \ldots, M$, of the continuous-time received signal $r_p(t)$ on to a set of M orthonormal directions specified by $\psi_1^{(p)}(t), \ldots, \psi_M^{(p)}(t)$,and letting $\mathbf{r}^{(p)} = (r_1^{(p)}, \ldots, r_M^{(p)})^T$, we obtain the following discrete-time representation of the received signal at the primary receiver:

$$\mathbf{r}^{(p)} = A_0 b_0 \mathbf{s}_0^{(p)} + \sum_{k=1}^{K} \Theta_k A_k b_k \mathbf{s}_k^{(p)} + \sigma_p \mathbf{n}^{(p)}$$

[2]The signal model below is general enough to allow for the extensions to more sophisticated multiuser receivers, and will be considered in a follow-up paper.

where $\mathbf{s}_k^{(p)} = (s_{k1}^{(p)}, \ldots, s_{kM}^{(p)})$, for $k = 0, 1, \ldots, K$, is the M-vector representation of $s_k(t)$ in the M-dimensional basis employed by the primary system, where $s_{km}^{(p)} = < s_k(t), \psi_m^{(p)}(t) >$, and $\mathbf{n}^{(p)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_M)$. Analogously a discrete-time representation of $r_s(t)$ with respect to an N-dimensional orthonormal basis $\psi_1^{(s)}(t), \ldots, \psi_N^{(s)}(t)$ used by the secondary system can be written as

$$\mathbf{r}^{(s)} = \sum_{k=1}^{K} B_k b_k \mathbf{s}_k^{(s)} + B_0 b_0 \mathbf{s}_0^{(s)} + \sigma_s \mathbf{n}^{(s)}$$

where $\mathbf{r}^{(s)} = (r_1^{(s)}, \ldots, r_N^{(s)})^T$, $r_n^{(s)} = < r_s(t), \psi_n^{(s)}(t) >$, for $n = 1, 2, \ldots, N$, is the projection of the received signal at the secondary receiver on to the the n-th orthonormal basis function $\psi_n^{(s)}(t)$, $\mathbf{s}_k^{(s)} = \left(s_{k1}^{(s)}, \ldots, s_{kN}^{(s)}\right)$, for $k = 0, 1, \ldots, K$, is the N-vector representation of $s_k(t)$ with respect to the N-dimensional basis employed by the secondary system with $s_{kn}^{(s)} =< s_k(t), \psi_n^{(s)}(t) >$, and $\mathbf{n}^{(s)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$.

With the conventional matched-filter (MF) detector at the primary and secondary receivers, we have that decisions are taken in base to the matched-filtered signals $y_0^{(p)} = (\mathbf{s}_0^{(p)})^T \mathbf{r}^{(p)}$ and $y_k^{(s)} = (\mathbf{s}_k^{(s)})^T \mathbf{r}^{(s)}$ respectively. Note that

$$y_0^{(p)} = A_0 b_0 + \sum_{k=1}^{K} \Theta_k \rho_{0k}^{(p)} A_k b_k + \sigma_p \eta^{(p)}$$

$$y_k^{(s)} = B_k b_k + \sum_{j=1, j\neq k}^{K} \rho_{kj}^{(s)} B_j b_j + \rho_{k0}^{(s)} B_0 b_0 + \sigma_s \eta_k^{(s)}$$

with $\rho_{0k}^{(p)} = (\mathbf{s}_0^{(p)})^T \mathbf{s}_k^{(p)}$, $\rho_{kj}^{(s)} = (\mathbf{s}_k^{(s)})^T \mathbf{s}_j^{(s)}$, for $j = 0, 1, \ldots, K$. Note that the noise terms $\eta^{(p)}$ and $\eta_k^{(s)}$ follow a $\mathcal{N}(0, 1)$.

It is straightforward to observe that the total secondary interference I_0 from all secondary transmissions to the primary-user is given by

$$I_0 = \sum_{k=1}^{K} \tilde{A}_k^2 p_k \quad (3)$$

where $\tilde{A}_k = \sqrt{q_k}\rho_{0k}^{(p)} h_{p_k}$ is the effective channel coefficient of the k-th secondary user as seen by the primary receiver. Similarly, the total interference from all secondary users to the k-th user signal, excluding the primary user, will be denoted by

$$i_k = \sum_{j=1, j\neq k}^{K} \left(\rho_{kj}^{(s)}\right)^2 h_{sj}^2 p_j.$$

III. Game Model for Dynamic Spectrum Leasing

A. Game model

In the proposed DSL-based cognitive radio network, the primary and secondary users interact with each other by adjusting their interference cap and transmit power levels, respectively, in order to maximize each others own utility. Hence, game theory provides a natural framework to model and analyze this system. In fact, we may formulate the above system as in the following noncooperative game $(\mathcal{K}, \mathcal{A}_k, u_k(.))$:

1) Players: $\mathcal{K} = \{0, 1, 2, \ldots, K\}$, where we assume that the 0-th user is the primary user and $k = 1, 2, \ldots, K$ represents the k-th secondary user.
2) Action space: $\mathcal{P} = \mathcal{A}_0 \times \mathcal{A}_1 \times \cdots \times \mathcal{A}_K$, where $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, \ldots, K$, represents the k-th secondary user's action set. Note that $\bar{Q}_0$ and $\bar{P}_k$ represent, respectively, the maximum possible interference cap of the primary user and the maximum transmission power of the k-th secondary user (as determined by the system and regulatory considerations). The lower limit of these actions sets being zero indicate that at times, a secondary user may turn-off its transmission or the primary user may not be willing to tolerate any interference from the secondary system at all. We denote the action vector of all users by $\mathbf{a} = (Q_0, p_1, \ldots, p_K)^T$, where $Q_0 \in \mathcal{Q}$ and $p_k \in \mathcal{P}_k$. It is customary to denote the action vector excluding the k-th user, for $k = 0, 1, 2, \ldots, K$, by $\mathbf{a}_{-k}$.
3) Utility function: We denote by $u_0\left(Q_0, \mathbf{a}_{-0}\right)$ the primary user's utility function, and by $u_k\left(p_k, \mathbf{a}_{-k}\right)$, for $k = 1, 2, \ldots, K$, the k-th secondary user's utility function.

At any given time, the primary user's target SINR is defined in terms of its assumed worst-case secondary interference:

$$\bar{\gamma}_0 = \frac{h_{p0}^2 p_0}{Q_0 + \sigma_p^2}, \tag{4}$$

Note that, since Q_0 is the maximum possible interference from secondary users the primary user is willing to tolerate, $\bar{\gamma}_0$ represents the least acceptable transmission quality of the primary user. On the other hand, the primary user's actual instantaneous SINR is given by

$$\begin{aligned}\gamma_0 &= \frac{h_{p0}^2 p_0}{\sum_{k=1}^{K} \tilde{A}_k^2 p_k + \sigma_p^2} \\ &= \bar{\gamma}_0 \left(1 + \frac{Q_0 - I_0}{I_0 + \sigma_p^2}\right).\end{aligned} \tag{5}$$

One of the main features of dynamic spectrum leasing approach is to take into account the coupling of primary system with the secondary-user system in terms of mutual interference. However, the awareness of the primary system to the secondary network must be kept low enough to avoid an excesive overhead and complexity of the network.

The primary user is expected to obtain a reward from the secondary network thus motivating the leasing of the owned spectrum. Moreover, the reward function for the primary system will be in general increasing with the demand seen from the secondary network, as it ocurs in the trade market. On the other hand, the reward is expected to grow with the allowed interference, since the secondary system has more resources in this case to exploit. With these points in mind, we introduce the following utility function for the primary user:

$$\begin{aligned}u_0\left(Q_0, \mathbf{a}_{-0}\right) &= \left(\bar{Q}_0 - \left(Q_0 - I_0(\mathbf{a}_{-0})\right)\right) Q_0 \\ &= u_0\left(Q_0, I_0\right).\end{aligned} \tag{6}$$

Note that (6) essentially assumes that the utility of the primary user is proportional to both demand and its interference cap Q_0. The demand is taken to be increasing when the extra interference margin $Q_0 - I_0$ decreases. This discourages the primary user to swamp all other transmissions (both primary and secondary), by setting too large an interference cap that will lead to higher transmission power according to (4). Additionally, the described reward function depends on just the parameter I_0 of the secondary system, which can be easily estimated as we will see later in this section avoiding the need of detailed channel state information from the secondary network. We believe that this model for primary utility is more sensible in a dynamic spectrum leasing cognitive radio network compared to, for example, what was used in [1]. It is also worth noting that this u_0 is continuous in $\mathbf{a}$ and concave in Q_0.

At the secondary receiver, the received SINR of the k-th secondary user, for $k = 1, 2, \ldots, K$, is given by

$$\begin{aligned}\gamma_k &= \frac{|h_{sk}|^2 p_k}{i_k + (\rho_{k0}^{(s)})^2 |h_{s0}|^2 p_0 + \sigma_s^2} \\ &= \frac{|h_{sk}|^2 p_k}{i_k + \tilde{\sigma}_s^2} = \frac{p_k}{N_k}\end{aligned}$$

where, as defined earlier i_k is the total secondary interference, $\tilde{\sigma}_s^2 = (\rho_{k0}^{(s)})^2 |h_{s0}|^2 p_0 + \sigma_s^2$ is the effective noise seen by the k-th user and $N_k = (i_k + \tilde{\sigma}_s^2)/|h_{sk}|^2$.

The (selfish) objective of each secondary user is to maximize a given utility function (for example, throughput) that depends on its own SINR without violating the primary user interference cap. Observe from (5) that as long as the secondary user interference I_0 is below the interference cap Q_0 set by the primary user, the required QoS of the primary user will be guaranteed. Therefore any utility function in a reasonable communication system will be a monotonically increasing function of the received SINR γ_k and it should be a fast decaying function of $I_0 - Q_0$ when this difference is positive. To ensure this the secondary utility function will be formed by two terms: (i) a shelfish reward function depending on the recived SINR and (ii) a penalization term depending on $I_0 - Q_0$. Motivated by these arguments we propose the following form for the secondary user utility function:

$$\begin{aligned}u_k\left(p_k, \mathbf{a}_{-k}\right) &= (Q_0 - \lambda_s I_0) f(p_k) \\ &= (Q_0 - \lambda_s I_{0,-k} - \lambda_s \tilde{A}_k^2 p_k) f(p_k)\end{aligned} \tag{7}$$

where $f(.)$ is a suitable, non-negative reward function, λ_s is a suitably chosen positive (pricing) coefficient, $I_{0,-k} = \sum_{j=1, j\neq k}^{K} \tilde{A}_j^2 p_j$ is the total secondary interference to the primary user excluding that from the k-th secondary user and $\tilde{A}_k$ is, the effective channel coefficient of the k-th secondary user at the primary receiver (see (3)). Note that the penalization term has been chosen linear on I_0 to allow simpler analytical derivations. However, the global system behavior is similar for other step like penalization functions. In (7) the coefficient λ_s esentially controls how strictly secondary users need to adhere to the primary user's interference cap, and allows the system designer to dimensionate the network for the maximum expected number of secondary users as we will see in the simulation section.

The proposed utility function (7) leaves the performance metrics of the secondary system to be arbitrary by allowing for any reasonable reward function $f(.)$ that will satisfy the conditions to be set forth in the next section. Without loss of generality, we may assume that the reward function $f(p_k)$ satisfies $f(0) = 0$ and $f'(0) > 0$, since when the received SINR of a user vanishes no useful communication is possible for that user.

B. Existence of a Nash equilibrium in the DSL game

In the following we investigate equilibrium strategies on the proposed DSL game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$, where users are interested in maximizing the following utility functions:

$$\begin{array}{ll} \text{primary-user utility:} & u_0(Q_0, \mathbf{a}_{-0}) \\ \text{secondary-user utility:} & u_k(p_k, \mathbf{a}_{-k}) \quad \text{for } k = 1, 2, \ldots, K. \end{array}$$

The most commonly used equilibrium concept in non-cooperative game theory is the Nash equilibrium:

Definition 1: A strategy vector $\mathbf{a} = (a_0, a_1, \ldots, a_k)$ is a Nash equilibrium of the primary-secondary user dynamic spectrum leasing game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$ if, for every $k \in \mathcal{K}$, $u_k(a_k, \mathbf{a}_{-k}) \geq u_k(a'_k, \mathbf{a}_{-k})$ for all $a'_k \in \mathcal{A}_k$.

In essence, at a Nash equilibrium no user has an incentive to unilaterally change its own strategy when all other users keep their strategies fixed. Hence, the Nash equilibrium can be viewed as a stable outcome where a game might end up when non-cooperative users adjust their strategies according to their self-interests. In fact, the best response correspondence of a user gives the best reaction strategy a rational user would choose in order to maximize its own utility, in response to the actions chosen by other users:

Definition 2: The user k's best response $r_k : \mathcal{A}_{-k} \longrightarrow \mathcal{A}_k$ is the set

$$r_k(\mathbf{a}_{-k}) = \{a_k \in \mathcal{A}_k : u_k(a_k, \mathbf{a}_{-k}) \geq u_k(a'_k, \mathbf{a}_{-k}) \\ \text{for all } a'_k \in \mathcal{A}_k\}$$

Note that the primary user action set is of the form of $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$, where $\bar{Q}_0$ is the maximum interference cap that is determined by the required minimum QoS and the maximum possible transmit power of primary user. Clearly $\mathcal{A}_0$ is both compact and convex. Similarly, for all $k = 1, \ldots, K$, the secondary user strategy sets are of the form of $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$. Again, it is easy to observe that all secondary user action sets are convex and compact (being closed and bounded real intervals). Further, both $u_0(\mathbf{a})$ and $u_k(\mathbf{a})$ are continuous in the action vector $\mathbf{a}$, and u_0 is concave in Q_0. For the existence of a Nash equilibrium, the only other condition that we need to ensure is the quasi-concavity of u_k's in p_k for $p_k \geq 0$, for $k = 1, 2, \ldots, K$.

Let us define

$$\phi_k(\gamma_k) = \frac{I_{0,-k}}{Q_0} + \frac{\tilde{A}_k^2 N_k}{Q_0}\left(\gamma_k + \frac{g(\gamma_k)}{g'(\gamma_k)}\right),$$

where $g(\gamma_k) = f(N_k \gamma_k)$ is the reward function with respect to SINR. Then, it can be seen that u_k has a local maximum that is indeed a global maximum if $\phi_k(\gamma_k) = \frac{1}{\lambda_s}$ has only one solution for $p_k \in \mathcal{P}_k$. Clearly, $\phi_k(\gamma_k) = \frac{1}{\lambda_s}$ has a solution if $\phi_k(0) \leq \frac{1}{\lambda_s} < \lim_{\gamma_k \to \infty} \phi_k(\gamma_k)$, and, moreover, this solution is indeed a global maximum if in addition $\phi'_k(\gamma_k) > 0$ for $\gamma_k > 0$. It can be easily verified that $\phi'_k(\gamma_k) > 0$ will be true if the reward function is such that $\frac{g(\gamma_k)g''(\gamma_k)}{(g'(\gamma_k))^2} < 2$ for all $\gamma_k > 0$. Note that, this is trivially true for any reward function that is concave in γ_k since in that case $g'' \leq 0$. Note also that $\phi_k(0) = \frac{I_{0,-k}}{Q_0}$ and $\lim_{\gamma_k \to \infty} \phi_k(\gamma_k) = \infty$ if $\lim_{\gamma_k \to \infty} \frac{g(\gamma_k)}{g'(\gamma_k)} > -\infty$. Hence, if reward function f (or, equivalently, g) and the coefficient λ_s satisfy the following conditions, u_k indeed has a local maximum that is a global maximum:

1. $g(0) = 0$, $g'(0) > 0$ and $\lim_{\gamma_k \to \infty} \frac{g(\gamma_k)}{g'(\gamma_k)} > -\infty$
2. $\frac{g(\gamma_k)g''(\gamma_k)}{(g'(\gamma_k))^2} < 2$ for all $\gamma_k > 0$
3. $0 < \lambda_s \leq \frac{Q_0}{I_{0,-k}}$

Theorem 1: With $\mathcal{A}_k$'s and u_k's as defined above the dynamic spectrum leasing game has a Nash equilibrium if conditions 1-3 are satisfied.

Proof: From the well-known result due to Debreu, Glicksberg and Fan [17], a Nash equilibrium exists in game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$, if, for all $k = 0, 1, \ldots, K$, $\mathcal{A}_k$ is a non-empty, convex and compact subset of some Euclidean space $\mathbb{R}^N$, $u_k(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k. Thus from the above discussion it follows that the above primary-secondary user dynamic spectrum leasing game G will have at least one Nash equilibrium. ■

Clearly, the above DSL game model is general enough to allow for various secondary reward functions g that may satisfy above conditions. In general, choosing the most suitable secondary user performance metric and the associated reward function in a cognitive radio network can itself be a non-trivial task [18]. While we do not delve into this issue here, for illustrative purposes, in the remainder of this paper we consider the following two specific reward functions:

$$\begin{aligned} g_k^{(1)}(\gamma_k) &= W_k \log(1 + \gamma_k) \quad \text{and} \\ g_k^{(2)}(\gamma_k) &= R_k \frac{C_{\text{BSC}}(P_e(\gamma_k))}{p_k}, \end{aligned}$$

where W_k and R_k are the bandwidth and data rate of user k, respectively, $P_e(\gamma_k)$ is the probability of bit error with received SINR of γ_k and $C_{\text{BSC}}(P_e)$ is the capacity of a binary symmetric channel with cross-over probability P_e which can be written in terms of the binary entropy function $H(P_e) = -P_e \log_2 P_e - (1-P_e)\log_2(1-P_e)$ as $C_{\text{BSC}}(P_e) = 1 - H(P_e)$. Both these reward functions can be justified in a wide variety of contexts. For example, $g^{(1)}$ is a measure of user k's capacity in the presence of all other users, and $g^{(2)}$ is a measure of its throughput per unit power. The reward function $g^{(1)}$ can be justified in a dynamic spectrum leasing application in which the secondary users are mainly concerned with getting access to the spectrum and their power consumption is not a major concern. On the other hand, $g^{(2)}$ is suitable when secondary users are interested in achieving best throughput per unit energy spent. Note that the function $g^{(2)}$ proposed here is arguably better than a similar utility function proposed in [19] and often used by many thereafter. For example, the utility function defined in [19] is based on an efficiency function that was defined in an ad-hoc way in order to avoid a degenerate behavior as the user transmit power vanishes. However, the proposed reward function $g^{(2)}$ avoids this degeneracy and has the natural meaning of throughput per unit energy as was intended in [19]. Indeed it can be shown that

$$\begin{aligned} g_k^{(2)}(0) &= \lim_{\gamma_k \to 0} g_k^{(2)}(\gamma_k) \\ &= -\lim_{\gamma_k \to 0} \frac{P_e'(\gamma_k) H'(P_e)}{N_k} = 0, \end{aligned}$$

since $H'(P_e) = \log_2 \frac{1-P_e}{P_e}$, $P_e(0) = \frac{1}{2}$ and $P_e'(0) < \infty$ for any practical communications receiver. For concreteness, throughout the remainder of this paper, we will assume that $P_e(\gamma_k) = \frac{1}{2}\exp(-\gamma_k)$ (i.e. BPSK modulation with a matched-filter receiver).

C. Best response adaptations and implementation issues

Primary user. Since the best response by a player in a game is a strategy that maximizes its own utility given all other players actions, the best response of the primary user in the above DSL game is obtained by setting $u_0'(Q_0) = 0$. The unique interior solution is given by

$$Q_0^*(I_0) = \frac{\bar{Q}_0 + I_0}{2}$$

Note that, since $u_0(Q_0)$ is monotonic increasing for $Q_0 < Q_0^*$, if the maximum interference cap is such that $\bar{Q}_0 < Q_0^*$, the best response of the primary user would be to set the interference cap to $Q_0 = \bar{Q}_0$. Hence, the primary user's best response is given by

$$r_0(\mathbf{a}_{-0}) = r_0(I_0) = \min\left\{\bar{Q}_0, Q_0^*(I_0)\right\}.$$

We observe that in order to determine its best response for a chosen power vector $\mathbf{a}_{-0}$ by the secondary users, the only quantity that the primary user needs to know is the total secondary interference at the primary receiver denoted by I_0 given in (3). This parameter can indeed be estimated at the primary receiver by using any standard SNR estimation algorithm, either data aided if the primary is able to decode its own signal or non-data aided in other case.

Secondary users. On the other hand, the best response of the k-th secondary user to the transmit powers of the other secondary users as well as interference cap set by the primary user is given by the (unique) solution $p_k = p_k^*(Q_0, I_{0,-k}, i_k)$ to the equation

$$\phi_k(\gamma_k) - \frac{1}{\lambda_s} = 0.$$

Since u_k is quasi-concave in p_k, if $p_k^*(Q_0, I_{p,-k}) > \bar{P}_k$ where $\bar{P}_k$ is the k-th user's maximum possible transmit power, its best response is to set its transmit power to $p_k = \bar{P}_k$. Hence, we have the best response of k-th secondary user, for $k = 1, 2, \ldots, K$:

$$r_k(\mathbf{a}_{-k}) = \min\left\{\bar{P}_k, p_k^*(Q_0, I_{0,-k}, i_k)\right\}.$$

Observe that, in general, the best response of the k-th secondary user is a function of the primary interference cap Q_0, the residual interference $I_{0,-k}$ from all other secondary users to the primary user, and the total interference from all secondary and primary users to the k-th user's received signal at the secondary receiver i_k. Like in the primary case, the secondary system can estimate i_k withoput much difficulty using standard SNR estimation algorithms. To obtain the knowledge of Q_0 and $I_{0,-k}$ we assume that the primary system periodically broadcasts Q_0 and I_0. Note that this is the only interaction that the primary system will need to have with the secondary system. Since these two quantities are readily available to the primary system, we believe that the periodic broadcast of these quantities, informing the secondary system what it needs to know in order to avoid severe conflicts with primary transmissions, is a reasonable expectation for a future cognitive radio system that expects to harvest spectrum leasing gains. Observe that knowing I_0, each secondary user can compute the residual interference $I_{0,-k} = I_0 - \tilde{A}_k^2 p_k$ if it can estimate the channel state information $\tilde{A}_k$. This quantity may be estimated if the reverse link signals are available in the same band. Otherwise the secondary receiver does not necessarily need the CSI of its link with the primary receiver, as we will demonstrate in our simulation results, since the approximation $I_{0,-k} \approx I_0$ performs well in practice, especially when the number of secondary users K is sufficiently large.

In the above discussion, we have assumed the quasi-static fading in which fading realizations stay fixed for a period of duration and then change to new values. This facilitated the Nash equilibrium analysis without having to deal with time-varying channel coefficients. While quasi-static assumption may be justified in certain channel environments, sometimes it is likely that the channel coefficients may slowly vary in time. It is easy to see that for the best-response adaptations to converge to a Nash equilibrium, the rate of adaptations need to be faster than the time-variations of the channel. One may expect that in the presence of channel variations, the convergence may be slowed, or even not occur. However, as we will demonstrate in the next section, the proposed DSL-game has the desired property of being tolerant towards slow time-variations of the channel state. Moreover, the Nash equilibrium of the proposed DSL-game is robust against small channel estimation errors. This is also a desired property since in practice the channel coefficients need to be estimated, and these estimations are almost always not perfect.

IV. Performance Analysis of a Dynamic Spectrum Leasing System

In the following we consider a dynamic spectrum leasing cognitive radio system that employs the proposed game-theoretic framework for their interactions. Our goal is to investigate the behavior of the primary and secondary systems at the equilibrium. It is to be noted that the Nash equilibrium can reasonably be expected to be the natural outcome of the system when it reaches steady-state. Thus, the performance of the system is to be considered as its performance at the Nash equilibrium.

To illustrate the characteristics of the Nash equilibrium in this primary-secondary user dynamic spectrum leasing game, we first consider a simplified scenario with identical secondary

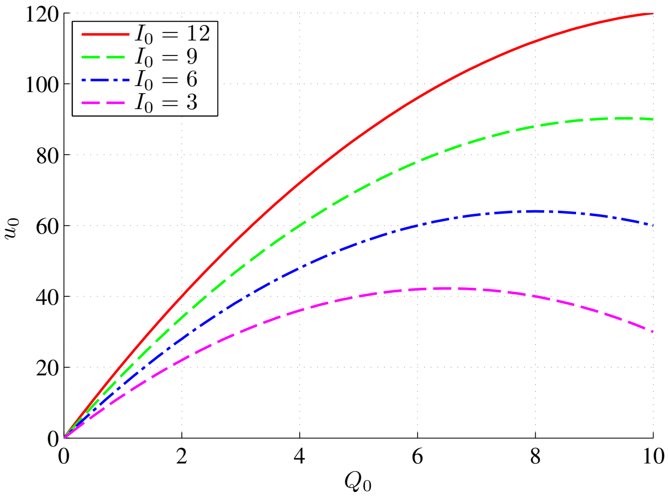
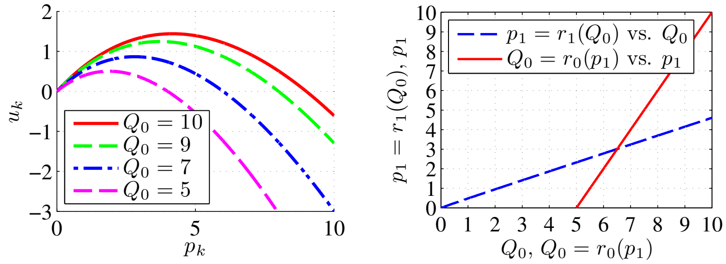
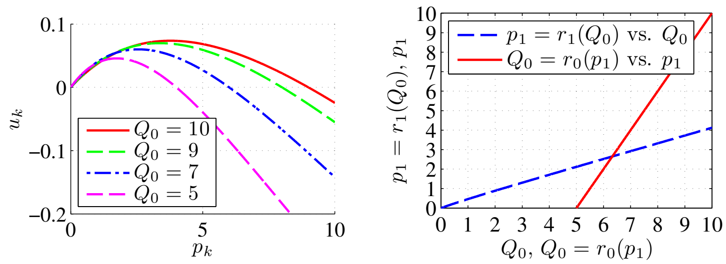


Fig. 1. Primary user utility u_0 for a fixed secondary interference I_0 in a single-user secondary system.



(a) (b)

Fig. 2. Secondary utility and the best response functions in a single secondary user dynamic spectrum leasing network with $f(\gamma) = f^{(1)}(\gamma) = \log(1+\gamma)$. (a) Secondary user utility u_k for a fixed primary interference cap Q_0. (b) Primary and secondary user best-response functions in a single secondary-user dynamic spectrum leasing system when $f(\gamma) = f^{(1)}(\gamma) = \log(1+\gamma)$ and $\lambda_s = 1$.



(a) (b)

Fig. 3. Secondary utility and the best response functions in a single secondary user dynamic spectrum leasing network when $f(p_1) = f^{(2)}(p_1) = \frac{R_1}{p_1} C_{BSC}\left(P_e(\gamma_1)\right)$ and $\lambda_s = 1$. (a) Secondary user utility for a fixed primary interference cap (b) Corresponding best-response functions.

users. This scenario allows to analytically determine the Nash equilibrium state and its general behavior. We analyze next a more general scenario with non-identical secondary users and fading channels by means of simulations.

A. Stationary system with identical secondary users

When all secondary user's have the same cross-correlation coefficients it is posible to characterize the best response correspondences of primary and secondary users to graphically visualize the Nash equilibrium. If $\rho_{0k}^{(p)} = \rho_0^{(p)}$, $\rho_{k0}^{(s)} = \rho_0^{(s)}$, $\rho_{kj}^{(s)} = \rho^{(s)}$, for all $k, j = 1, 2, \ldots, K$, same collision probabilities $q_k = q$, for all k and all channels are additive white Gaussian noise (AWGN): $h_{sk} = h_{pk} = 1$ for all $k = 0, 1, \ldots, K$. Then $\tilde{A}_k = \tilde{A}$ for all k. By symmetry, in this case all secondary users must have the same power $p_k = p^*$ at the Nash equilibrium (equivalently, the same SINR $\gamma_k = \gamma^*$). Thus the Nash equilibrium is characterized by the intersection (Q_0^*, p^*) of the following two curves:

$$
\begin{aligned}
Q_0 &= r_0(p) = \frac{\bar{Q}_0 + K\tilde{A}^2 p}{2} && (8)\\
p &= r_s\left(Q_0\right) && (9)\\
&= (\text{solution to equation } \psi_{Q_0}(p) = 0)
\end{aligned}
$$

where

$$
\psi_{Q_0}(p) = Kp + \frac{f(p)}{f'(p)} - \frac{Q_0}{\lambda_s \tilde{A}^2}. \qquad (10)
$$

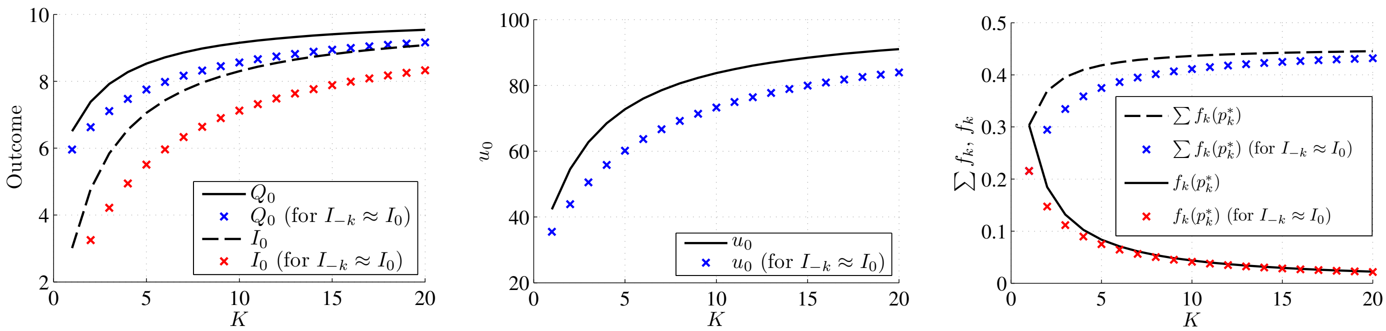
Combining (8) and (9), the Nash power p^* of the secondary users is given by the solution to the equation

$$
K\left(1 - \frac{1}{2\lambda_s}\right) p + \frac{f(p)}{f'(p)} - \frac{\bar{Q}_0}{2\tilde{A}^2\lambda_s} = 0 \qquad (11)
$$

Figure 1 shows the primary utility function for fixed secondary network actions in a single secondary user system, that is $K = 1$, assuming that $\bar{Q} = Q_{\max} = 10$, $\bar{P}_1 = 12$, $W_1 = 1$, $q_1 = 1$, $\rho_{01}^{(p)} = \rho_{10}^{(s)} = 1$, $\lambda_s = 1$, $\bar{\gamma}_0 = 1$, $q_1 = 1$, $h_{p1} = 1 = h_{p0} = h_{s0} = h_{s1} = 1$ and $\sigma_s^2 = \sigma_p^2 = 1$.

On the other hand, for the setup described, secondary utility and best response depends on the considered reward function $g(\gamma)$. First Figs. 2(a) and 2(b) assume the secondary reward function $g(\gamma) = g^{(1)}(\gamma) = \log(1+\gamma)$. In Fig. 2(a) we can see the concavity of the secondary utility function for fixed primary response, and thus the existence of a best response. The primary and secondary best response curves $Q_0 = r_0(p_1)$ and $p_1 = r_1(Q_0)$ for the setup described are presented in Fig. 2(b). Of course, the intersection of these two best response curves specifies the Nash equilibrium for this system: $(Q^*, p_1^*) = (6.505, 3.010)$.

Similarly, Figs. 3(a) and 3(b) show the secondary user utility for a fixed primary interference cap and the best response functions, respectively, when the secondary utility function is chosen to be $g(\gamma) = g^{(2)}(\gamma) = R\frac{C_{\text{BSC}}(P_e(\gamma))}{p}$ with $R_1 = 1$ and all other parameters being the same as in the previous figures. From 3(a) we observe that the secondary utility function is still concave in secondary power. The best response curves in Fig. 3(b) are characterized by (9) and (8) where, now, $g(\gamma) = g^{(2)}(\gamma)$. Figure 3b shows that the Nash equilibrium in this system is $(Q^*, p_1^*) = (6.325, 2.650)$. Note that this NE shows that due to the penalty for increasing transmit power in the secondary system, the secondary user now settles for a slightly lower transmit power level compared to the earlier situation in which it was not concerned with power expenditure. As a result, the primary user is also better off by slightly lowering its interference cap so that it keeps the demand high.



(a) (b) (c)

Fig. 4. System performance of the DSL game at the Nash equilibrium, with both exact CSI and using the approximation $I_{0,-k} \approx I_0$, as a function of secondary system size K, assuming identical secondary users, when $f(p) = f^{(1)}(p)$. (a) Game outcome. (b) Primary user utility. (c) Sum-rate and the per-user rate achieved by the secondary system at the Nash equilibrium.

It is of interest to investigate the equilibrium behavior of this dynamic spectrum leasing system as a function of the secondary system size K. In Fig. 4(a) we show the allowed interference cap Q_0 and the actual secondary interference I_0 at the system equilibrium for a system such that $f(p) = f^{(1)}(p) = \log(1 + \frac{p}{N(p)})$ where $N(p) = N_k = (K-1)p + \rho_{sp}(Q_0 + \sigma_p^2) + \sigma_s^2$, $\bar{Q} = 10$, $\bar{P}_k = 10$, $W_k = 1$, $R_k = 1$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$, $h_{pk} = h_{sk} = 1$ for all k, and $\sigma_s^2 = \sigma_p^2 = 1$. From Fig. 4(a) we can observe how the total interference I_0 increases with increasing K, and how, in turn, the primary user also increases its interference cap to maximize its utility. It is also of interest to note that the safety margin $Q_0 - I_0$ is large for smaller number of users, and seems to monotonically decrease with increasing K. This, we believe, is essentially due to the fact that the number of degrees of freedom in a multiuser system is being proportional to the number of users. When the number of secondary users K is large, the interference generated by the secondary system I_0 is close to the interference cap Q_0, yet, as desired, is always below it. Figure 4(a) shows the game outcomes when exact channel state information for the primary system is available at each secondary user (via estimation) so the exact $I_{0,-k}$ is used in its best response adaption, as well as when this channel state information to the primary is not available, so that the secondary user employs the approximation $I_{0,-k} \approx I_0$. As we may observe from Fig. 4(a), the system that does not rely on the knowledge of channel state information demonstrates the same performance trends at the equilibrium. In particular, still the DSL game converges to a Nash equilibrium that does not violate the primary interference cap. It seems that the only effect of not having the exact $I_{0,-k}$ is that the safety margin $Q_0 - I_0$ at the equilibrium is slightly larger. This is essentially due to the fact that each secondary user believes an exaggerated residual interference $I_{0,-k}$ making it to decrease its power.

Figures 4(b) and 4(c) show the primary and secondary utilities at the Nash equilibrium of the system considered in Fig. 4(a) as a function of the secondary system size. Again we have shown the utilities achieved when exact channel state information for the primary system is available at each secondary user (via estimation) so the exact $I_{0,-k}$ is used in its best response adaptation, as well as when this channel state information to the primary is not available, so that the secondary user employs the approximation $I_{0,-k} \approx I_0$. In particular, as seen by Fig. 4(b) the primary utility u_0^* at the Nash equilibrium typically increases with the number of secondary users K. However, the rate of increase decreases with increasing K. Thus, from a design point of view we may argue that the primary user might prefer the system to operate at a point where its rate of utility increase is above a certain threshold value. However, the primary system cannot impose this explicitly on the secondary system and indeed it is not a requirement. The only requirement is that $I_0 \leq Q_0$. However, as we see next from Fig. 4(c) the secondary system has the incentive to keep K not too high. It is also observed from Fig. 4(b) that the equilibrium utility of the primary user is decreased when exact channel state information is not available at the secondary users.

Fig. 4(c) shows both the sum-rate $\sum_{k=1}^{K} f_k(p_k^*)$ as well as the the per-user rate $\frac{1}{K}\sum_{k=1}^{K} f_k(p_k^*)$ achieved by the secondary system, with and without exact channel state information. As was the case with primary utility, the secondary utilities are also reduced slightly in the absence of channel state information. However, as we observe from Fig. 4(c), this performance degradation seems to be small when the secondary system size is sufficiently large. Note that, from a system point of view the secondary system would prefer to maximize the sum-rate. As we see from Fig. 4(c), the sum-rate monotonically increases with K both with and without CSI. Thus, at a first glance, allowing more secondary users to operate simultaneously seems to be the preferred solution. However, Fig. 4(c) also shows that the per-user rate is monotonically decreasing in K, leading to decreasing incremental gains in sum-rate as additional secondary users are added to the system. Depending on the application and the QoS requirement of the secondary system, each secondary user will have a minimum required rate (in bits per transmission) below which the transmissions would be useless. Thus we note that this QoS requirement will determine the maximum number of secondary users K the secondary system would want to support at any given time. For example, if the minimum per-user rate required is 0.1 bps, the optimal K would be $K^* = 4$,

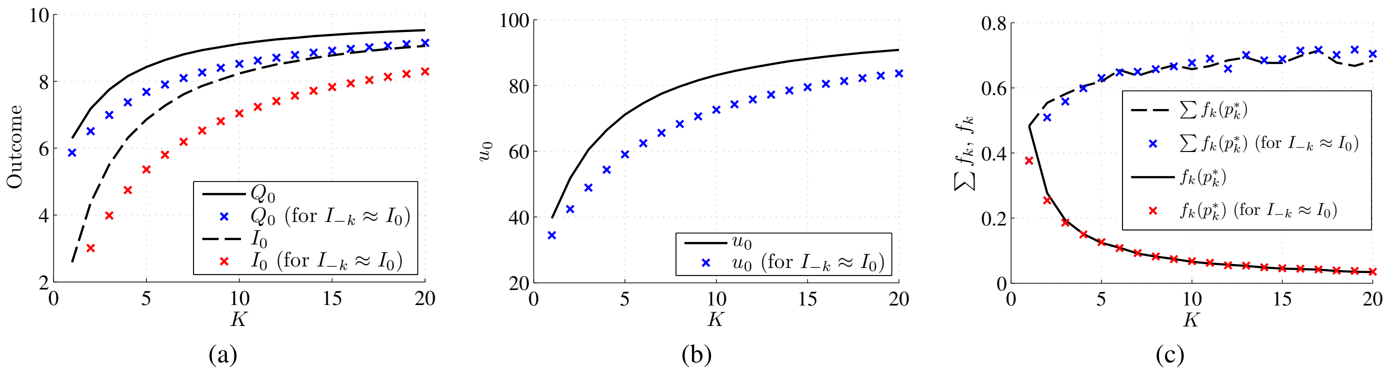


Fig. 5. System performance of the DSL game at the Nash equilibrium, with both exact CSI and using the approximation $I_{0,-k} \approx I_0$, as a function of secondary system size K in the presence of Rayleigh distributed quasi-static channel fading when $f(p) = f^{(1)}(p)$ and $\lambda_s = 1$. (a) Game outcome. (b) Primary user utility. (c) Sum-rate and the per-user rate achieved by the secondary system at the Nash equilibrium.

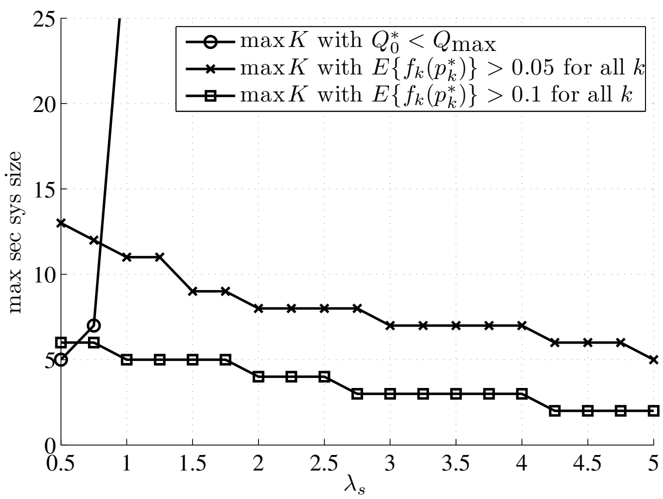


Fig. 6. Maximum secondary system size for a required Quality-of-Service requirement $f_{\min}$ for all secondary users and the maximum secondary system size supportable without violating the primary interference cap as a function of the weighting coefficient λ_s assuming channel fading.

assuming exact CSI. If, on the other hand, the rate threshold was reduced to 0.025 bps, the secondary system may allow up to $K = 18$ secondary users to simultaneously operate.

B. DSL network under quasi-static fading channels

In the presence of wireless channel fading, the Nash equilibrium power profile of the dynamic spectrum leasing system will depend on the observed channel state realization. In particular, it is expected that in this case the Nash equilibrium transmit powers of individual secondary users will be different for each user. In Fig. 5(a) we have shown the game outcome at the Nash equilibrium in the presence of channel fading as a function of number of secondary users K, both with and without CSI (when there is no channel state information, again, we use the approximation $I_{0,-k} \approx I_0$). Note that Fig. 5(a) assumes $f(p_k) = f^{(1)}(p_k)$ with $\bar{Q} = 10$, $\bar{P}_k = 10$, $W_k = 1$, $R_k = 1$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$, and $\sigma_s^2 = \sigma_p^2 = 1$ as before. Figures 5(b) and 5(c) show the corresponding primary and secondary user utilities achieved at the Nash equilibrium in the presence of channel fading. In obtaining Fig. 5 we have assumed all channel gains in the system to be Rayleigh distributed with all channel coefficients normalized so that $\mathbb{E}\{h^2\} = 1$. This essentially allows us to consider, without any loss in generality, the transmit powers p_k to be equal to the average received power (averaged over fading). Note that, due to interference averaging in the presence of fading, in this case the secondary system is able to achieve better sum- and per-user rates compared to those with non-fading channels.

Note that, when the reward function $f = f^{(1)}$, the reward for a secondary user is the capacity (in bps) it can achieve assuming all other transmissions (both primary and secondary) are purely noise. In the presence of channel fading, this capacity is a random quantity determined by the fading coefficients of all users. As we saw earlier with identical users, the per-user reward is typically a decreasing function of the increasing secondary system size. The interpretation is simple: Essentially, all secondary users in the system must share the allowed interference level set by the primary system. As we mentioned earlier, a secondary user may require a minimum capacity to ensure at least an acceptable QoS for its applications. In Fig. 6 we show the maximum secondary system size (i.e. K) in the presence of fading for different quality of service requirements in the secondary system as a function of the (weighting) coefficient λ_s. Note that in Fig. 6 we have set $W_k = W = 1$ so that the secondary reward with $f = f^{(1)}$ has the meaning of spectral efficiency in bits-per-second-per-Hertz (or the normalized capacity). All other parameter values are the same as that assumed in Fig. 5. The minimum transmission quality for the secondary system is defined as the average (over fading) minimum reward achieved by a user at the equilibrium. We denote this minimum required QoS for user k as $f_{\min,k}$ and in all simulation results below assume that $f_{\min,k} = f_{\min}$ for all secondary users.

As one would expect, as the minimum QoS requirement $f_{\min}$ increases, the number of secondary users who can simultaneously transmit decreases. In addition, the maximum secondary system size also decreases, albeit slowly, as the pricing coefficient λ_s increases. As we may observe from Fig. 6 the greatest impact of the coefficient λ_s is on the primary system. We have included in Fig. 6 the maximum tolerable secondary system size by the primary system before

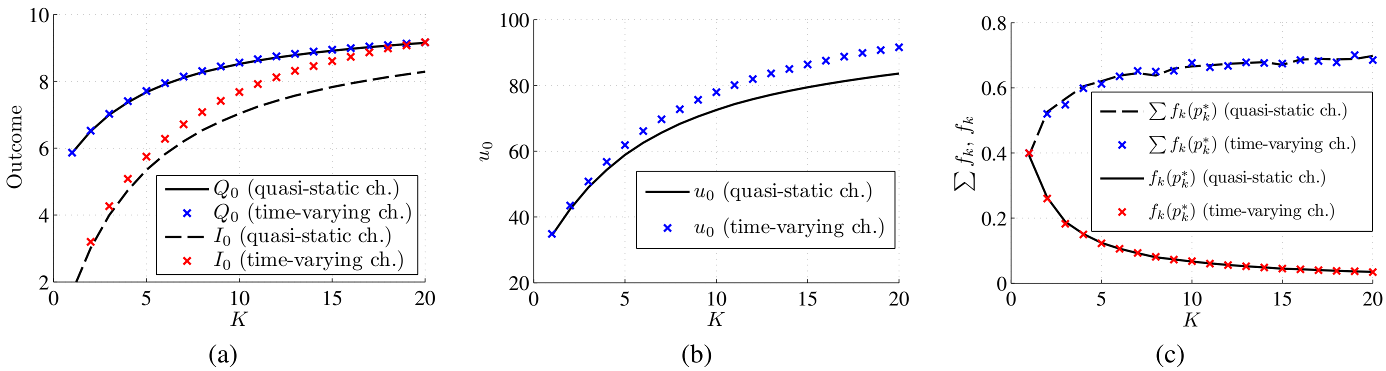


Fig. 7. System performance of the DSL game at the Nash equilibrium in the presence of slow time-varying channel fading when $f(p) = f^{(1)}(p)$ using the approximation $I_{0,-k} \approx I_0$. (a) Game outcome. (b) Primary user utility. (c) Sum-rate and the per-user rate achieved by the secondary system at the Nash equilibrium.

the interference cap is exceeded at the equilibrium. Figure 6 shows that when $\lambda_s < 1$ there is a high likelihood that the interference cap might be exceeded by even a relatively smaller size secondary systems. While smaller λ_s would result in higher utilities for the primary system (we have not shown these plots to save space), this comes at the price of violating the interference condition. Thus the risk with smaller λ_s values is that, depending on the secondary QoS requirement $f_{\min}$, the secondary system may opt to operate at a number of simultaneous users K that could easily violate the interference condition. However, as we observe from Fig. 6 when $\lambda_s \geq 1$, the number of secondary users who simultaneously transmit without violating the primary interference condition dramatically increases, leaving the primary system with enough safety margin in case the secondary system opts for large number of simultaneous users. Thus we believe that in a proposed DSL network, the primary system must set the pricing coefficient λ_s based on how strictly it want the secondary users to adhere to the maximum interference cap condition. If the primary system is also based on a certain amount of cognition, it is reasonable to expect that it may adjust its (pricing) coefficient λ_s to maximize its profits by dynamically adapting optimal λ_s based on its estimation of how many secondary users are in the secondary system.

On the other hand Fig. 6 shows the maximum number of secondary users who can on average coexist while achieving a minimum required transmission quality. However, at times depending on the fading statistics a particular user may or may not meet the minimum transmission quality at the system equilibrium. When this occurs we say that the user is in outage and thus the probability of outage for user k is defined as $Pr\left(f_k(p_k^*) < f_{\min,k}\right)$.

C. DSL network under time varying fading channels

In the previous section we have assumed that the fading coefficients are essentially quasi-static so that they remain constant during the best-response adaptations. However, in practice these fading coefficients may slowly change during the best-response iterations. In these circumstances, transceivers may need to employ a channel tracking algorithm to update the estimated fading coefficients. In Fig. 7 we investigate the effect of slowly-varying channel coefficients on the DSL game. We model the variations of the channel coefficients with a first order Gauss-Markov process [20], so that the fading coefficients of the $(n+1)$-th best-response adaptation are related to those of the n-th iteration as below:"

$$h_{\cdot k}^{(n+1)} = \sqrt{1-\epsilon^2}\, h_{\cdot k}^{(n)} + \epsilon w_{\cdot k}^{(n)}, \qquad (12)$$

where $w_{\cdot k}^{(n)}$ is a complex white Gaussian random process of variance $\sigma^2_{h_{\cdot k}}$, independent among channel coefficients $h_{\cdot k}$, ϵ is a parameter indicating the temporal variation rate of the channel and initial $h_{\cdot k}^{(0)}$ is chosen to be complex Gaussian. It is easy to verify that $\sqrt{1-\epsilon^2}$ represents the temporal correlation of the channel coefficients between two best-response iterations. Figure 7(a) shows the DSL game outcome when the coefficients are time-varying with an $\epsilon = 0.1$ as compared to a quasi-static system in which fading is constant throughout (i.e. $\epsilon = 0$). It is assumed that the slowly time-varying system only updates the fading coefficients once in every $L = 10$ iterations (of course, the quasi-static system always has exact coefficients since they stay fixed throughout the iterations and thus correspond to $L = 1$). Figure 7(a) shows that the primary interference cap is basically insensitive against assumed slow channel variations. However, the corresponding secondary interference I_0 at the NE is usually larger in the presence of channel variations, especially for large number of secondary users. The reason for this is that the game response falls somewhat behind compared to the channel variations. Of course, this effect could be reduced by allowing more frequent channel adaptations (i.e. small L). The effect of this increased I_0 is to reduce the safety margin $(Q_0 - I_0)$ the primary receiver has in terms of its tolerable interference level. However, as Fig. 7(a) shows, unless the number of secondary users is relatively large, still the interference cap Q_0 is not violated by the increased interference I_0. Thus, we conclude that as long as the channel variations are sufficiently slow and/or coefficient adaptations are fast enough, the DSL game can still reach an acceptable equilibrium state.

Figures 7(b) and (c) show the corresponding primary and secondary utilities in the presence of slowly time-varying channel fading at the NE outcome shown in Fig. 7(a). Figure 7(b) shows that the primary utility at the Nash equilibrium is

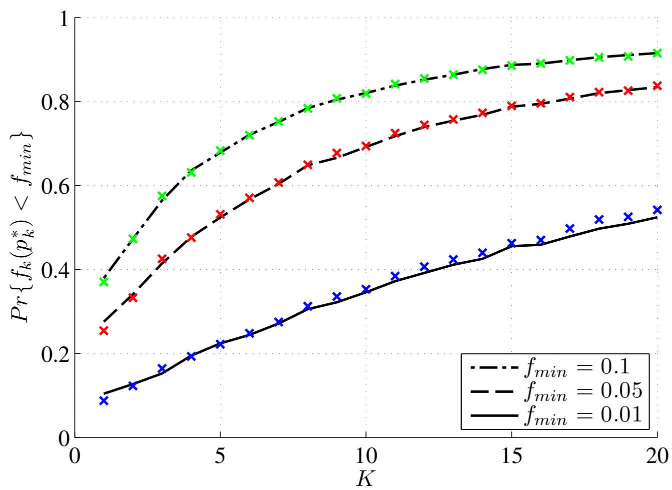


Fig. 8. Outage probability $Pr\left(f_k(p_k^*) < f_{\min}\right)$ of a typical secondary user at the Nash equilibrium of the DSL game in both quasi-static (lines) and slow time-varying (marks) fading channels, as a function of secondary system size K for a required Quality-of-Service requirement $f_{\min}$.

slightly increased when fading is slow time-varying compared to that with static channels. This is a direct consequence of the increased interference level I_0 seen in Fig. 7(a) that reduced the safety margin. From primary utility function (6) we can see that the reduced safety margin may lead to increased primary utility. However, it is to be noted that this itself may not be a good outcome if the number of secondary users is too large since the secondary interference may violate the interference cap. As can be seen from Fig. 7(c), both the sum and per-user rates achieved by the secondary system is reasonably robust against slow time variations in fading.

Figure 8 shows the outage probability of a typical secondary user as the system size increases with both quasi-static as well as slow time-varying (according to (12)) channel fading. It is seen from Fig. 8 that the outage probability increases with K as well as with the minimum QoS requirement. However, as one may have predicted from the insensitivity of secondary user utilities to slow channel variations observed in Fig. 7(c), the outage probabilities are robust against the channel time-variations. The maximum secondary system size which can be supported according to Fig. 6 thus needs to be interpreted in conjunction with the outage probabilities shown in Fig. 8. For example, although as shown by Fig. 6 about 5 secondary users can on average meet the $f_{\min=} = 0.1$ QoS requirement, according to Fig. 8 each of these users may be in outage about 70% of time. This of course is the price of operating as a secondary system.

V. Conclusion

In this paper we have proposed the concept of dynamic spectrum leasing as a new paradigm for dynamic spectrum access in cognitive radio networks. As opposed to the hierarchical dynamic spectrum access networks, the proposed dynamic spectrum leasing networks provide an incentive for the primary users who owns the spectrum to actively allow secondary spectrum access whenever it is feasible. In our proposed framework, this is achieved by defining a utility function for the primary system that is proportional to both demand (for interference) as well as the amount of total interference it is willing to tolerate. The rationale behind the proposed utility is that the more the secondary interference the primary user is willing to tolerate, the higher must be its reward. On the other hand, if the interference cap set by the primary user is higher than the actual secondary interference that exists in the system, then the demand for interference by the secondary system must decrease, and the primary utility must be proportional to this demand. For the secondary users, their utility must be proportional to a suitably chosen reward function f as well as the achieved interference margin with respect to the primary system. The higher the interference margin, the safer the secondary operation without violating the primary QoS. Hence, the rationale for its utility to be proportional to the interference margin. We formulated the dynamic spectrum leasing cognitive system as a non-cooperative DSL game between the primary and the secondary users and established a basic result on the existence of a unique Nash equilibrium. Specifically, we have established the general condition on the reward function f so as to ensure the existence of an equilibrium.

Next, we considered several example cognitive radio DSL networks in detail to investigate the behavior of the proposed system. In particular, we showed that in the case of identical users the proposed DSL game can be solved to obtain the Nash equilibrium action profile as the solution to a single equation. In such a system we observed that the proposed dynamic spectrum leasing naturally leads to a design that will determine the maximum number of secondary users based on required minimum QoS criteria. In the presence of fading, we observed that the achieved secondary sum-rate could be considerably higher than that without fading. This was due to interference averaging effect due to fading that de-emphasized the interference among users leading to better SINR.

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