SPECTRUM SENSING IN TIME-VARYING CHANNELS USING MULTIPLE ANTENNAS

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ABSTRACT

The problem of detecting a Gaussian signal received with multiple antennas through a time-varying channel and corrupted by noise of unknown power is addressed by deriving the Generalized Likelihood Ratio test. The channel is described by means of a basis expansion model with deterministic unknown coefficients. Since the detection rule requires the knowledge of the Maximum Likelihood estimates of the channel and noise power, which lack closed-form expressions, an Expectation-Maximization algorithm is proposed to carry out such computation. Finally, simulations reveal that the proposed detector successfully exploits spatial correlation and time variation of the channel.

***Index Terms*—** Spectrum sensing, cognitive radio, detection theory, time-varying channels

1. INTRODUCTION

The problem of detecting the presence of transmissions in a particular frequency channel has been the subject of many research works in the past, but interest has grown considerably due to the proposal of dynamic spectrum access (DSA) [1], commonly referred to as cognitive radio, in the last decade. The presence of noise forces us to exploit properties of the signal such as power, spatial correlation or cyclostationarity [2]. In this work we exploit spatial correlation and time variation of the channel.

Most existing detectors in the literature assume time-invariant channels, but this may be unrealistic in cases like those where the symbol period is comparable to the coherence time of the channel (e.g. in narrowband communications) or those where the signal-to-noise ratio (SNR) is very low so that long observation windows are required. Conversely, most works in time varying channels have not addressed activity detection, with the exception of [3–5]. However, these works do not exploit the spatial information that may be present in multiantenna sensors.

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In this paper, we assume that the signal has been transmitted using a single antenna and that its amplitude follows a Gaussian distribution. The latter assumption is widespread in the literature since it is well-motivated in communication scenarios. The channel is assumed frequency flat, and time variations are modelled using a basis expansion model (BEM) [6], which considers that the time evolution of the channel coefficient at a particular antenna can be described in terms of a set of basis functions, which can be formed by discrete prolate spheroidal sequences [7], complex exponentials [6, 8, 9], Karhunen-Loeve orthogonal expansion functions [10] or simply polynomials [11]. The model used in this paper is the multiantenna BEM in [12], which uses decoupled coefficients for the time variations at each antenna.

In applications like radar or DSA [1], we are interested in decision rules maximizing the probability of detection for a target probability of false alarm [13]. Unfortunately, in the presence of unknown parameters no such rule can be found and we are forced to resort to suboptimal schemes. There exist sets of guidelines to design this class of detectors, one of them being the Generalized Likelihood Ratio (GLR) Test [13], which is known to exhibit good detection performance in many cases. However, in order to evaluate this test we need the Maximum Likelihood (ML) estimates of the channel coefficients and noise power, which must be obtained by numerical means. We propose the utilization of an Expectation-Maximization (EM) algorithm [14] to find those estimates. The resulting detector is seen to exploit both time variations of the channel and the rank-one space structure of the transmitted signal.

This paper is structured as follows: first, the observation model is described in Sec. 2. Next, the Generalized Likelihood Ratio is presented in Sec. 3 and an EM algorithm is proposed in Sec. 4 for its computation. Finally, performance assessment is provided in Sec. 5 and some final remarks discussed in Sec. 6.

2. OBSERVATION MODEL

Assume that a spectrum sensor is monitoring a particular frequency channel using M antennas and that, after sampling and down-conversion, N samples per antenna are processed to decide whether they contain a signal transmitted by a single-antenna user, which is known as primary signal. We collect the received samples in the matrix $\boldsymbol{Y} \in \mathbb{C}^{M\times N}$, where the m-th row corresponds to the m-th antenna and the n-th column with the n-th time instant. The decision problem is hindered due to the presence of noise. More specifically, when a primary waveform is present, we may write the observations $\boldsymbol{Y}$ as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \sigma\boldsymbol{W} \tag{1}$$

where $\boldsymbol{H}\boldsymbol{X}$ and $\sigma\boldsymbol{W}$ are, respectively, the signal and noise terms, which are discussed next. First, $\boldsymbol{X} = \operatorname{diag}\{\boldsymbol{x}\} \in \mathbb{C}^{N\times N}$ is a diagonal matrix containing the transmitted signal $\boldsymbol{x}$, which is a white complex Gaussian circularly symmetric random vector with zero mean, i.e., $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_N)$. Second, $\boldsymbol{H} \in \mathbb{C}^{M\times N}$ is a matrix whose (m,n)-th element $h_{m,n}$ is the channel coefficient at antenna m and time sample n. Finally, the noise is assumed spatially and temporally white, so that the entries $w_{m,n}$ of the noise matrix $\boldsymbol{W}$ are independent and identically distributed (i.i.d.) with $w_{m,n} \sim \mathcal{CN}(0,1)$. Since the elements of $\boldsymbol{W}$ are normalized to unit variance, σ^2 represents the (unknown) noise power.

Each row of $\boldsymbol{H}$ represents the evolution of the channel at a particular antenna over time. This evolution is modeled here using a BEM which is decoupled from antenna to antenna [12]. In particular, if the m-th row of $\boldsymbol{H}$ is denoted as $\boldsymbol{h}_m^H$, where the superscritp ^{H} represents conjugate transpose, we may apply the expansion

$$\boldsymbol{h}_m = \sum_{k=0}^{K-1} \boldsymbol{f}_k c_{km} \tag{2}$$

where $\boldsymbol{f}_k \in \mathbb{C}^N$ are the basis functions (or vectors), which are known to the sensor, and c_{km} is the associated coefficient, which is regarded as a deterministic unknown parameter. Expression (2) can also be written as $\boldsymbol{h}_m = \boldsymbol{F}\boldsymbol{c}_m$, where $\boldsymbol{F} = [\boldsymbol{f}_0, \boldsymbol{f}_1, \ldots, \boldsymbol{f}_{K-1}] \in \mathbb{C}^{N\times K}$ and $\boldsymbol{c}_m = [c_{0m}, c_{1m}, \ldots, c_{K-1,m}]^T \in \mathbb{C}^K$, or more compactly as $\boldsymbol{H}^H = \boldsymbol{F}\boldsymbol{C}$, where $\boldsymbol{C} = [\boldsymbol{c}_0, \boldsymbol{c}_1, \ldots, \boldsymbol{c}_{M-1}] \in \mathbb{C}^{K\times M}$.

It is also convenient to rewrite (1) by defining the column-wise vectorizations $\boldsymbol{y} = \operatorname{vec}\boldsymbol{Y}$ and $\boldsymbol{w} = \operatorname{vec}\boldsymbol{W}$ so that we obtain the equivalent

$$\boldsymbol{y} = \boldsymbol{G}\boldsymbol{x} + \sigma\boldsymbol{w} \tag{3}$$

where $\boldsymbol{G} \in \mathbb{C}^{MN\times N}$ is given by

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g}_0 & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{g}_1 & \ldots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \ldots & \boldsymbol{g}_{N-1} \end{bmatrix}, \tag{4}$$

with $\boldsymbol{g}_n \in \mathbb{C}^M$ representing the n-th column of $\boldsymbol{H}$. This enables us to write the covariance matrix of the observations as $\boldsymbol{\Sigma}_{\boldsymbol{y}} = \operatorname{E}\{\boldsymbol{y}\boldsymbol{y}^H\} = \boldsymbol{G}\boldsymbol{G}^H + \sigma^2\boldsymbol{I}_{MN}$, so that the likelihood function of the parameters is given by

$$p(\boldsymbol{y};\boldsymbol{\theta}) = \frac{\exp\{-\boldsymbol{y}^H\boldsymbol{\Sigma}_{\boldsymbol{y}}^{-1}\boldsymbol{y}\}}{\pi^{NM}|\boldsymbol{\Sigma}_{\boldsymbol{y}}|}, \tag{5}$$

where $\boldsymbol{\theta} = [(\operatorname{vec}\boldsymbol{C})^T, \sigma^2]^T$ is a vector collecting all the unknown parameters of the distribution.

3. GENERALIZED LIKELIHOOD RATIO

The problem of deciding over the presence of the transmitted waveform can be stated as a binary hypothesis test with hypotheses:

$$\mathcal{H}_0: \ \boldsymbol{Y} = \sigma\boldsymbol{W} \qquad \mathcal{H}_1: \ \boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \sigma\boldsymbol{W}. \tag{6}$$

The Generalized Likelihood Ratio test is constructed by comparing the GLR statistic $\mathcal{T}$ against a threshold γ, which is fixed so as to satisfy certain probability of false alarm/detection constraint, deciding $\mathcal{H}_1$ when $\mathcal{T} > \gamma$ and $\mathcal{H}_0$ otherwise. The GLR test can thus be written as

$$\mathcal{T} = \frac{p(\boldsymbol{y};\hat{\boldsymbol{\theta}}_1)}{p(\boldsymbol{y};\hat{\boldsymbol{\theta}}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{7}$$

where $\hat{\boldsymbol{\theta}}_i$ is the ML estimate of the unknown parameters under hypothesis $\mathcal{H}_i$, i.e., $\hat{\boldsymbol{\theta}}_i$ is the maximizer of (5) under $\mathcal{H}_i$.

Since under $\mathcal{H}_0$ we have that $\boldsymbol{G} = \boldsymbol{0}$, we only need to find the ML estimate of σ^2, which can be easily seen to be $\sigma^2 = (MN)^{-1}\boldsymbol{y}^H\boldsymbol{y} = (MN)^{-1}\operatorname{Tr}(\boldsymbol{Y}\boldsymbol{Y}^H)$. On the other hand, the computation of the ML estimates under $\mathcal{H}_1$ is not so simple and we are forced to resort to numerical methods. In the next section we propose an EM algorithm to perform such computation.

4. EM ALGORITHM

The EM algorithm is an iterative method proposed by Dempster *et al* [15] that allows numerical ML estimation and enjoys, as its most appealing property, local convergence. Moreover, as opposed to other methods like gradient descent, no stepsize parameter needs to be tuned. The key observation is the fact that the optimization of (5) with respect to $\boldsymbol{\theta}$ would be much easier if we knew the transmitted sequence $\boldsymbol{x}$. We thus may form the vector with the so-called complete data $\boldsymbol{z}$:

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0}_N & \boldsymbol{I}_N \\ \sigma\boldsymbol{I}_{MN} & \boldsymbol{G} \end{bmatrix} \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{x} \end{bmatrix} \tag{8}$$

which is clearly Gaussian distributed with zero mean and covariance matrix given by

$$\boldsymbol{\Sigma}_{\boldsymbol{z}} = \operatorname{E}\{\boldsymbol{z}\boldsymbol{z}^H\} = \begin{bmatrix} \boldsymbol{I}_N & \boldsymbol{G}^H \\ \boldsymbol{G} & \sigma^2\boldsymbol{I}_{MN} + \boldsymbol{G}\boldsymbol{G}^H \end{bmatrix} \tag{9}$$

and the associated likelihood function can be written as

$$p(\boldsymbol{z};\boldsymbol{\theta}) = \frac{\exp\left\{-\boldsymbol{z}^H\boldsymbol{\Sigma}_{\boldsymbol{z}}^{-1}\boldsymbol{z}\right\}}{\pi^{N(M+1)}|\boldsymbol{\Sigma}_{\boldsymbol{z}}|}. \tag{10}$$

Given a guess $\bar{\boldsymbol{\theta}}$ for the vector of true parameters, every iteration of the EM algorithm obtains a refined estimate $\boldsymbol{\theta}_*$ as $\boldsymbol{\theta}_* = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}})$, where $Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}}) = \mathrm{E}\left\{\log p(\boldsymbol{z}|\boldsymbol{\theta})|\boldsymbol{y};\bar{\boldsymbol{\theta}}\right\}$. This procedure is repeated by taking the output $\boldsymbol{\theta}_*$ of each iteration as the input $\bar{\boldsymbol{\theta}}$ of the next one. Throughout, the notation with the bar $^-$ will be used to denote input parameter, and the notation with the asterisk $_*$ to denote output parameter. The two above operations are referred to as the expectation step (or E-step) and the maximization step (or M-step).

4.1. Expectation Step

In view of (10), it is clear that the expectation in $Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}})$ can be expanded as

$$Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}}) = -N(M+1)\log\pi - \log|\boldsymbol{\Sigma}_{\boldsymbol{z}}| - \mathrm{E}\left\{\boldsymbol{z}^H\boldsymbol{\Sigma}_{\boldsymbol{z}}^{-1}\boldsymbol{z}|\boldsymbol{y};\bar{\boldsymbol{\theta}}\right\}$$

On the other hand, using the properties of the Schur complement [16] we find that $|\boldsymbol{\Sigma}_{\boldsymbol{z}}| = \sigma^{2MN}$, and it can also be seen that the inverse of $\boldsymbol{\Sigma}_{\boldsymbol{z}}$ is given by

$$\boldsymbol{\Sigma}_{\boldsymbol{z}}^{-1} = \frac{1}{\sigma^2}\begin{bmatrix} \sigma^2\boldsymbol{I}_N + \boldsymbol{G}^H\boldsymbol{G} & -\boldsymbol{G}^H \\ -\boldsymbol{G} & \boldsymbol{I}_{MN}\end{bmatrix}. \tag{11}$$

Combining these expressions we obtain, after some algebra,

$$Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}}) \propto -MN\log\sigma^2 - \frac{\mathrm{Tr}\left(\boldsymbol{G}^H\boldsymbol{G}\boldsymbol{\Upsilon}_{\boldsymbol{x}|\boldsymbol{y}}\right) - 2\,\mathrm{Re}\left\{\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}^H\boldsymbol{G}^H\boldsymbol{y}\right\} + \boldsymbol{y}^H\boldsymbol{y}}{\sigma^2} \tag{12}$$

where $\boldsymbol{\Upsilon}_{\boldsymbol{x}|\boldsymbol{y}} = \mathrm{E}\left\{\boldsymbol{x}\boldsymbol{x}^H|\boldsymbol{y},\bar{\boldsymbol{\theta}}\right\}$ and $\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}} = \mathrm{E}\left\{\boldsymbol{x}|\boldsymbol{y},\bar{\boldsymbol{\theta}}\right\}$. Since $\boldsymbol{y}$ and $\boldsymbol{z}$ are jointly Gaussian, the values of $\boldsymbol{\Upsilon}_{\boldsymbol{x}|\boldsymbol{y}}$ and $\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}$ can be easily computed [14, Sec. 10.5]: on the one hand,

$$\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}} = \bar{\boldsymbol{G}}^H(\bar{\sigma}^2\boldsymbol{I}_{MN} + \bar{\boldsymbol{G}}\bar{\boldsymbol{G}}^H)^{-1}\boldsymbol{y}. \tag{13}$$

Applying the Matrix Inversion Lemma [16] we obtain

$$(\bar{\sigma}^2\boldsymbol{I}_M + \bar{\boldsymbol{g}}_n\bar{\boldsymbol{g}}_n^H)^{-1} = \frac{1}{\bar{\sigma}^2}\left[\boldsymbol{I}_M - \frac{\bar{\boldsymbol{g}}_n\bar{\boldsymbol{g}}_n^H}{\bar{\sigma}^2 + ||\bar{\boldsymbol{g}}_n||^2}\right] \tag{14}$$

so that (13) is also

$$\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}} = (\bar{\sigma}^2\boldsymbol{I}_N + \bar{\boldsymbol{G}}^H\bar{\boldsymbol{G}})^{-1}\bar{\boldsymbol{G}}^H\boldsymbol{y}. \tag{15}$$

For convenience, let us also define $\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}} = \mathrm{diag}\,\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}$ and

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{y}_0 & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{y}_1 & \dots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{y}_{N-1}\end{bmatrix}, \tag{16}$$

where $\boldsymbol{y}_n$ is the n-th column in $\boldsymbol{Y}$. Then, it is clear that

$$\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}} = (\bar{\sigma}^2\boldsymbol{I}_N + \bar{\boldsymbol{G}}^H\bar{\boldsymbol{G}})^{-1}\bar{\boldsymbol{G}}^H\boldsymbol{Z}. \tag{17}$$

On the other hand

$$\boldsymbol{\Upsilon}_{\boldsymbol{x}|\boldsymbol{y}} = \boldsymbol{\Sigma}_{\boldsymbol{x}|\boldsymbol{y}} + \boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}^H \tag{18}$$

where [14, Sec. 10.5]

$$\begin{aligned}\boldsymbol{\Sigma}_{\boldsymbol{x}|\boldsymbol{y}} &= \mathrm{E}\left\{(\boldsymbol{x}-\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}})(\boldsymbol{x}-\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}})^H|\boldsymbol{y},\bar{\boldsymbol{\theta}}\right\} \\ &= \boldsymbol{I}_N - \bar{\boldsymbol{G}}^H(\bar{\sigma}^2\boldsymbol{I}_{MN} + \bar{\boldsymbol{G}}\bar{\boldsymbol{G}}^H)^{-1}\bar{\boldsymbol{G}},\end{aligned} \tag{19}$$

although, using (14) again, (19) simplifies to

$$\boldsymbol{\Sigma}_{\boldsymbol{x}|\boldsymbol{y}} = \bar{\sigma}^2(\bar{\sigma}^2\boldsymbol{I}_N + \bar{\boldsymbol{G}}^H\bar{\boldsymbol{G}})^{-1}, \tag{20}$$

which is much easier to compute. Finally, since the cost function $Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}})$ only depends on the values on the diagonal of $\boldsymbol{\Upsilon}_{\boldsymbol{x}|\boldsymbol{y}}$, we define

$$\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}} = \boldsymbol{\Sigma}_{\boldsymbol{x}|\boldsymbol{y}} + \boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}^H \tag{21}$$

which is equal to $\boldsymbol{\Upsilon}_{\boldsymbol{x}|\boldsymbol{y}}$ on the diagonal and zero elsewhere.

4.2. Maximization Step

In this section we shall maximize (12) with respect to (w.r.t.) σ^2 and $\boldsymbol{G}$, subject to the constraints that $\sigma^2 \geq 0$ and $\boldsymbol{H} = \boldsymbol{C}^H\boldsymbol{F}^H$ for some $\boldsymbol{C}$. For a given $\boldsymbol{G}$, the maximizer w.r.t. σ^2 is given by

$$\sigma_*^2 = \frac{\mathrm{Tr}\left(\boldsymbol{G}^H\boldsymbol{G}\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}}\right) - 2\,\mathrm{Re}\left\{\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}^H\boldsymbol{G}^H\boldsymbol{y}\right\} + \boldsymbol{y}^H\boldsymbol{y}}{MN} \tag{22}$$

and results in

$$\max_{\sigma^2} Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}}) \propto -\mathrm{Tr}\left(\boldsymbol{G}^H\boldsymbol{G}\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}}\right) + 2\,\mathrm{Re}\left\{\boldsymbol{\mu}_{\boldsymbol{x}|\boldsymbol{y}}^H\boldsymbol{G}^H\boldsymbol{y}\right\},$$

which can be written as

$$\max_{\sigma^2} Q(\boldsymbol{\theta}|\bar{\boldsymbol{\theta}}) \propto -\sum_{m=0}^{M-1}\left[\boldsymbol{h}_m^H\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{h}_m - 2\,\mathrm{Re}\left\{\boldsymbol{r}_m^H\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}^H\boldsymbol{h}_m\right\}\right],$$

where $\boldsymbol{r}_m^H$ denotes the m-th row in $\boldsymbol{Y}$. Now, recalling that $\boldsymbol{h}_m = \boldsymbol{F}\boldsymbol{c}_m$ enables us to write the right hand side as

$$-\sum_{m=0}^{M-1}\left[\boldsymbol{c}_m^H\boldsymbol{F}^H\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{F}\boldsymbol{c}_m - 2\,\mathrm{Re}\left\{\boldsymbol{r}_m^H\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}^H\boldsymbol{F}\boldsymbol{c}_m\right\}\right]$$

Maximizing the previous expression with respect to $\boldsymbol{c}_m$ yields

$$\boldsymbol{c}_{m,*} = (\boldsymbol{F}^H\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{F})^{-1}\boldsymbol{F}^H\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{r}_m \tag{23}$$

or, alternatively,

$$\boldsymbol{C}_* = (\boldsymbol{F}^H\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{F})^{-1}\boldsymbol{F}^H\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}\boldsymbol{Y}^H. \tag{24}$$

The resulting algorithm is summarized as Algorithm 1.

Algorithm 1 Expectation-Maximization

Initialize $\bar{\sigma}^2$ and $\bar{\boldsymbol{H}}$
while stopping_criterion==FALSE **do**
• E-STEP:
$\boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}} = (\bar{\sigma}^2 \boldsymbol{I}_N + \bar{\boldsymbol{G}}^H \bar{\boldsymbol{G}})^{-1} \bar{\boldsymbol{G}}^H \boldsymbol{Z}$
$\tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}} = \bar{\sigma}^2 (\bar{\sigma}^2 \boldsymbol{I}_N + \bar{\boldsymbol{G}}^H \bar{\boldsymbol{G}})^{-1} + \boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}} \boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}^H$
• M-STEP:
$\boldsymbol{H} = \boldsymbol{Y} \boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}^H \boldsymbol{F} (\boldsymbol{F}^H \tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}} \boldsymbol{F})^{-1} \boldsymbol{F}^H$
$\delta = \operatorname{Tr}\left(\boldsymbol{G}^H \boldsymbol{G} \tilde{\boldsymbol{\Upsilon}}_{\boldsymbol{x}|\boldsymbol{y}} - 2 \operatorname{Re}\left\{ \boldsymbol{U}_{\boldsymbol{x}|\boldsymbol{y}}^H \boldsymbol{G}^H \boldsymbol{Z} \right\} + \boldsymbol{Z}^H \boldsymbol{Z} \right)$
$\sigma^2 = \delta/(MN)$
• UPDATE:
$\bar{\sigma}^2 \leftarrow \sigma^2$
$\bar{\boldsymbol{H}} \leftarrow \boldsymbol{H}$
end while

4.3. Initialization

The EM iteration described above can be initialized in a number of different ways. Fortunately, we have observed in all our experiments that the detection performance does not depend meaningfully on the particular choice among those described here. One possibility stems from the assumption that $\boldsymbol{X} = \boldsymbol{I}$. In that case, the minimum-variance unbiased estimate for $\boldsymbol{C}$ is given by $\bar{\boldsymbol{C}} = (\boldsymbol{F}^H \boldsymbol{F})^{-1} \boldsymbol{F}^H \boldsymbol{Y}^H$. A different option is to assume that the channel is time invariant, i.e., if the first basis function $\boldsymbol{f}_0$ corresponds to a constant channel, that is, $\boldsymbol{f}_0 \propto \boldsymbol{1}_N$, then the matrix $\boldsymbol{C}$ of a time invariant channel is of the form $\boldsymbol{C} = [\boldsymbol{c}_0, \boldsymbol{0}, \ldots, \boldsymbol{0}]^H$. In that case, it is known [17] that the ML estimate of $\boldsymbol{c}_0$ is given by $\hat{\boldsymbol{c}}_0 = \nu_0 \cdot \boldsymbol{v}_0$, where ν_0 is a constant depending on the trace and largest eigenvalue of the spatial sample correlation matrix $\hat{\boldsymbol{R}} = \boldsymbol{Y}\boldsymbol{Y}^H/N$, and $\boldsymbol{v}_0$ is the principal eigenvector. As an initial value for σ^2, it seems reasonable to take

$$\bar{\sigma}^2 = \max\left[\epsilon, \frac{||\boldsymbol{Y}||_F^2 - ||\boldsymbol{F}\bar{\boldsymbol{C}}||_F^2}{MN} \right], \tag{25}$$

where $||\cdot||_F$ denotes Frobenius norm and ϵ is a small positive constant, required to avoid negative variance estimates.

5. SIMULATIONS

Since analytical evaluation of the performance of the algorithm seems an extremely difficult task, in this section we resort to Monte Carlo (MC) simulation. In all the experiments here we consider that $\boldsymbol{F}$ is composed of the first $(K+1)/2$ and the last $(K-1)/2$ columns of the unitary Fourier matrix, where K is an odd integer. The reason for doing so is to equally consider positive and negative frequencies. Clearly, this corresponds to a BEM with complex exponentials. The coefficients of matrix $\boldsymbol{C}$ are generated at each MC run as realizations of independent zero-mean complex Gaussian random variables with variance α^2, where α^2 is a constant adjusted to set a given SNR. More specifically, the energy of the signal term is given by

$$\begin{aligned}
\mathrm{E}\left\{||\boldsymbol{H}\boldsymbol{X}||_F^2\right\} &= \mathrm{E}\left\{\operatorname{Tr}\left(\boldsymbol{C}^H\boldsymbol{F}^H\boldsymbol{X}\boldsymbol{X}^H\boldsymbol{F}\boldsymbol{C}\right)\right\} \\
&= \mathrm{E}\left\{\operatorname{Tr}\left(\boldsymbol{F}^H\boldsymbol{X}\boldsymbol{X}^H\boldsymbol{F}\boldsymbol{C}\boldsymbol{C}^H\right)\right\} \\
&= \alpha^2 M \operatorname{Tr}\left(\boldsymbol{F}^H\boldsymbol{I}_N\boldsymbol{F}\boldsymbol{I}_K\right) = \alpha^2 MK
\end{aligned}$$

since $\boldsymbol{X}$ and $\boldsymbol{C}$ are uncorrelated. The average SNR per antenna is thus given by

$$\frac{\mathrm{E}\left\{||\boldsymbol{H}\boldsymbol{X}||_F^2\right\}}{\mathrm{E}\left\{||\sigma\boldsymbol{W}||_F^2\right\}} = \frac{\alpha^2 MK}{\sigma^2 MN} = \frac{\alpha^2 K}{\sigma^2 N}. \tag{26}$$

Due to the mechanism used to generate $\boldsymbol{C}$, the channel $\boldsymbol{H}$ is actually complex Gaussian or, in other words, it is a Rayleigh channel. It can also be seen that the Doppler spectrum of the channel is flat, with bandwidth (i.e. Doppler spread) proportional to K/N. Moreover, in order for the results to be as general as possible, a different channel is generated for each MC iteration. Since the probability of false alarm (P_{FA}) does not depend on the channel, this fact does not affect the choice of the threshold γ and naturally averages the probability of detection (P_D) over the different realizations.

Fig. 1 shows the probability of detection *vs.* the parameter K controlling the Doppler spread of the channel. The case $K = 1$ corresponds to time-invariant channels. The results are compared with those obtained with the GLR test for time-invariant channels [17] (TI-GLRT) and with the detectors for single-antenna time-varying channels from [5]: the *Generalized Kurtosis* (GK) detector with parameter $\rho = \infty$, the *eigenvalue* (EV) detector and the *Arithmetic Mean/Geometric Mean* (AM/GM) detector. We observe that when $K > 1$ the time variation of the channel is exploited and the performance is improved with respect to the TI-GLRT detector, which does not exploit it. The gain resulting from using several antennas is also noticed. However, for $M = 1$ and the parameters chosen, the detector proposed here is not better than all the detectors from [5], highlighting the fact that no detector exists that is optimal for the whole range of unknown parameters.

Finally, Fig. 2 shows the Receiver Operating Characteristics (ROC) of the proposed detector and the TI-GLRT detector from [17], for $N = 20$ samples, $M = 4$ antennas and $K = 5$, and three different SNR values. The improvement entailed by exploiting channel time variations is clear.

6. CONCLUSIONS

A detector exploiting time variations in the channel and space correlation was derived using the GLR approach. The channel was modeled using a BEM and the signal was assumed Gaussian. Since the ML estimates required for evaluation of the GLR statistic lack closed-form expression, an EM algorithm was proposed to perform such computation. Finally, MC simulations illustrated the advantage of exploiting both temporal and spatial features of the received signal.

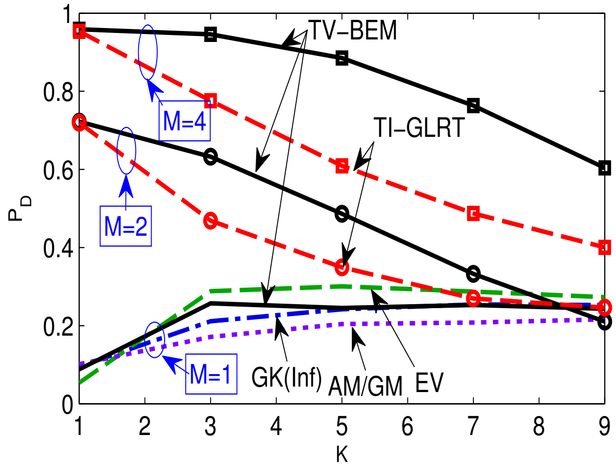

Fig. 1: P_D for fixed $P_{FA} = 0.1$ when N = 20, SNR = 0 dB

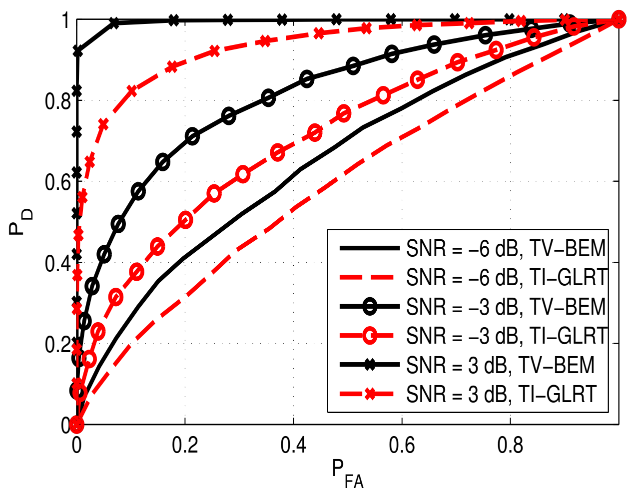

Fig. 2: Receiver Operating Characteristics for N = 20, M = 4, K = 5

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