ON THE APPLICABILITY TO CORRELATED SOURCES OF A BLIND CHANNEL EQUALIZATION METHOD ROBUST TO ORDER OVERESTIMATION

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ABSTRACT

We consider the blind equalization problem in single-user, FIR multichannel models from the second-order statistics of the channel output. In particular, we focus on the case of a correlated channel input, whose statistics are known to the receiver. The few algorithms that are able to handle colored sources usually require exact knowledge of the channel order. This is a major drawback since channel order determination is a difficult issue. Recently, Gazzah *et al.* have presented an SOS-based multichannel estimation technique which is robust to order overestimation. Although their method was derived assuming uncorrelated sources, we show that it can be suitably modified in order to handle colored channel inputs. In particular, it is shown that the algorithm is still able to blindly compute a bank of FIR pre-equalizers such that the overall channel - pre-equalizer response reduces to an FIR transfer function. This transfer function depends only on the source statistics, and in fact it is known *a priori* by the receiver, up to a complex phase rotation. Therefore a post-equalizer can be designed in a blind, straightforward manner. As expected, the method remains robust to order overmodeling in the correlated source case.

1. INTRODUCTION

Intersymbol interference (ISI) is the factor that limits performance in many digital communication systems, and hence adequate processing by an equalizer becomes necessary at the receiver. Blind channel equalization addresses those techniques that estimate the equalizer parameters based only on the channel output waveform and knowledge of the statistics of the transmitted signal. This eliminates the need for training sequences, which decrease system throughput. Second-order statistics (SOS) based methods are particularly attractive, as SOS can be more accurately estimated from finite data records than their higher-order counterparts. The seminal work in [11] showed that under certain conditions, finite impulse response (FIR) single-input multiple-output (SIMO) channels can be equalized by a bank of FIR filters, whose coefficients can be computed from the channel output SOS. Following [11], many SOS-based blind methods have been developed; see [10] and the references therein.

We consider communication systems in which the channel input statistics *are colored but known to the receiver.* Correlated sources may arise for instance as a result of channel encoding [7], or from the use of nonlinear modulation formats such as continuous-phase modulation (CPM) [9]. In either case, knowledge of the transmitter structure will provide the required source statistics to the receiver.

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Most blind equalization techniques have been designed based on the assumption of a white channel input, and therefore fail if the source is correlated. Other methods, such as the one in [8], require no knowledge of input statistics whatsoever, which clearly could be an advantage in certain situations. However, this knowledge is often available, and its use should intuitively improve performance. Among the algorithms that exploit source correlation information one finds those in [1, 4, 6]. One common drawback of these methods is that they require precise knowledge of the channel order. In practice, channel order estimation becomes a delicate task [5], and therefore blind algorithms robust to the effects of channel order overestimation are of clear interest. Recently, Gazzah *et al.* [2] have proposed an SOS-based method for channel identification which is able to accurately estimate the channel impulse response when the assumed order is greater than the exact channel order. Their method, however, was originally devised for uncorrelated sources.

The goal of this paper is to show that Gazzah's approach can be suitably modified in order to allow for colored channel inputs. Although blind channel identification seems no longer feasible, it will be shown that the technique is able to blindly determine a 'pre-equalizer' whose convolution with the unknown channel reduces to an FIR filter which depends solely on the source statistics, and most importantly, *it is known a priori up to a complex phase rotation.* Hence, a 'post-equalizer' can be easily (and blindly) designed, and its output provides the desired estimate of the channel input. Even with correlated sources, the method remains robust to channel order overmodeling.

The following notation is adopted: bold lowercase letters denote vectors, while matrices are denoted by bold uppercase or calligraphic letters. $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, denote conjugate, transpose and transpose conjugate respectively. $\mathbf{J}$ is the downshift square matrix with ones in the first subdiagonal and zeros elsewhere, $\mathbf{X}$ is the reversal square matrix with ones in the main antidiagonal and zeros elsewhere,and $\mathbf{e}_k$ is the k-th unit vector (dimensions should be clear from the context).

2. CHANNEL MODEL

The precise model to be considered is the FIR SIMO model

$$\bar{\mathbf{x}}_n = \sum_{i=0}^{l} \mathbf{h}_i a_{n-i} + \bar{\mathbf{w}}_n, \qquad (1)$$

where $\{a_n\}$ is the zero mean, wide sense stationary sequence of transmitted symbols, $\{\bar{\mathbf{x}}_n\}$ is the $p \times 1$ vector of channel outputs, $\{\bar{\mathbf{w}}_n\}$ is a $p \times 1$ zero mean white noise vector with covariance $\sigma_w^2 \mathbf{I}$, and the $p \times 1$ vectors $\{\mathbf{h}_i\}$ represent the channel impulse response; the number of subchannels is thus p. This multi-channel model may arise if multiple sensors are deployed or if the channel output is oversampled. The channel relation (1) can be recast as

$$\mathbf{x}_n = \mathcal{H}\mathbf{s}_n + \mathbf{w}_n, \tag{2}$$

where, with m the equalizer length,

$$\begin{aligned}
\mathbf{x}_n &= \begin{bmatrix} \bar{\mathbf{x}}_n^T & \bar{\mathbf{x}}_{n-1}^T & \cdots & \bar{\mathbf{x}}_{n-m+1}^T \end{bmatrix}^T, \\
\mathbf{w}_n &= \begin{bmatrix} \bar{\mathbf{w}}_n^T & \bar{\mathbf{w}}_{n-1}^T & \cdots & \bar{\mathbf{w}}_{n-m+1}^T \end{bmatrix}^T, \\
\mathbf{s}_n &= \begin{bmatrix} a_n & a_{n-1} & \cdots & a_{n-m-l+1} \end{bmatrix}^T,
\end{aligned}$$

and $\mathcal{H}$ is an $mp \times (m+l)$ generalized Sylvester matrix constructed from the channel impulse response [11]. A linear equalizer consists of an $mp \times 1$ vector $\mathbf{g}$ whose output is just

$$\mathbf{g}^H \mathbf{x}_n = \mathbf{g}^H \mathcal{H}\mathbf{s}_n + \mathbf{g}^H \mathbf{w}_n.$$

Thus the row vector $\mathbf{q}^H = \mathbf{g}^H \mathcal{H}$ contains the taps of the combined channel-equalizer impulse response. The vector $\mathbf{g}$ is said to constitute a zero-forcing (ZF) equalizer with associated delay δ if $\mathbf{q} = \mathcal{H}^H \mathbf{g}$ has a single nonzero tap at the $\delta + 1$ position.

The following standard assumption is adopted.

Assumption 1. The channel matrix $\mathcal{H}$ is tall and has full column rank. ■

For this, the equalizer length m must satisfy $mp > m + l$ (for which $p \geq 2$ is required). Then, if the transfer functions of the p available subchannels do not present any common root, a celebrated result from [11] states that $\mathcal{H}$ will have full column rank. In that case, it is readily seen that any combined channel-equalizer response $\mathbf{q}$ can be attained by suitably choosing the equalizer vector $\mathbf{g}$. In particular, ZF equalizers of delays 0 through $m + l - 1$ exist and are given by the rows of the pseudoinverse $\mathcal{H}^{\#}$.

3. PRELIMINARIES

Let us introduce

$$\mathcal{R}_x(k) = E[\mathbf{x}_n \mathbf{x}_{n-k}^H], \qquad \mathcal{R}_s(k) = E[\mathbf{s}_n \mathbf{s}_{n-k}^H],$$

as the lag k autocorrelation matrices of the channel output and the source symbols respectively. In the sequel we will assume that the noise is zero, as the noise component can be subtracted from the output autocorrelation matrices $\mathcal{R}_x(k)$ using a standard device [11]: note that

$$\mathcal{R}_x(0) = \mathcal{H}\mathcal{R}_s(0)\mathcal{H}^H + \sigma_w^2 \mathbf{I}.$$

Since it is assumed that $\mathcal{H}$ is strictly tall, the smallest eigenvalue of $\mathcal{R}_x(0)$ is seen to be σ_w^2 and therefore it can be estimated. Then we can construct the *denoised* output autocorrelation matrices

$$\tilde{\mathcal{R}}_x(k) = \mathcal{R}_x(k) - \sigma_w^2 \mathbf{J}^{kp} = \mathcal{H}\mathcal{R}_s(k)\mathcal{H}^H. \tag{3}$$

For convenience, let us define $d = m + l$. The following assumption on the source SOS is made:

Assumption 2. The $(d+1) \times (d+1)$ autocorrelation matrix of the source process

$$E\left\{ \begin{bmatrix} \mathbf{s}_n \\ a_{n-d} \end{bmatrix} \begin{bmatrix} \mathbf{s}_n^H & a_{n-d}^* \end{bmatrix} \right\} \tag{4}$$

is positive definite. ■

Following the approach in [6], we introduce the Cholesky factorization

$$\mathcal{R}_s(0) = \mathcal{L}\mathcal{L}^H, \tag{5}$$

with $\mathcal{L}$ lower triangular with positive diagonal elements (note that $\mathcal{R}_s(0) > \mathbf{0}$ due to Assumption 2) which is known to the receiver. With this, we can introduce the normalized matrices

$$\mathbf{H} = \mathcal{H}\mathcal{L}, \qquad \mathbf{R}_s(1) = \mathcal{L}^{-1}\mathcal{R}_s(1)\mathcal{L}^{-H}. \tag{6}$$

Then from (3) one has

$$\tilde{\mathcal{R}}_x(0) = \mathbf{H}\mathbf{H}^H, \qquad \tilde{\mathcal{R}}_x(1) = \mathbf{H}\mathbf{R}_s(1)\mathbf{H}^H. \tag{7}$$

4. REVIEW OF GAZZAH'S APPROACH

Gazzah's approach [2] can be briefly described as follows. Consider vectors $\mathbf{g}_1$, $\mathbf{g}_d$ satisfying

$$\tilde{\mathcal{R}}_x(1)^H \mathbf{g}_1 = \mathbf{0}, \qquad \tilde{\mathcal{R}}_x(1)\mathbf{g}_d = \mathbf{0}. \tag{8}$$

Under our assumptions, $\mathbf{H} = \mathcal{H}\mathcal{L}$ has full column rank, which equals d. Therefore, from (7), (8) yields

$$\mathbf{R}_s(1)^H \mathbf{H}^H \mathbf{g}_1 = \mathbf{0}, \qquad \mathbf{R}_s(1)\mathbf{H}^H \mathbf{g}_d = \mathbf{0}. \tag{9}$$

Observe that

$$\mathbf{H}^H \mathbf{g}_1 = \mathcal{L}^H \mathbf{q}_1, \qquad \mathbf{H}^H \mathbf{g}_d = \mathcal{L}^H \mathbf{q}_d$$

where

$$\mathbf{q}_1 = \mathcal{H}^H \mathbf{g}_1, \qquad \mathbf{q}_d = \mathcal{H}^H \mathbf{g}_d$$

are the corresponding channel-equalizer combined responses.

Now, if the source is white with variance σ_a^2, then one has

$$\mathcal{L} = \sigma_a \mathbf{I}, \qquad \mathbf{R}_s(1) = \mathbf{J}. \tag{10}$$

Therefore $\mathbf{q}_1$ and $\mathbf{q}_d$ lie in the null spaces of $\mathbf{J}^H$ and $\mathbf{J}$ respectively. We conclude that

$$\begin{aligned}
\mathbf{q}_1 &= \begin{bmatrix} c_1 & 0 & \cdots & 0 \end{bmatrix}^T = c_1 \mathbf{e}_1, && (11) \\
\mathbf{q}_d &= \begin{bmatrix} 0 & \cdots & 0 & c_d \end{bmatrix}^T = c_d \mathbf{e}_d, && (12)
\end{aligned}$$

with c_1, c_d some constants which need not be nonzero. If the zero solution could be avoided, then it is seen that $\mathbf{g}_1$, $\mathbf{g}_d$ would constitute ZF equalizers with associated delays 0 (minimal) and $d-1$ (maximal). To do so, [2] proposes to maximize the SNR at the equalizer output. Since the signal and noise terms are respectively $\mathbf{g}^H \mathcal{H}\mathbf{s}_n$ and $\mathbf{g}^H \mathbf{w}_n$, this output SNR is seen to be given by

$$\mathrm{SNR} = \frac{\mathbf{g}^H \mathcal{H}\mathcal{R}_s(0)\mathcal{H}^H \mathbf{g}}{\sigma_w^2 \mathbf{g}^H \mathbf{g}}. \tag{13}$$

Thus the problem can be cast as

$$\text{maximize } \frac{\mathbf{g}^H \tilde{\mathcal{R}}_x(0)\mathbf{g}}{\mathbf{g}^H \mathbf{g}} \text{ subject to } \left\{ \begin{array}{c} \tilde{\mathcal{R}}_x(1)^H \mathbf{g} = \mathbf{0} \\ \text{or} \\ \tilde{\mathcal{R}}_x(1)\mathbf{g} = \mathbf{0} \end{array} \right. \tag{14}$$

Observe that the null space of $\tilde{\mathcal{R}}_x(1)$ (or $\tilde{\mathcal{R}}_x(1)^H$) has dimension $mp-d+1$. Let us focus on the first constraint in (14), as the problem with the second constraint is completely analogous. Let $\mathcal{U}_1$ be an $mp\times(mp-d+1)$ matrix whose columns form an orthonormal basis of the null space of $\tilde{\mathcal{R}}_x(1)^H$. Thus, if $\tilde{\mathcal{R}}_x(1)^H\mathbf{g}=\mathbf{0}$, then $\mathbf{g}$ satisfies $\mathbf{g}=\mathcal{U}_1\mathbf{f}$ for some vector $\mathbf{f}$. Therefore, the output SNR (13) becomes

$$\mathrm{SNR}=\frac{\mathbf{f}^H\mathcal{U}_1^H\tilde{\mathcal{R}}_x(0)\mathcal{U}_1\mathbf{f}}{\sigma_w^2\mathbf{f}^H\mathbf{f}},$$

since $\mathcal{U}_1^H\mathcal{U}_1=\mathbf{I}$. The solution $\mathbf{f}_1$ maximizing this SNR is the eigenvector of $\mathcal{U}_1^H\tilde{\mathcal{R}}_x(0)\mathcal{U}_1$ corresponding to its largest eigenvalue. The resulting equalizer $\mathbf{g}_1=\mathcal{U}_1\mathbf{f}_1$ is a ZF equalizer with zero delay such that the resulting overall response is of the form (11). The constant c_1 in (11) can be determined (within a phase rotation ambiguity, inherent to the blind nature of the problem) by noting that $\mathbf{g}_1^H\tilde{\mathcal{R}}_x(0)\mathbf{g}_1=\sigma_a^2|c_1|^2$.

5. THE COLORED SOURCE CASE

In the general case, one has $\mathcal{L}\neq\sigma_a\mathbf{I}$ and $\mathbf{R}_s(1)\neq\mathbf{J}$. However, as seen in [6], these matrices present a rich structure which can be exploited. In particular, they are related by

$$\mathbf{R}_s(1)=\mathcal{L}^{-1}(\mathbf{J}-\mathbf{e}_1\boldsymbol{\alpha}^H)\mathcal{L}, \tag{15}$$

where $\boldsymbol{\alpha}=[\begin{array}{ccc}\alpha_1 & \cdots & \alpha_d\end{array}]^T$ is the coefficient vector of the d-th order forward prediction error filter (FPEF) for the source process $\{a_n\}$. It is given by

$$\boldsymbol{\alpha}=-\mathcal{R}_s(0)^{-1}\mathcal{R}_s(1)^H\mathbf{e}_1.$$

The transfer function of the d-th order FPEF is then

$$\alpha(z)=1+\sum_{k=1}^{d}\alpha_k^*z^{-k}. \tag{16}$$

Note that $\mathbf{J}-\mathbf{e}_1\boldsymbol{\alpha}^H$ is a companion matrix whose eigenvalues coincide with the zeros of $\alpha(z)$.

Now, if we require again that $\mathbf{g}_1$, $\mathbf{g}_d$ satisfy (8), we see from (9) that $\mathbf{H}^H\mathbf{g}_1$ and $\mathbf{H}^H\mathbf{g}_d$ lie in the null spaces of the matrices $\mathbf{R}_s(1)^H$ and $\mathbf{R}_s(1)$ respectively. A potentially problematic situation may arise if $\mathbf{R}_s(1)$ is nonsingular, in which case these null spaces reduce to $\{\mathbf{0}\}$. Then the corresponding equalizer output has no signal component, which is clearly undesirable. To avoid this situation, we shall make an additional assumption.

Assumption 3. The coefficient α_d is equal to zero. ■

When this holds, $\mathbf{R}_s(1)$ becomes singular in view of (15). Although this assumption is critical to the following analysis, simulations in Section 7 will show that the method does not break down when $\alpha_d\neq 0$.

Null space of $\mathbf{R}_s(1)^H$

Let us examine the null space of $\mathbf{R}_s(1)^H$ when $\alpha_d=0$:

$$\begin{aligned}\mathbf{R}_s(1)^H\mathbf{v}=\mathbf{0} \quad &\Rightarrow \quad \mathcal{L}^H(\mathbf{J}^H-\boldsymbol{\alpha}\mathbf{e}_1^H)\mathcal{L}^{-H}\mathbf{v}=\mathbf{0}\\ &\Rightarrow \quad (\mathbf{J}^H-\boldsymbol{\alpha}\mathbf{e}_1^H)\mathcal{L}^{-H}\mathbf{v}=\mathbf{0}\\ &\Rightarrow \quad \mathcal{L}^{-H}\mathbf{v}=c_1[\begin{array}{cccc}1 & \alpha_1 & \cdots & \alpha_{d-1}\end{array}]^T\end{aligned}$$

where the last line follows from Assumption 3, and c_1 is a constant. Thus we conclude that if $\tilde{\mathcal{R}}_x(1)^H\mathbf{g}_1=\mathbf{0}$, then

$$\mathcal{L}^{-H}\mathbf{H}^H\mathbf{g}_1=\mathcal{H}^H\mathbf{g}_1=c_1[\begin{array}{cccc}1 & \alpha_1 & \cdots & \alpha_{d-1}\end{array}]^T. \tag{17}$$

Hence the overall channel-equalizer response $\mathbf{q}_1^H=\mathbf{g}_1^H\mathcal{H}$ reduces to a multiple of the FPEF response, whose transfer function is $\alpha(z)$ in (16).

Null space of $\mathbf{R}_s(1)$

On the other hand, if a vector $\mathbf{v}$ lies in the null space of $\mathbf{R}_s(1)$ under Assumption 3, then

$$\begin{aligned}\mathbf{R}_s(1)\mathbf{v}=\mathbf{0} \quad &\Rightarrow \quad \mathcal{L}^{-1}(\mathbf{J}-\mathbf{e}_1\boldsymbol{\alpha}^H)\mathcal{L}\mathbf{v}=\mathbf{0}\\ &\Rightarrow \quad (\mathbf{J}-\mathbf{e}_1\boldsymbol{\alpha}^H)\mathcal{L}\mathbf{v}=\mathbf{0}\\ &\Rightarrow \quad \mathcal{L}\mathbf{v}=c_d\mathbf{e}_d\\ &\Rightarrow \quad \mathbf{v}=c_d\beta_0\mathbf{e}_d.\end{aligned}$$

The third line follows now from the fact that the null space of a singular $d\times d$ companion matrix is the span of the vector $\mathbf{e}_d$, and the last one is a consequence of $\mathcal{L}^{-1}$ being lower triangular; β_0 denotes the (d,d) entry of $\mathcal{L}^{-1}$. Again, c_d is an unknown constant.

Hence, if $\tilde{\mathcal{R}}_x(1)\mathbf{g}_d=\mathbf{0}$, then

$$\mathbf{H}^H\mathbf{g}_d=c_d\beta_0\mathbf{e}_d \qquad \Rightarrow \qquad \mathcal{H}^H\mathbf{g}_d=c_d\beta_0\mathcal{L}^{-H}\mathbf{e}_d. \tag{18}$$

Thus, in this case the overall channel-equalizer response $\mathbf{q}_d^H=\mathbf{g}_d^H\mathcal{H}$ is proportional to the last row of the inverse Cholesky factor $\mathcal{L}^{-1}$. It is known from linear prediction theory [3] that this row contains the coefficients of the FPEF of order $d-1$ for the process $\{a_n\}$. Specifically, if we denote

$$\begin{aligned}\mathbf{e}_d^H\mathcal{L}^{-1} &= [\begin{array}{cccc}\beta_{d-1} & \cdots & \beta_1 & \beta_0\end{array}]\\ &= \boldsymbol{\beta}^T,\end{aligned} \tag{19}$$

then the transfer function of the $(d-1)$-th order FPEF is given by

$$\beta(z)=1+\sum_{k=1}^{d-1}\frac{\beta_k^*}{\beta_0}z^{-k} \tag{20}$$

(note that β_0 is positive real). However, due to the order-update property of prediction filters [3], under Assumption 3 it turns out that the FPEFs of orders d and $d-1$ coincide, i.e.

$$\alpha_k=\frac{\beta_k}{\beta_0}, \quad k=1,\ldots,d-1 \quad \Rightarrow \quad \alpha(z)=\beta(z). \tag{21}$$

Observe, however, that the β_k coefficients in (19) appear in reverse order. Hence, the transfer function of the combined response $\mathbf{q}_d^H=\mathbf{g}_d^H\mathcal{H}$ is proportional to $z^{-(d-1)}\alpha^*(1/z^*)$. This, of course, is the transfer function of the *backward* prediction error filter (BPEF) for the process $\{a_n\}$.

The magnitudes of the constants c_1, c_d can be easily obtained. First, from (18) it follows that

$$\mathbf{g}_d^H\tilde{\mathcal{R}}_x(0)\mathbf{g}_d=\mathbf{g}_d^H\mathbf{H}\mathbf{H}^H\mathbf{g}_d=|c_d|^2\beta_0^2. \tag{22}$$

Obtaining $|c_1|^2$ is slightly more involved, but the approach is the same. Noting that the vector in the right-hand side of (17) is just $\beta_0^{-1}\mathbf{X}\boldsymbol{\beta}$, one has

$$\begin{aligned}\mathbf{g}_1^H\tilde{\mathcal{R}}_x(0)\mathbf{g}_1 &= \mathbf{g}_1^H\mathbf{H}\mathbf{H}^H\mathbf{g}_1\\ &= \frac{|c_1|^2}{\beta_0^2}||\mathcal{L}^H\mathbf{X}\boldsymbol{\beta}||^2.\end{aligned} \tag{23}$$

One can show that the vector $\mathcal{L}^H \mathbf{X}\boldsymbol{\beta}$ has unit norm:

$$\begin{aligned}\boldsymbol{\beta}^H \mathbf{X}^H \mathcal{L}\mathcal{L}^H \mathbf{X}\boldsymbol{\beta} &= \boldsymbol{\beta}^H \mathbf{X}^H \mathcal{R}_s(0)\mathbf{X}\boldsymbol{\beta} \\ &= \boldsymbol{\beta}^H \mathcal{R}_s(0)^* \boldsymbol{\beta} \\ &= \boldsymbol{\beta}^H \mathcal{L}^* \mathcal{L}^T \boldsymbol{\beta} \\ &= \mathbf{e}_d^H \mathbf{e}_d \\ &= 1,\end{aligned}$$

where the second line follows from the fact that $\mathbf{X}\mathcal{R}_s(0) = \mathcal{R}_s(0)^* \mathbf{X}$ because $\mathcal{R}_s(0)$ is Hermitian Toeplitz, and the fourth from the definition of $\boldsymbol{\beta}$ in (19). Therefore, one has $|c_1|^2 = \beta_0^2(\mathbf{g}_1^H \tilde{\mathcal{R}}_x(0)\mathbf{g}_1)$.

In both cases ($\mathbf{g}_1$ or $\mathbf{g}_d$) an output SNR maximization approach can be undertaken exactly in the same manner as in Section 4, in order to avoid the zero solution.

6. POST-EQUALIZATION

In the preceding section we have shown that in the colored source case, and under Assumption 3, the equalizers chosen from left and right null vectors of the lag-1 denoised autocorrelation matrix $\tilde{\mathcal{R}}_x(1)$ of the channel output result in combined responses that are proportional, respectively, to the forward and backward prediction error filters for the source process. The FPEF and BPEF are determined by the source statistics alone, and therefore they are available to the receiver. As a consequence, by virtue of the 'pre-equalizers' $\mathbf{g}_1$, $\mathbf{g}_d$, the (unknown) SIMO channel is reduced to a known (up to a scaling) SISO one. This means that a 'post-equalizer' can be designed. Several approaches seem possible:

1. By using the *two* pre-equalizers $\mathbf{g}_1$, $\mathbf{g}_d$, a single-input double-output channel would be obtained, the FPEF and BPEF being the two subchannels. These two subchannels do not have common zeros, since Assumption 3 guarantees the FPEF's transfer function to be minimum phase, while the BPEF's will be maximum phase[1]. Hence, a double-input single-output ZF post-equalizer could be readily designed.

   The problem with this approach is that the residual phase ambiguities in each subchannel (cf. the constants c_1, c_d in (17) and (18)) are independent, which precludes direct design of the post-equalizer. It is possible, in principle, to find the relative phase difference between the two subchannels by noting that

   $$\mathbf{g}_1^H \tilde{\mathcal{R}}_x(0)\mathbf{g}_d = (c_1^* c_d)(\mathbf{e}_1^H \mathcal{R}_s(0)\mathbf{e}_d). \qquad (24)$$

   However, in most practical cases the lag-$(d-1)$ autocorrelation coefficient of the source, $\mathbf{e}_1^H \mathcal{R}_s(0)\mathbf{e}_d$, will have very small magnitude. This will make estimation of $c_1^* c_d$ via (24) numerically troublesome.

2. Another option is to use a single pre-equalizer ($\mathbf{g}_1$ or $\mathbf{g}_d$), and then design a Minimum Mean Squared Error (MMSE) FIR postequalizer. This design can be carried out because the combined response ($\mathbf{g}_1$ or $\mathbf{g}_d$) is known in advance, together with the autocorrelation of the source sequence. Since it is assumed that the noise vector at the pre-equalizer input is white with covariance σ_w^2, the autocorrelation function of the noise at its output is also known: the lag k autocorrelation coefficient is given by $\sigma_w^2(\mathbf{g}^H \mathbf{J}^{kp}\mathbf{g})$.

3. The simplest approach follows from noting that the transfer function $\alpha(z)$ of the FPEF is minimum phase, and therefore it can be stably and causally inverted. This suggests using $\mathbf{g}_1$ as pre-equalizer and an all-pole filter $1/\alpha(z)$ as post-equalizer. This has the advantage of computational simplicity over the MMSE approach, which requires computation and inversion of the covariance matrix of the pre-equalizer output. On the other hand, this straightfoward design does not take noise into account, so that noise enhancement problems could appear at low SNRs.

[1] In fact, having $|\alpha_d| < 1$ is enough for these to hold.

Using this last approach, the algorith can be summarized as follows:

1. Estimate $\mathcal{R}_x(0)$, $\mathcal{R}_x(1)$ from the received data.
2. Estimate σ_w^2 as the smallest eigenvalue of $\mathcal{R}_x(0)$, and construct the denoised matrices $\tilde{\mathcal{R}}_x(0)$, $\tilde{\mathcal{R}}_x(1)$ in (3).
3. Perform a Singular Value Decomposition (SVD) of $\tilde{\mathcal{R}}_x(1)$ and retain in the columns of $\mathcal{U}_1$ the $mp-d+1$ left singular vectors corresponding to the $mp-d+1$ smallest singular values.
4. Compute $\mathbf{f}_1$ as the unit-norm eigenvector of $\mathcal{U}_1^H \tilde{\mathcal{R}}_x(0)\mathcal{U}_1$ corresponding to its largest eigenvalue $\lambda_{\max}$, and let $\mathbf{g}_1 = \mathcal{U}_1 \mathbf{f}_1$. Also let $c_1 = \beta_0\sqrt{\lambda_{\max}}$.
5. Obtain the pre-equalizer output $y_n = (\mathbf{g}_1^H \mathbf{x}_n)/c_1$.
6. Estimate the source data via $\hat{a}_n = [1/\alpha(z)]y_n$.

This extension of Gazzah's algorithm remains robust to channel order overmodeling. Note that the only point in the algorithm above at which the (estimated) channel lenght l intervenes is step 3 (since $d = m + l$). If l is overestimated, then fewer singular vectors will be retained in $\mathcal{U}_1$. However, the resulting equalizer will still perform acceptably (except in the highly unlikely case in which all the singular vectors retained happen to be orthogonal to the signal subspace), although some degradation is expected.

7. SIMULATION RESULTS

In the simulations, a correlated QPSK source $\{a_n\}$ was used. The generation process, which was chosen for its simplicity and with illustrative purposes, is as follows:

$$a_n = \begin{cases} -1 + j & \text{if} \quad (b_n \, b_{n-2}) = (0\,0) \\ +1 + j & \text{if} \quad (b_n \, b_{n-2}) = (0\,1) \\ -1 - j & \text{if} \quad (b_n \, b_{n-2}) = (1\,0) \\ +1 - j & \text{if} \quad (b_n \, b_{n-2}) = (1\,1) \end{cases}$$

where $\{b_n\}$ is the input stream of i.i.d. bits ($b_n = 0$ or 1 with equal probability). The resulting autocorrelation coefficients are

$$E[a_n a_{n-k}^*] = \begin{cases} 2, & k = 0, \\ \pm j, & k = \pm 2, \\ 0, & \text{else.} \end{cases}$$

It can be checked that the resulting coefficients α_k of the order d FPEF are, for $d = 2q$ or $d = 2q + 1$,

$$\alpha_k = \begin{cases} 0, & k \text{ odd}, \\ \left(1 - \frac{k/2}{q+1}\right) j^{k/2}, & k \text{ even}. \end{cases}$$

Note that Assumption 3, i.e. $\alpha_d = 0$, is satisfied for odd d, but not for d even.

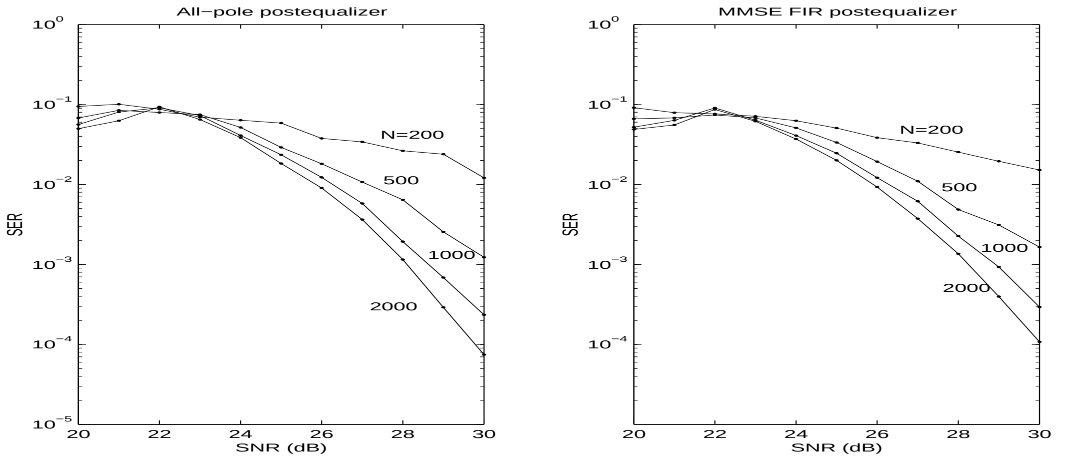

Figure 1: SER vs. SNR. Pre-equalizer length $m = 10$, assumed channel order $l = 3$.

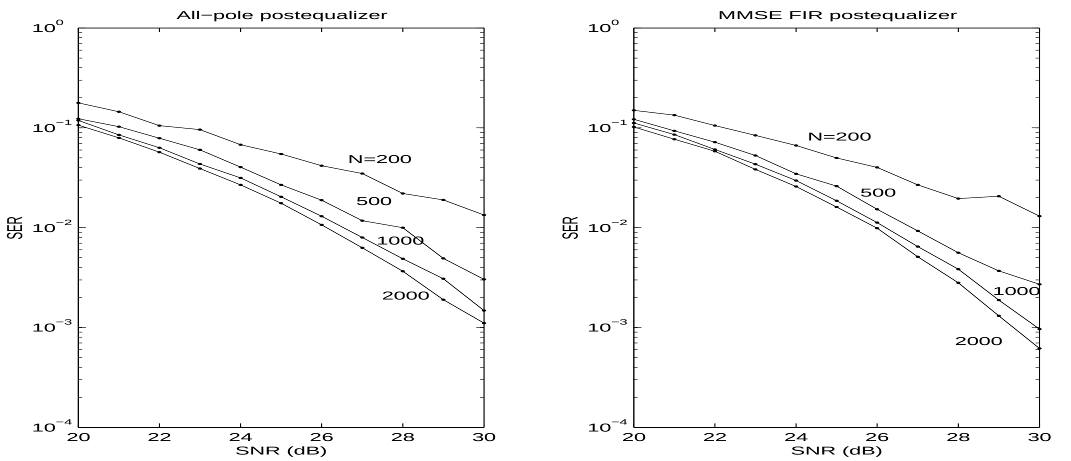

Figure 2: SER vs. SNR. Pre-equalizer length $m = 5$, assumed channel order $l = 3$.

Consider the following $p = 2$ subchannels with order $l = 3$ and coefficients $[\ \mathbf{h}_0 \quad \mathbf{h}_1 \quad \mathbf{h}_2 \quad \mathbf{h}_3\ ] =$

$$\begin{bmatrix} 0.3 & 0.4 - j0.1 & 0.1 + j0.2 & -0.2 + j0.6 \\ -0.2 - j0.1 & 0.5 - j0.2 & -0.5 - j0.1 & 0.4 + j0.2 \end{bmatrix}$$

We computed the pre-equalizer $\mathbf{g}_1$ by the method described in Section 6 for several choices of the parameters. For performance comparison purposes, the residual phase ambiguity is removed before evaluating the symbol error rate (SER) for each case. This evaluation was made using 10^5 symbols and averaging the results over 100 independent trials.

Fig. 1 shows the symbol error rate obtained with the proposed method. The vector $\mathbf{g}_1$ was computed and used as pre-equalizer, using several record lengths ($N = 200$, 500, 1000 and 2000 received samples per subchannel) for the estimation of the autocorrelation matrices. The pre-equalizer order was $m = 10$, and $l = 3$ (correct channel order) was assumed. Observe that for these values, $d = m + l = 13$ is odd, so that Assumption 3 is satisfied.

The first set of curves in Fig. 1 were computed by using an all-pole post-equalizer $1/\alpha(z)$, while an FIR post-equalization filter designed under the MMSE criterion was used for the second set (the value of σ_w^2 was estimated as in step 2 of the pre-equalizer computation algorithm in Section 6). For a fair comparison, the same number of taps was taken for the FIR and IIR post-equalizers. Since it is known *a priori* that the overall channel - pre-equalizer transfer function is minimum phase, an associated delay of zero can be taken in the FIR post-equalizer design without performance loss.

The results obtained with the two approaches are quite similar, which makes the ‘quick and dirty’ all-pole post-equalizer design more attractive given its smaller computational load. In both cases, performance improves as more samples are used for pre-equalizer computation, as expected. These effects are also evident in Fig. 2, which shows the results obtained under the same conditions, the only difference being that the pre-equalizer order has been reduced to $m = 5$. In that case, one has $d = m + l = 8$ which is even, so that Assumption 3 is violated (one has $\alpha_d = 0.2$). Still, significant reduction of the SER is observed, so that the algorithm seems to be robust to this effect.

Finally, Fig. 3 compares the SER obtained with exact and overestimated channel orders, for the two cases of a short ($m = 5$) and long ($m = 10$) pre-equalizers, both with an all-pole post-

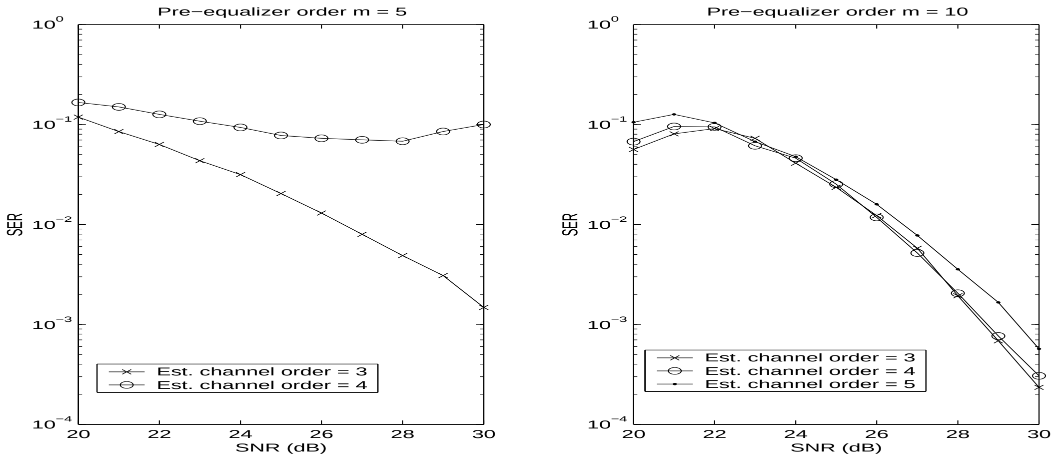

Figure 3: SER vs. SNR. All-pole post-equalizers, $N = 1000$ symbols used for estimation.

equalizer. It is seen that with a short pre-equalizer, performance degrades when the channel order is overestimated, although the scheme is still able to equalize the channel, i.e. it does not break down. If a longer pre-equalizer is used, channel order overestimation does not translate into performance degradation. This is as expected, since the dimension of the subspace from which the pre-equalizer is extracted is $m(p-1) - l + 1$, which grows linearly with m.

8. CONCLUSIONS

We have shown how the algorithm of Gazzah *et al.* [2] for blind identification of FIR SIMO channels can be appropiately modified in order to account for source correlation (assuming that the source statistics are known to the receiver). More precisely, although channel identification is no longer feasible, the method is still able to blindly obtain pre-equalizers whose convolution with the channel is known to the receiver (up to a phase rotation). Therefore, design of a post-equalizer is straightforward. The algorithm uses second-order statistics only of the observed signal, and it preserves the appealing property of the original method [2] of being robust to channel order overmodeling. The computational complexity of the proposed extension is comparable to that of the original approach [2].

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