DISTRIBUTED SPECTRUM SENSING WITH MULTIANTENNA SENSORS UNDER CALIBRATION ERRORS

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ABSTRACT

Spectrum sensing design for Cognitive Radio systems is challenged by the nature of the wireless medium, which makes the detection requirements difficult to achieve by standalone sensors. To combat shadowing and fading, distributed strategies are usually proposed. However, most distributed approaches are based on the energy detector, which is not robust to noise uncertainty. This phenomenon can be overcome by multiantenna sensors exploiting spatial independence of the noise process. We combine both ideas to develop distributed detectors for multiantenna sensors. Fusion rules are provided for sensors based on the Generalized Likelihood Ratio as well as for *ad hoc* detectors derived from geometric considerations. Simulation results are provided comparing the performance of the different strategies under lognormal shadowing and Ricean fading.

Index Terms— Cognitive radio, spectrum sensing, distributed detection, fading channels.

1. INTRODUCTION

Spectrum sensing in Cognitive Radio is a critical issue which drastically affects the achievable throughput of the secondary network [1]. To satisfy the detection requirements imposed by regulatory bodies to protect primary users from interference, the detectors should operate at very low Signal-to-Noise Ratio (SNR) conditions, and an efficient solution in terms of sensing time, false alarm rate and control overhead must be provided.

Due to its simplicity, the most popular approach to primary detection is the *energy detector* (ED) [2]. Since practical environments demand some robustness under channel impairments such as shadowing that a standalone sensor cannot achieve, several distributed rules based on ED have been proposed [3, 4]. However, ED performance is very sensitive to uncertainty about the power of the background noise [5]. This motivates standalone multiantenna sensors that overcome this problem by exploiting spatial whiteness of the noise [6–11].

Combining both ideas, we derive distributed detectors based on multiantenna sensors, in order to mitigate the detrimental effects of both noise uncertainty and shadowing.

In distributed settings, the information collected by each sensor is sent to a fusion center (FC) that makes the final decision. With *decision-fusion* rules, which minimize the required bandwidth for control traffic, only one-bit local decisions are sent to the FC. Deriving the optimal thresholds for the local and global decisions is difficult, and a popular suboptimal scheme is the OR rule. In *data-fusion* schemes, a locally computed statistic is sent to the FC, where all of them are somehow combined and the result is then compared against a threshold. These strategies typically outperform decision-fusion ones, at the expense of larger bandwidths.

Most standalone multiantenna detectors [6,7,10] assume that the unknown noise power is the same at all antennas. In practice this assumption may not hold true, due to differences in the electronics of the analog front-ends. A few multiantenna detectors robust to this effect have been derived [8,9,11]. In a distributed scheme, different sensors will experience different (and unknown) noise levels, which makes it necessary to cope with both inter-antenna and inter-sensor noise uncertainties.

We focus on two kinds of standalone multiantenna detectors as the basis of the distributed spectrum sensing network, respectively assuming calibrated and uncalibrated frontends. In the first class are the detectors derived from the Generalized Likelihood Ratio (GLR) approach [2] and require the computation of the largest eigenvalues of the sample spatial covariance or coherence matrices. Assuming independence of the observations at different sensors, the data-fusion rules for these GLR detectors are easily found. The detectors from the second class measure the angle between these matrices and the identity or a generic diagonal matrix, respectively, and have the advantage of not requiring eigenvalue extraction. This geometric approach to the design of these detectors also provides the means to design the corresponding data-fusion rules.

The paper is organized as follows. After setting the problem in Sec. 2, standalone multiantenna detectors are briefly summarized in Sec. 3. Extensions to distributed settings are given in Sec. 4. Simulation results under lognormal shadow-

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ing and Ricean fading are given in Sec. 5. Finally, conclusions are drawn in Sec. 6.

2. SIGNAL MODEL

Consider a network of N spectrum sensors, in which sensor n is equipped with M_n antennas. The observed signal in a given frequency channel is downconverted to baseband and I/Q sampled. No synchronization with the primary signal is assumed, as this would be unrealistic in practical settings (detectors must operate under SNR conditions well below decodability levels in order to overcome the hidden node problem). Sensor n gathers K_n samples per antenna in the sensing window allocated for the current channel, collected in the $K_n \times M_n$ complex-valued data matrix \mathbf{Y}_n :

$$Y_n = x_n h_n^H + W_n \Sigma_n, \qquad 1 \le n \le N.$$
 (1)

 \boldsymbol{x}_n is a $K_n \times 1$ vector with the samples of the primary signal at sensor n, assumed zero-mean and normalized to unit variance. \boldsymbol{h}_n^* is an $M_n \times 1$ vector with the channel gains from the primary transmitter to the antennas of sensor n. \boldsymbol{W}_n is a $K_n \times M_n$ matrix of noise samples, assumed zero-mean Gaussian, temporally and spatially white: $E\{[\boldsymbol{W}_n]_{ki}[\boldsymbol{W}_n]_{lj}^*\} = \delta_{kl}\delta_{ij}$. The $M_n \times M_n$ diagonal matrix $\boldsymbol{\Sigma}_n^2$ collects the variances of the noises at each antenna.

The primary signal is modeled as circularly symmetric and temporally white Gaussian process. (This Gaussian assumption leads to tractable models and useful detectors. In addition, multicarrier signals can be accurately modeled as Gaussian if the number of subcarriers is reasonably large). The channel is assumed to be frequency-flat and to remain constant within the sensing interval. The frequency channel under scrutiny is idle iff $h_n = \mathbf{0}$ for all n (representing a transmission opportunity for the secondary network) and busy otherwise.

Under this model, the observations at sensor n are temporally white and Gaussian, with pdf given by

$$f_n(\mathbf{Y}_n; \mathbf{R}_n) = \left[\frac{1}{\pi \det \mathbf{R}_n} \exp\{-\operatorname{Tr}(\mathbf{R}_n^{-1} \hat{\mathbf{R}}_n)\} \right]^{K_n}, \quad (2)$$

where $\hat{\boldsymbol{R}}_n \doteq \frac{1}{K_n} \boldsymbol{Y}_n^H \boldsymbol{Y}_n$ is the sample spatial covariance matrix, and $\boldsymbol{R}_n \doteq E\{\hat{\boldsymbol{R}}_n\} = \boldsymbol{\Sigma}_n^2 + \boldsymbol{h}_n \boldsymbol{h}_n^H$ is the true covariance.

3. STANDALONE MULTIANTENNA DETECTORS

We briefly summarize four multiantenna detectors for non-collaborative settings: sensor n makes a local decision based on its data, according to $T_n^{(i)} \geqslant_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma$, where $T_n^{(i)}$ is the corresponding statistic and γ is a threshold. A more comprehensive review of multiantenna detectors can be found in [8].

3.1. Detectors for calibrated receivers

These detectors assume that the noise variance, though unknown, is the same at the M_n antennas, i.e. $\Sigma_n^2 = \sigma_n^2 I$.

Mean/max eigenvalue detector [7]: This test is derived from a Generalized Likelihood Ratio (GLR) approach for this hypothesis testing problem:

$$\mathcal{H}_0: \mathbf{R}_n = \sigma_n^2 \mathbf{I}, \qquad \mathcal{H}_1: \mathbf{R}_n = \sigma_n^2 \mathbf{I} + \mathbf{h}_n \mathbf{h}_n^H, \quad (3)$$

where both σ_n^2 and h_n are unknown. The GLR test statistic is given by

$$T_{n} \doteq \log \frac{\max_{\boldsymbol{h},\sigma^{2}} f_{n}(\boldsymbol{Y}_{n};\boldsymbol{h},\sigma^{2} \mid \mathcal{H}_{1})}{\max_{\sigma^{2}} f_{n}(\boldsymbol{Y}_{n};\sigma^{2} \mid \mathcal{H}_{0})}$$
(4)

$$= 2K_n \log \frac{(M_n - 1)^{M_n - 1} \mu_n^{M_n}}{(M_n \mu_n - 1)^{M_n - 1}}, \tag{5}$$

where

$$\mu_n \doteq \frac{\frac{1}{M_n} \operatorname{Tr} \hat{\boldsymbol{R}}_n}{\lambda_1(\hat{\boldsymbol{R}}_n)},\tag{6}$$

and $\lambda_m(\mathbf{A})$ represents the m-th largest eigenvalue of \mathbf{A} . Note that (5) is a monotonic function of μ_n , which is the ratio of the mean of the eigenvalues of $\hat{\mathbf{R}}_n$ to the largest one.

Correlation-identity detector [8]: This *ad hoc* test does not require eigenvalue computations. Its statistic is the inverse of the squared correlation coefficient between \hat{R}_n and a scaled identity $\alpha^2 I$, under the standard inner product in matrix space $\langle A, B \rangle \doteq \text{Tr}(B^H A)$:

$$T_n \doteq \frac{\langle \hat{\boldsymbol{R}}_n, \hat{\boldsymbol{R}}_n \rangle \cdot \langle \alpha^2 \boldsymbol{I}, \alpha^2 \boldsymbol{I} \rangle}{|\langle \hat{\boldsymbol{R}}_n, \alpha^2 \boldsymbol{I} \rangle|^2} = \frac{M_n \operatorname{Tr}(\hat{\boldsymbol{R}}_n^H \hat{\boldsymbol{R}}_n)}{\operatorname{Tr}^2(\hat{\boldsymbol{R}}_n)}, \quad (7)$$

which is independent of α^2 .

3.2. Detectors for uncalibrated receivers

These schemes do not assume the same noise power across all antennas. Thus, Σ_n^2 is an unknown diagonal matrix.

 λ_1 **detector** [8]: The GLR test applied to the problem

$$\mathcal{H}_0: \mathbf{R}_n = \mathbf{\Sigma}_n^2, \qquad \mathcal{H}_1: \mathbf{R}_n = \mathbf{\Sigma}_n^2 + \mathbf{h}_n \mathbf{h}_n^H, \quad (8)$$

with Σ_n^2 diagonal, and Σ_n^2 , h_n unknown, results in the following statistic as the SNR goes to zero:

$$T_n \approx 2K_n[-1 + \lambda_1(\hat{\boldsymbol{C}}_n) - \log \lambda_1(\hat{\boldsymbol{C}}_n)], \tag{9}$$

where the spatial coherence matrix \hat{C}_n is defined as follows. Let $\hat{D}_n = \operatorname{diag}(\hat{R}_n)$ be a diagonal matrix retaining the diagonal of matrix \hat{R}_n . Then

$$\hat{C}_n \doteq \hat{D}_n^{-1/2} \hat{R}_n \hat{D}_n^{-1/2}. \tag{10}$$

Correlation-diagonal detector: Following the same philosophy as with the derivation of (7), a test statistic could be

obtained from the (inverse squared) correlation coefficient between \hat{R}_n and a generic diagonal matrix D (rather than a scaled identity). However, in this case such coefficient depends on the particular reference matrix D. One possibility is to take as reference the diagonal matrix maximizing the correlation coefficient, i.e.

$$\tilde{D}_n \doteq \arg\max_{D} \frac{|\langle \hat{R}_n, D \rangle|^2}{\langle \hat{R}_n, \hat{R}_n \rangle \langle D, D \rangle}.$$
 (11)

It is readily seen that the maximum is achieved when $\tilde{D}_n = \operatorname{diag}(\hat{R}_n) = \hat{D}_n$ (up to an irrelevant scaling). The resulting statistic is therefore

$$T_n \doteq \frac{\operatorname{Tr}(\hat{R}_n^H \hat{R}_n)}{\operatorname{Tr}(\hat{D}_n^H \hat{D}_n)},\tag{12}$$

and the test turns out to be a particular case of the family of "covariance-based" detectors from [9].

4. DISTRIBUTED DATA-FUSION RULES

In a heterogeneous distributed setting, different sensors may have different number of antennas M_n , collect different number of samples K_n , and be affected by different amounts of noise. Their listening intervals need not be perfectly time-synchronized, and propagation delays from the primary transmitter to each sensor will be different in general. These considerations motivate a model in which the observations $\{Y_n\}$ at different sensors are regarded as independent. Note that attempting to exploit any potential correlation between observations at different sensors would require transmitting those data to the FC, which is clearly impractical and undesirable.

4.1. Fusion rule for GLR detectors

Under the independence assumption on the observed data at different sensors, it is clear that the joint pdf becomes

$$f(\mathcal{Y}; \mathcal{R}) = \prod_{n=1}^{N} f_n(\mathbf{Y}_n; \mathbf{R}_n), \tag{13}$$

where $\mathcal{Y}=\{\boldsymbol{Y}_n\}_{n=1}^N$ and $\mathcal{R}=\{\boldsymbol{R}_n\}_{n=1}^N$. The log-GLR statistic is given by:

$$T = \log \frac{\max_{\mathcal{R}} f(\mathcal{Y}; \mathcal{R} \mid \mathcal{H}_1)}{\max_{\mathcal{R}} f(\mathcal{Y}; \mathcal{R} \mid \mathcal{H}_0)}.$$
 (14)

Due to the factorization in (13),

$$T = \sum_{n=1}^{N} \log \frac{\max_{\boldsymbol{R}} f_n(\boldsymbol{Y}_n; \boldsymbol{R} \mid \mathcal{H}_1)}{\max_{\boldsymbol{R}} f_n(\boldsymbol{Y}_n; \boldsymbol{R} \mid \mathcal{H}_0)} = \sum_{n=1}^{N} T_n.$$
 (15)

Therefore, the global GLR detector only requires the transmission of the local statistics T_n to the FC. Note that this

fusion rule applies also to other GLR-based detectors, such as the *sphericity test* [10] and the *Hadamard ratio test* [11], which assume that the spatial covariance matrix under \mathcal{H}_1 is unstructured, and were derived for calibrated and uncalibrated frontends respectively.

4.2. Fusion rule for ad hoc detectors

The *ad hoc* standalone detectors based on the statistics from (7) and (12) are not GLR tests, and thus it is not obvious in principle which fusion rules to use. A possible approach is to consider the global block-diagonal sample covariance matrix

$$\hat{\boldsymbol{R}} \doteq \begin{bmatrix} g(K_1)\hat{\boldsymbol{R}}_1 & & & \\ & \ddots & & \\ & & g(K_N)\hat{\boldsymbol{R}}_N \end{bmatrix}, \qquad (16)$$

in which each block \hat{R}_n has been weighted by a monotonically increasing function g of the number of samples K_n used at sensor n, in order to take into account the different estimation accuracies across sensors. Now, if calibrated sensors are assumed, one may consider the correlation coefficient between \hat{R} and a reference block diagonal matrix

$$\boldsymbol{D} = \begin{bmatrix} \sigma_1^2 \boldsymbol{I}_{M_1} & & & \\ & \ddots & & \\ & & \sigma_N^2 \boldsymbol{I}_{M_N} \end{bmatrix} . \tag{17}$$

Maximizing this coefficient w.r.t. $\sigma_1^2, \ldots, \sigma_N^2$, the following test statistic is obtained:

$$T = \frac{\sum_{n=1}^{N} g^{2}(K_{n}) \operatorname{Tr}(\hat{R}_{n}^{H} \hat{R}_{n})}{\sum_{n=1}^{N} \frac{g^{2}(K_{n})}{M_{n}} \operatorname{Tr}^{2}(\hat{R}_{n})},$$
(18)

which reduces to (7) if N=1. In the case of uncalibrated sensors, one should maximize the correlation coefficient between \hat{R} and a generic diagonal matrix. The corresponding test statistic thus obtained is:

$$T = \frac{\sum_{n=1}^{N} g^{2}(K_{n}) \operatorname{Tr}(\hat{\mathbf{R}}_{n}^{H} \hat{\mathbf{R}}_{n})}{\sum_{n=1}^{N} g^{2}(K_{n}) \operatorname{Tr}(\hat{\mathbf{D}}_{n}^{H} \hat{\mathbf{D}}_{n})},$$
(19)

which reduces to (12) for N=1. Note that with the fusion rules (18)-(19), each sensor must transmit two quantities to the FC.

5. SIMULATION RESULTS

The motivation for distributed spectrum sensing is its potential resilience against shadowing; with multiantenna sensors, robustness against fast fading can also be expected. To evaluate these schemes, we consider both effects in the model for the channel coefficient vector of sensor n:

$$\boldsymbol{h}_n = u_n \cdot \left(\sqrt{\frac{\kappa_r}{1 + \kappa_r}} \bar{\boldsymbol{h}}_n + \sqrt{\frac{1}{1 + \kappa_r}} \tilde{\boldsymbol{h}}_n \right), \tag{20}$$

where:

- u_n is a log-normally distributed scalar random variable modeling slow fading (log-normal shadowing);
- $\bar{h}_n = [1 \ e^{j\theta_n} \ \cdots \ e^{j(M_n-1)\theta_n}]^T$, with $\theta_n \sim \mathcal{U}(0,\pi)$ modeling the relative phase of the signal at the antennas of a uniform linear array;
- $\tilde{\boldsymbol{h}}_n$ is an $M_n \times 1$ zero-mean complex Gaussian vector, independent of u_n and θ_n , with $E\{\tilde{\boldsymbol{h}}_n\tilde{\boldsymbol{h}}_n^H\} = \boldsymbol{I}_{M_n}$.

In this way, \bar{h}_n accounts for the line-of-sight (LOS) component, whereas \tilde{h}_n models the scattering (Rayleigh contribution). κ_r is the Rice factor, i.e., the ratio between LOS and scattered powers. Independent Rayleigh fading is justified for sufficiently separated antennas at the sensors [12].

It is assumed that all sensors are at roughly the same distance from the primary transmitter, so that the mean pathloss is the same for all of them, and $20\log_{10}u_n$ is normally distributed with dB-spread $\sigma_{\rm dB}$. Sensors are assumed sufficiently far away from each other, so that they experience independent shadowing, i.e. the variables $\{u_n\}_{n=1}^N$ are uncorrelated¹. Note that, from (20), $E\{\boldsymbol{h}_n\boldsymbol{h}_n^H\}=E\{|u_n|^2\}\boldsymbol{I}_{M_n}$. Therefore, with $\sigma_{n,m}^2$ the noise power at the m-th antenna of sensor n, the average SNR at that antenna is $E\{|u_n|^2\}/\sigma_{n,m}^2$. For simplicity, all sensors have similar characteristics; we fix $M_n=M=4$ antennas per sensor, $K_n=K=128$ samples, $\sigma_{\rm dB}=3$ dB, and $P_{\rm FA}=0.15$ throughout.

Fig. 1 shows the variation of the average probability of miss $P_{\rm MD}$ with the number of sensors N, for the different detectors considered, and for both fusion strategies: data-fusion (DF), as derived in Sec. 4; and decision-fusion or hard combining, as exemplified by the OR (i.e. "1-out-of-N") rule with equal thresholds, as commonly used in practice [3]. The Rice factor is $\kappa_r=0$ dB, i.e. the LOS and NLOS channel components have equal weight. In all cases $P_{\rm MD}$ decreases exponentially with N, and the decay rate is faster for the Mean/Max and Correlation-identity detectors, which exploit more efficiently the data model since there are no noise mismatches in this case. On the other hand, the OR versions exhibit a significant performance loss with respect to their data-fusion counterparts, as could be expected.

Next we consider the effect of noise mismatches in the antennas of the different sensors. These are modeled by drawing the values of the noise powers, when measured in dB, from a normal distribution with the same mean values as in the preceding experiment and with standard deviation $\sigma_{\rm NP}=0.5$ dB. Such a model takes into account the multiple sources of noise that are present in a given branch (antenna, thermal, quantization, etc.) which can result in noise power variations. It is seen in Fig. 2 that the Mean/Max and Correlation-identity detectors degrade considerably with respect to the case of uniform noise powers. On the other hand, the performance of the λ_1 and Correlation-diagonal detectors does not change appre-

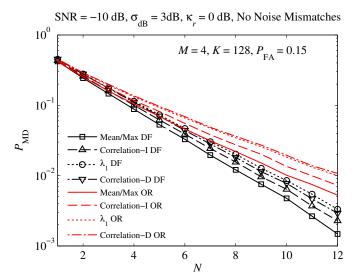


Fig. 1. Probability of miss vs. number of sensors.

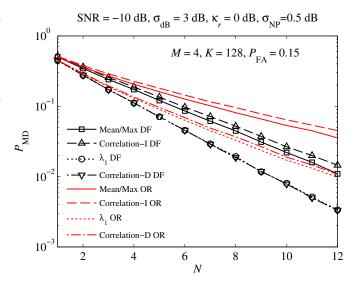


Fig. 2. Influence of noise mismatches on $P_{\rm MD}$.

ciably. This is true for both the data-fusion and OR versions of the distributed detectors.

Lastly, we study the influence of the Rice factor on distributed detector performance. In Fig. 3 we fix the number of sensors at N=8, and no noise mismatches are introduced. In an NLOS environment ($\kappa_r \to 0$) the Mean/Max and Correlation-identity detectors outperform the λ_1 and Correlation-diagonal schemes, as expected. However, this situation is reversed as $\kappa_r \to \infty$, i.e. in settings in which a strong LOS component is present (such as e.g. rural environments): surprisingly, the performance of the λ_1 and Correlation-diagonal detectors improves as κ_r increases, eventually outperforming the Mean/Max and Correlation-

¹Correlated shadowing reduces spatial diversity and is expected to degrade the performance of the distributed detectors [13].

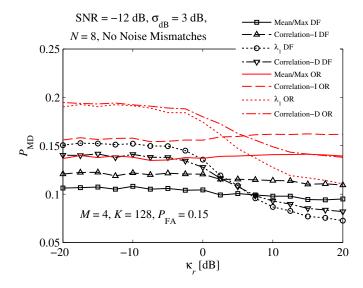


Fig. 3. Probability of miss vs. Rice factor.

identity detectors even though the noise power is the same at each antenna of any given sensor. The performance of the last two schemes is seen to be almost independent of κ_r . Another interesting observation is that in NLOS scenarios, the performance of the ad hoc Correlation-diagonal detector is better than (for data-fusion) or equal to (for the OR rule) that of the GLR-based λ_1 detector, which fares better in LOS environments.

6. CONCLUSION

We studied the problem of devising data-fusion strategies for distributed spectrum sensing with multiantenna devices. Adopting a Gaussian model, four standalone detectors were considered, for the case of calibrated as well as uncalibrated receivers. Two of these were obtained from a GLR perspective, whereas the other two were derived from geometric considerations and constitute low complexity alternatives, as they do not require eigenvalue computations. These schemes have the potential to counteract noise uncertainty as well as slow and fast fading. The data-fusion methods presented outperform other decision-fusion schemes, such as the OR rule, which in turn requires less communication bandwidth. The performance of the detectors derived under the assumption of uniform noise power across all antennas of any given sensor substantially degrades in the presence of noise mismatches. Besides being robust to this phenomenon, the detectors derived for uncalibrated receivers turn out to outperform the other two in strong LOS environments, even in the absence of noise mismatches. Future work will be devoted to the statistical analysis of the schemes discussed here, as well as to the derivation of detectors exploiting the structure of the Rice channel model.

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