SINR Optimization in Wideband Full-Duplex MIMO Relays under Limited Dynamic Range

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Abstract—We develop a method for self-interference mitigation in wideband full-duplex multiple-input multiple-output regenerative relays, taking into account transmitter impairments and limited receiver dynamic range. The method combines a cancellation and suppression architecture by incorporating feedforward filters at both sides of the relay. The design criterion is to maximize the signal-to-interference-plus-noise ratio (SINR) at the relay input. On the transmit side, linear constraints are imposed in order to control the effect on the information signal at destination. Simulation results show the effectiveness of the proposed method both in terms of self-interference reduction and SINR improvement.

Index Terms—full-duplex, MIMO, regenerative relay, relaying, self-interference, cancellation, suppression.

I. INTRODUCTION

Achieving full-duplex communication in a relay network requires each element of the network to be carefully designed. At the relay, simultaneous transmission and reception in the same frequency results in self-interference distortion. This distortion can be tens of dB higher than the information signal, which can prompt an inadmissible interference level [1]–[5].

To overcome this problem, passive physical isolation between transmit and receive antenna arrays as well as analog cancellation are provided during the relay design. Those techniques are able to attenuate the self-interference in approximately 60-70 dB [1], [6], which is usually insufficient for optimal relay performance. Residual interference reduces the available dynamic range of the information signal, and consequently, the SINR. Further mitigation of the self-interference is attained in the relay, after analog-to-digital conversion, by using digital cancellation [7], [8]. However, the performance of digital-domain cancellation, in which a digital estimate of the interference signal is subtracted from the relay input, is fundamentally limited by the aforementioned reduction of the dynamic range due to the residual self-interference [2].

We present a design for wideband full-duplex regenerative MIMO relays which outperforms baseline cancellation techniques by introducing additional feedforward filters at the receive and transmit sides of the relay [9]. To maximize the SINR, the transmit filter is designed to reduce the self-interference signal before digital conversion, while the receive filter is designed to mitigate other noise sources. In contrast to other works, our SINR maximization design takes into account nonlinear distortion and transmission impairments at the relay transmit side and limited dynamic range at the relay receive side using the noise model presented in [2].

II. SYSTEM MODEL

We consider a full-duplex MIMO relay link consisting of a source node (S) with \( M_s \) antennas, a destination node (D) with \( M_r \) antennas and a relay (R) with \( N_r \) receive and \( N_t \) transmit antennas, respectively. Node \( S \) transmits signal \( s_i[n] \), \( D \) receives signal \( d_i[n] \), and \( R \) receives \( r_i[n] \) while simultaneously transmitting \( r_t[n] \). The number of independent streams transmitted by \( S \) and \( R \) are \( m_s \) and \( m_r \), respectively. The received signals at \( R \) and \( D \) are given by

\[
\begin{align*}
\mathbf{r}_r[n] &= \mathbf{H}_{sr}[n] \star \mathbf{s}_r[n] + \mathbf{H}_{rr}[n] \star \mathbf{r}_r[n] + \mathbf{n}_r[n] \\
\mathbf{d}_r[n] &= \mathbf{H}_{ar}[n] \star \mathbf{s}_r[n] + \mathbf{H}_{dr}[n] \star \mathbf{r}_r[n] + \mathbf{n}_d[n]
\end{align*}
\]

where operator \( \star \) denotes convolution and \( \mathbf{H}_{ij}[n], i \in \{s,r\} \) and \( j \in \{r,d\} \), is the channel impulse response matrix, of order \( L_{ij} \), between nodes \( i \) and \( j \). Vectors \( \mathbf{n}_d[n] \) and \( \mathbf{n}_r[n] \) denote the noise components at \( D \) and \( R \), respectively, with \( \mathbf{n}_r[n] \) containing the receiver input noise, the transmitter noise that couples back through \( \mathbf{H}_{rr}[n] \), and the noise due to limited dynamic range of the receiver, i.e.,

\[
\mathbf{n}_r[n] = \mathbf{n}_i[n] + \mathbf{H}_{rr}[n] \star \mathbf{v}_r[n] + \mathbf{v}_r[n]
\]

where the input noise \( \mathbf{n}_i[n] \sim \mathcal{CN}(0,\sigma^2 \mathbf{I}) \), and the transmitter noise, denoted by \( \mathbf{v}_r[n] \), is statistically independent of \( \mathbf{r}_r[n] \), temporally white and \( \mathbf{v}_r[n] \sim \mathcal{CN}(0,\delta \mathbf{I} \mathbf{E} \{\mathbf{r}_r[n] \mathbf{r}_r^H[n]\}) \), with \( 0 < \delta \ll 1 \), models transmitter imperfections [2]. Limited receiver dynamic range is modeled by injecting a noise-like signal \( \mathbf{v}_r[n] \), which is statistically independent of \( \mathbf{r}_r[n] \), temporally white and \( \mathbf{v}_r[n] \sim \mathcal{CN}(0,\gamma \mathbf{I} \mathbf{E} \{\mathbf{r}_r[n] \mathbf{r}_r^H[n]\}) \), with \( 0 < \gamma \ll 1 \) and \( \mathbf{r}_r[n] = \mathbf{r}_r[n] - \mathbf{v}_r[n] \) the signal before digital conversion [2].

The relay implements a decode-and-forward protocol, which will introduce enough processing delay to assume that samples of the received and transmitted signals are uncorrelated, i.e.,

\[
\mathbf{E}\{\mathbf{r}_r[n] \mathbf{r}_r^H[n-k]\} = \mathbf{E}\{\mathbf{r}_r[n] \mathbf{n}_r^H[n-k]\} = 0, \quad k \geq 0
\]

with \( \mathbf{r}_r[n] = \mathbf{H}_{rr}[n] \star \mathbf{s}_r[n] \) the information signal arriving at \( R \) from \( S \) (cf. Fig. 1). We define SINR at the relay input as

\[
\text{SINR}_R = \frac{\mathbf{E}\{||\mathbf{r}_r[n]||^2\}}{\mathbf{E}\{||\mathbf{i}_r[n] + \mathbf{n}_r[n]||^2\}}
\]

where the self-interference signal, \( \mathbf{i}_r[n] = \mathbf{H}_{rr}[n] \star \mathbf{r}_t[n] \) (cf. Fig. 1), constitutes the major source of distortion at \( R \) [1]–[5]. Moreover, when \( \mathbf{H}_{rr}[n] \) has high gain and \( \mathbf{r}_t[n] \) is weak, \( \mathbf{v}_r[n] \) has a significant impact on the performance [9].
Next we present the cancellation-suppression architecture, whereby we aim at maximizing the post-processing SINR. We assume that channel state information (CSI) of $H_{rr}[n]$, $H_{rd}[n]$ and $H_{rt}[n]$ as well as $\gamma$, $\delta$ and $\sigma^2$ are available at $\mathcal{R}$.

### A. Architecture

Figure 1 depicts the relay node incorporating the interference mitigation scheme, which consists of the following components: the $L_r$-th order feedforward filter $G_r[n]$ of size $m_r \times N_r$, the $L_t$-th order feedforward filter $G_t[n]$ of size $N_t \times m_t$, and the $L_a$-th order cancellation filter $A[n]$ of size $N_r \times N_t$. In the relay, with $A[n]$ takes place first, whereas filtering with $G_r[n]$ is subsequently performed. After being processed by $p_r(\cdot)$ and before analog conversion, the information signal, $r_t[n]$, is filtered with $G_t[n]$. Function $p_r(\cdot)$, see Fig. 1, aggregates all the operations associated with the relay protocol, such as demodulation, equalization and data decoding.

The post-processing SINR is measured at the input of the $p_r(\cdot)$ block, and is therefore

$$\text{SINR}_{eq} = \frac{\mathbb{E}\{\|r_t^{eq}[n]\|^2\}}{\mathbb{E}\{\|r_r^{eq}[n] + n_r^{eq}[n]\|^2\}} \tag{5}$$

where $r_t^{eq}[n] = G_t[n] \ast r_t[n]$, $r_r^{eq}[n] = G_r[n] \ast (H_{rr}[n] + A[n]) \ast G_t[n] \ast r_t[n]$ and $n_r^{eq}[n] = n_r[n]$. The relay performance depends directly on (5), thus filters $G_t[n]$, $G_r[n]$ and $A[n]$ should be designed in order to maximize (5):

$$\text{maximize } A[n], G_t[n], G_r[n] \text{ subject to } \mathbb{E}\{\|r_t[n]\|^2\} \leq P_{\text{max}} \tag{6}$$

with $P_{\text{max}} > 0$ the maximum transmit power. In view of (5), the optimum value of $A[n]$ is $A[n] = -H_{rr}[n]$, which renders $r_r^{eq}[n] = 0$ and requires that $L_a \geq L_{rr}$, i.e., the order of $A[n]$ should be, at least, equal to the order of $H_{rr}[n]$. The optimization of $G_t[n]$ and $G_r[n]$ is discussed in the following sections.

### B. Maximization of $\text{SINR}_{eq}$ with respect to $G_t[n]$

When $A[n] = -H_{rr}[n]$, the residual self-interference $i_r^{eq}[n]$ vanishes, and the only dependence of (5) with $G_t[n]$ is through $n_r^{eq}[n]$. Therefore, problem (6) amounts to minimizing $\mathbb{E}\{\|n_r^{eq}[n]\|^2\}$, which also depends on $G_r[n]$, so designing both filters is a coupled problem. To circumvent this, and since $n_r^{eq}[n]$ is a function of both $i_r[n]$ and $v_r[n]$, we slightly modify the optimization criterion in (6). Concretely, $G_t[n]$ is designed as the solution to the following optimization problem

$$\text{minimize } G_t[n] \text{ subject to } \mathbb{E}\{\|r_t[n]\|^2\} \leq P_{\text{max}} \tag{7}$$

$$\text{minimize } G_t[n] \text{ subject to } \mathbb{E}\{\|r_t[n]\|^2\} \leq P_{\text{max}} \tag{8}$$

The design criterion in (7) aims to minimizing self-interference and transmitter noise arriving at $\mathcal{R}$. As a result of that, not only is (7) decoupled from $G_r[n]$ but it is also formulated as a quadratic minimization problem, allowing us to use ordinary convex optimization techniques.

The inequality constraint in (7), i.e., $\mathbb{E}\{\|r_t[n]\|^2\} \leq P_{\text{max}}$, limits the maximum transmitted power to $P_{\text{max}} > 0$, which results in $G_t[n] = 0$ or disruption of the $\mathcal{R}$-$D$ link, i.e., $r_t[n] = 0$, if no additional constraints are imposed. In order to preclude trivial solutions and avoid excessive distortion in the received signal at $D$, we introduce the linear equality constraints $H_{rd}[n] \ast G_t[n] = H_{rd}^{eq}[n]$, where $H_{rd}^{eq}[n]$ denotes the target $(L_t + L_{rd})$-th order channel between $\mathcal{R}$ and $D$. The constraints will ensure that the information signal at $D$ undergoes the controlled and predictable distortion specified in $H_{rd}^{eq}[n]$, which is typically selected based on precoding or equalization techniques [10], [11].

By introducing the vector $\mathbf{g}_t = \text{vec}(G_t[0] \ldots G_t[L_t])$, which stacks the columns of $G_t[n]$ into a vector of size $m_r N_t (L_t + 1)$, problem (7) is reformulated as

$$\text{minimize } \mathbf{g}_t^H (\mathbf{P}_t + \mathbf{R}_t) \mathbf{g}_t \text{ subject to } \mathbf{H}_{rd} \mathbf{g}_t = \mathbf{h}_{rd}^{eq} \tag{9}$$

where matrices $\mathbf{P}_t$ and $\mathbf{R}_t$ are easily obtained by expressing $\mathbb{E}\{\|r_t[n]\|^2\}$ and $\mathbb{E}\{\|r_r[n] + n_r[n]\|^2\}$ in terms of $\mathbf{g}_t$, respectively. The linear constraints $\mathbf{H}_{rd} \mathbf{g}_t = \mathbf{h}_{rd}^{eq}$ are expressed by $H_{rd}[n] \ast G_t[n] = H_{rd}^{eq}[n]$, while $\mathbf{H}_{rd} \mathbf{g}_t$, which depends on the autocorrelation of $r_t[n]$, is obtained by expressing $\mathbb{E}\{\|r_t[n]\|^2\}$ in terms of $\mathbf{g}_t$.

In order to solve (8), note first that the degrees of freedom in $\mathbf{g}_t$ should be sufficiently large so as to allow for a feasible linear equality constraint in (8). Noting that $\mathbf{H}_{rd}$ is of size $M_r m_r (L_t + L_{rd} + 1) \times m_r N_t (L_t + 1)$, this means that we must have $m_r N_t (L_t + 1) > \text{rank}(\mathbf{H}_{rd})$, or, in general, $N_t > M_r$ and $L_t > (M_r L_{rd} / (N_t - M_r)) - 1$. Therefore, the relay should have more transmit antennas than $D$ and the required order of $G_t[n]$ grows linearly with the order of $H_{rd}[n]$. All the possible
g_t belong to the manifold given by

\[ g_t = \mathbf{H}_{rd}^{(eq)} h_{rd}^{(eq)} + Nw \]  

(9)

with w is an arbitrary vector of size \( m_r N_r (L_t + 1) - \text{rank}\{ \mathbf{H}_{rd} \} \), \# denotes pseudo-inversion and N represents the basis of the nullspace of \( \mathbf{H}_{rd} \).

The problem in (8) is reduced to finding the vector w minimizing \( g_t^H (P_t + \mathbf{R}_t) g_t \) subject to \( g_t^H \tilde{R} g_t \leq P_{\text{max}} \). Using the square root factorization \( P_t + \mathbf{R}_t = \mathbf{Q}^H \mathbf{Q} \), and \( \mathbf{R} = \mathbf{S}^H \mathbf{S} \), we rewrite (7) as

\[
\begin{align*}
\text{minimize} & \quad \| \mathbf{A} \mathbf{w} - \mathbf{b} \|^2 \\
\text{subject to} & \quad \| \mathbf{C} \mathbf{w} - \mathbf{d} \|^2 \leq P_{\text{max}}
\end{align*}
\]  

(10)

where matrices \( \mathbf{A} = \mathbf{Q} \mathbf{N} \), \( \mathbf{C} = \mathbf{S} \mathbf{N} \) and vectors \( \mathbf{b} = -\mathbf{Q} \mathbf{H}_{rd}^H h_{rd}^{(eq)} \), \( \mathbf{d} = -\mathbf{S} \mathbf{H}_{rd}^H h_{rd}^{(eq)} \).

The problem in (10) has the form of a standard least-squares problem with a quadratic inequality constraint which can be solved semi-analytically by diagonalizing both \( \mathbf{A} \) and \( \mathbf{B} \) using their generalized singular value decomposition [12]. After the optimal \( \mathbf{w} \) is computed, \( \mathbf{G}_r[n] \) can be recovered by unvectoring the \( g_t \) computed via (9).

C. Maximization of SINR_{\text{ave}} with respect to \( \mathbf{G}_r[n] \)

With \( \mathbf{A}[n] \), \( \mathbf{G}_r[n] \) designed as above, \( \mathbf{r}_r[n] \) consists of the information signal \( \mathbf{r}_t[n] \), and the noise signal \( \mathbf{n}_r[n] \). Consequently, see (6), the filter \( \mathbf{G}_r[n] \) is the solution to the following problem

\[
\begin{align*}
\text{maximize} & \quad \frac{\mathbb{E}\{\|\mathbf{r}_r^{(eq)}[n]\|^2\}}{\mathbb{E}\{\|\mathbf{n}_r^{(eq)}[n]\|^2\}} \\
\text{subject to} & \quad \| \mathbf{g}_r^H \mathbf{P}_t \mathbf{g}_r \|_2 \leq \rho \| \mathbf{L}^t \|_2 \max
\end{align*}
\]  

(11)

where, as in Sec. III-B, we have substituted \( \mathbf{G}_r[n] \) with \( \mathbf{g}_r = \text{vec}\{ \mathbf{G}_r[0] \ldots \mathbf{G}_r[L_t] \} \) of size \( m_s N_r (L_t + 1) \). Note that the term \( \mathbb{E}\{\|\mathbf{n}_r^{(eq)}[n]\|^2\} \) depends on the covariance of \( \mathbf{r}_r[n], \mathbf{n}_r[n] \) and \( \mathbf{I}_r[n] \), therefore on \( \mathbf{g}_r \), which means that \( g_t \) needs to be computed first. Finally, matrices \( \mathbf{P}_r \) and \( \mathbf{P}_n \) are obtained respectively by expressing \( \mathbf{r}_r^{(eq)}[n] \) and \( \mathbf{n}_r^{(eq)}[n] \) in terms of \( \mathbf{g}_r \).

Problem (11) is recognized as a generalized eigenvalue problem [12]. Using the square root factorization \( \mathbf{P}_n = \mathbf{L}^H \mathbf{L} \), its solution is given by

\[ \mathbf{g}_r = \rho \mathbf{L}^{-1} \mathbf{v}_{\text{max}} \]  

(12)

where \( \mathbf{v}_{\text{max}} \) is the principal eigenvector of \( \mathbf{L}^{-H} \mathbf{P}_r \mathbf{L}^{-1} \) and \( \rho \) is an arbitrary constant.

IV. SIMULATION RESULTS AND DISCUSSION

Next we present numerical results from simulations of a relay link with the following characteristics: \( \mathcal{S} \) and \( \mathcal{D} \) transmit, respectively, \( m_s = 2 \) and \( m_r = 2 \) independent streams, while \( M_t = m_s \) and \( M_r = m_r \). The relay protocol \( p_t() \) regenerates the received signal using the same modulation scheme as \( \mathcal{S} \). Each stream consists of a 64-QAM modulated OFDM signal with 8192 subcarriers and a normalized cyclic prefix length of 1/4. The oversampling factor is 2, i.e., the data signal covers approximately half of the available bandwidth. Channels \( \mathbf{H}_{sr}[n], \mathbf{H}_{eq}[n] \) and \( \mathbf{H}_{rr}[n] \) are drawn from a complex Gaussian distribution, have orders \( L_{sr} = L_{rd} = L_{rr} = 2 \) and gains of 0 dB, 0 dB and 30 dB, respectively. Additionally, \( \mathbb{E}\{\|\mathbf{s}_t[n]\|^2\} = 0 \) dB and \( \mathbb{E}\{\|\mathbf{r}_t[n]\|^2\} = 0 \) dB, while \( L_a = L_{rr}, L_t = L_r = 2, \) and \( P_{\text{max}} = 20 \) dB. The target channel is

\[ \mathbf{H}_{rd}^{(eq)}[n] = \begin{cases} 
\mathbf{I}, & n = 0 \\
\mathbf{0}, & n \neq 0
\end{cases} \]  

(13)

Note that with (13) and if no direct link between \( \mathcal{S} \) and \( \mathcal{D} \) exists, i.e., \( \mathbf{H}_{sr}[n] = \mathbf{0} \), no channel equalization is needed at \( \mathcal{D} \). Finally, \( \sigma^2 = -20 \) dB, while parameters \( N_t, N_r, \delta \) and \( \gamma \) are varied across simulations.

Fig. 2 shows the self-interference isolation as a function of the transmitter noise level for different antenna configurations. Isolation is defined as \( P_{\text{iso}} = \mathbb{E}\{\|\mathbf{I}_{ref}[n]\|^2\}/\mathbb{E}\{\|\mathbf{I}_r[n]\|^2\} \), where \( \mathbf{I}_{ref}[n] \) is obtained by using the reference system \( \mathbf{G}_t[n] = \mu \mathbf{1} \).
with $L_t = 0$ and $1$ an all-ones matrix of size $N_t \times m_r$, i.e., $G_t[n]$ equally distributes the data streams over the different antennas. Constant $\mu$ matches the transmitted power of the reference system to the power of the actual system. In view of Fig. 2, the achieved isolation roughly depends on the relay transmission redundancy $N_t/N_r$, achieving higher isolation for larger values of this ratio. When $N_t/N_r \approx 1$ or less, isolation tends to $0$ dB, because $G_t[n]$ does not have enough degrees of freedom. On the other hand, when the redundancy is large, e.g., $N_t = 8$, $G_t[n]$ has enough degrees of freedom to mitigate the self-interference up to several tens of dB. Therefore, in terms of isolation, having more transmit antennas than receive antennas is preferable while designing the relay. Additional isolation can be achieved by increasing the order of $G_t[n]$, which is denoted by $L_t$. Fig. 3 represents the additional isolation in terms of $L_t$ and $N_r$ when $N_t = 4$, i.e., values above $0$ dB represent additional isolation with respect to the case of $L_t = 2$. Larger values of $L_t/N_r$ provide better additional isolation, e.g., up to $10$ dB for $L_t = 6$ and $N_r = 2$.

Fig. 4 shows the SINR improvement, $\text{SNIR}_{req}/\text{SNIR}_R$, in terms of $\gamma$, i.e., the receiver dynamic range for different antenna configurations. To compute $\text{SNIR}_R$ we use the reference system and $\delta = -30$ dB. We can approximately distinguish two cases based on the receiver dynamic range: wide range for $\gamma \in (-45, -30)$ dB, and narrow range for $\gamma \in (-30, -15)$ dB. When the receiver has wide dynamic range, the noise due to the self-interference is relatively low and, consequently, the isolation level is not critical. In fact, as seen in Fig. 4, the best performance is achieved with a large number of receive antennas, see case $N_r = 6$. The performance improves when the number of transmit antennas is reduced, and, hence, the receiver redundancy $N_r/N_t$ is a good indicator of the achievable performance. On the other hand, when the receiver has narrow dynamic range, the noise caused by the self-interference is relatively high, and larger isolation is required to obtain a good performance. As a result, configurations with a larger number of transmit antennas, see cases $N_t = \{6, 8\}$, perform better. Thus, in contrast to the wide dynamic range case, a large number of transmit antennas is preferable while designing the relay.

Note that due to the tradeoff between noise and self-interference, different configurations can result in the same performance. For example, when $\gamma \approx -35$ dB, configurations with $N_r = 6$ yield about the same SINR.

V. Conclusions

A method for SINR maximization in wideband full-duplex MIMO relays under limited dynamic range has been presented, making use of a combined cancellation-suppression architecture incorporating feedforward filters at both sides of the relay together with a feedback filter. Each of these three elements is designed by solving a convex optimization problem. On the transmit side, additional linear constraints are imposed to limit the distortion in the information signal at the destination. Results from simulations show that the method is able to reduce the self-interference at $R$ up to several tens of decibels, and the overall SINR improvement is, for a typical example case, over $40$ dB.

REFERENCES


