Set-Membership Identification of Resampled Signals

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***Abstract*—The problem of resampling factor estimation as a means for tampering detection has been largely investigated. Most of the existing techniques rely on the analysis of cyclic correlations induced in the resampled signal. However, in this paper, a new direction is explored by addressing the same problem in terms of the set-membership estimation theory. The proposed technique constructs a model of the problem using available a priori knowledge and in consonance with a finite number of observations that comes from the resampled signal under study. With this information, the proposed technique is able to provide an estimate of the resampling factor applied to the original signal and, if required, an estimate of such signal and an estimate of the interpolation filter. The performance in terms of accuracy and MSE of the proposed approach is evaluated and comparative results with state-of-the-art methods are reported.**

I. Introduction

Multimedia contents, such as digital images, audio or video, have become the most extensively used vehicle for communication during last years. The massive proliferation of these digital contents over the Internet, across all of the media or through social networks has converted them on an appreciated and valuable asset. At the same time, the rapid growth of editing tools that enable an unskilled person to easily manipulate any of these multimedia contents, has boosted an important concern about their authenticity. In particular, when a multimedia file is used as a proof of facts in a legal proceeding, it is imperative to know its origin and also to be able to trace back the processing history of its content, in order to justify whether the file can be admitted as a legal evidence.

In the past few years a number of techniques have been developed to verify the authenticity or integrity of multimedia contents in a blind way, i.e., without using any known signal like a digital watermark. These tools rely on the analysis of traces left by the capturing device during the acquisition process or any other operation applied after its creation, such as compression and/or edition. These traces, also known as digital footprints, have been broadly investigated in the case of images [1], and increasing attention is given to audio [2], and video [3]. Therefore, nowadays, a forensic analyst can find a considerably large set of tools to determine the processing history of a multimedia content.

One of the widely known methodologies to detect forgeries on multimedia contents consists in the analysis of the resampling factor of small portions across the whole content, which should be constant if no manipulation has been performed [4-8]. For instance, when an image splicing is carried out, it is very likely that one of the pasted regions has been transformed geometrically to adapt the content to the scene, thus introducing an inconsistency on the resampling factor in that part of the image. In the same way, when two audio signals with different sampling rates are mixed, then at least the sampling rate of one of them must be adjusted in order to avoid audible distortions.

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The proposed techniques in [4-8] work remarkably well when uncompressed signals are used, but the corresponding detectors can be easily deluded when a post-processing or simply a lossy-compression is applied to their content, as it is described in [9]. Furthermore, all these approaches are based on the study of the periodic correlation that is inherently induced in the resulting signals after applying a resampling operation. The main drawbacks of the frequency analysis for resampling factor estimation were pointed out in [10]. Despite the unavoidable ambiguity in the identification of the resampling factor due to frequency aliasing, the main issues are: 1) a considerably large number of samples are necessary to circumvent the windowing effect in the frequency domain; 2) the presence of periodic patterns in the content usually leads to a wrong detection or estimation. By relying on the rounding operation applied after resampling, the estimator derived in [10] is able to sort out these problems. However, its applicability is quite limited since only a fixed linear interpolation filter is considered through the definition of the estimator.

To overcome these deficiencies and pursuing the idea behind the work in [10], which gave important insights about how to perform resampling factor estimation, a new approach for the identification of resampled signals is provided in this paper. The procedure derived in [10], where a vector of observations coming from a linearly resampled signal is tested against a set of plausible resampling factors to find the correct one, is able to quickly discard the tested resampling factors that lead to an empty feasible set for the original signal. This formulation of the problem can be linked to the set-membership estimation theory (a.k.a., set-theoretic estimation), which is well known in the field of automatic control and also in certain signal processing areas [11,12].

Set-membership estimation is commanded by the concept of feasibility and provides solutions whose singular characteristic is to be consistent with all information arising from the observed data and the a priori knowledge about the problem to solve. As it was stated above, frequency-based methods cannot always provide reliable solutions. Indeed, such solutions could infringe known constraints about the problem. However, when the problem is approached in set-membership terms, the provided solution will be consistent with all the known constraints, according to the observed data. This is very important from the point of view of a forensic analyst that must always provide objective judgements on the identification of forgeries, basing his decision on evidences, i.e., on the observed data, and on the prior knowledge about the problem under analysis.

To this extent, by relying on the set-membership theory and generalizing the work carried out in [10] to any interpolation filter and also to a wider range of resampling factors, we propose a new methodology for resampling factor estimation.

The structure of the paper is as follows: Section II describes the formulation of the problem in mathematical terms and following the set-membership framework; Section III introduces a practical implementation to solve the derived problem; Section IV shows the experimental results obtained under different settings; and finally, Section V concludes the paper and future lines of work are pointed out.

II. PROBLEM FORMULATION

Before introducing the set-membership formulation, the description of all the steps involved in the sampling rate conversion by a factor ξ of a 1-D signal will be presented. Note that we will only focus the analysis on 1-D signals to keep the definition of the problem more tractable, but the 2-D extension can be straightforwardly obtained. The following notational conventions will be used along the paper: boldface capital letters will denote matrices, while boldface lowercase letters will represent column vectors. Non-boldface letters will refer to scalar variables and, finally, calligraphic letters will be only used for denoting sets.

Let $\mathbf{x}^{(0)}$ be a column vector that contains $N_x^{(0)}$ samples from the original signal before being resampled. The applied resampling factor is defined as $\xi \triangleq \frac{L}{M}$, i.e., the ratio between the upsampling factor $L \in \mathbb{N}^+$ and downsampling factor $M \in \mathbb{N}^+$. Regarding the interpolation filter, denoted by the column vector $\mathbf{h}^{(0)}$, we consider a freely designed low-pass FIR filter of order $N_h^{(0)} - 1$ with cutoff frequency $\omega_c = \min\left(\frac{\pi}{M}, \frac{\pi}{L}\right)$ in order to avoid aliasing. Under these premises, the resampled version of $\mathbf{x}^{(0)}$, can be written as

$$\mathbf{y}^{(0)} = \mathbf{X}^{(0)}\mathbf{h}^{(0)},$$

where $\mathbf{X}^{(0)}$ is a matrix of size $N_z^{(0)} \times N_h^{(0)}$ with $N_z^{(0)} = \frac{L}{M} N_x^{(0)}$,[1] which is constructed from the samples of $\mathbf{x}^{(0)}$, i.e., $x_i^{(0)}$ with $i = 0, \ldots, N_x^{(0)} - 1$, and as a function of the employed resampling factor ξ. Each element (i, j) of the matrix $\mathbf{X}^{(0)}$ is denoted by $X_{ij}^{(0)}$ and is defined as:

$$X_{ij}^{(0)} \triangleq \begin{cases} x^{(0)}_{\frac{iM+k-j}{L}}, & \text{if } \frac{iM+k-j}{L} \in \left(\left\lceil \frac{iM-k}{L} \right\rceil, \left\lfloor \frac{iM+k}{L} \right\rfloor\right) \cap \mathbb{Z} \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

with $k \triangleq \frac{N_h^{(0)}-1}{2}$, $i = 0, \ldots, N_z^{(0)} - 1$ and $j = 0, \ldots, N_h^{(0)} - 1$. In the above expression, $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote the ceiling and floor functions, respectively.

The interpolated values of $\mathbf{y}^{(0)}$ will be generally represented with more bits than for the original signal $\mathbf{x}^{(0)}$, hence a requantization to the original precision is commonly done prior to saving the resulting signal. This quantized version of the resampled signal, denoted by $\mathbf{z}^{(0)}$, is expressed as

$$\mathbf{z}^{(0)} = Q_\Delta\left(\mathbf{y}^{(0)}\right) = Q_\Delta\left(\mathbf{X}^{(0)}\mathbf{h}^{(0)}\right), \tag{2}$$

where $Q_\Delta(\cdot)$ represents a uniform scalar quantization with step size Δ (i.e., the same one used for the original signal).[2]

A. Set-membership formulation

As it was pointed out in the Introduction, the set-membership theory is governed by the concept of feasibility; hence, once applied to a particular problem, its main goal is to find a solution that satisfies simultaneously all the constraints defined through the observed data and the a priori knowledge about the problem. In those cases where there exists no solution fulfilling all the requirements at the same time, the problem does not have a feasible solution.

Let us first introduce the set-membership formulation of a general problem whose solution belongs to a space Ξ. Each piece of information from the observed data, i.e., each i-th observation, is associated with a property set $\mathcal{S}_i$ in the solution space Ξ and can be defined as follows

$$\mathcal{S}_i = \{a \in \Xi \,:\, a \text{ satisfies } \Psi_i\},$$

where Ψ_i represents a constraint of the problem and a is an arbitrary point of the solution space Ξ. Each subset $\mathcal{S}_i$ represents all the estimates that are consistent with the i-th observation. Therefore, the feasible set of solutions for the problem will be composed by the intersection of all the property sets that are obtained with N available observations, thus having $\mathcal{S} = \cap_{i=0}^{N-1} \mathcal{S}_i$, where $\mathcal{S}$ is also commonly known as the solution set. If the solution set is empty, i.e., $\cap_{i=0}^{N-1} \mathcal{S}_i = \emptyset$, then the problem is designated as *infeasible*. Otherwise, the problem is *feasible* and a set-membership estimate consists in choosing any point $\hat{a} \in \mathcal{S}$.

Set-membership theory allows us to define a feasibility problem for checking whether a vector of observations $\mathbf{z}$ of length N_z has been resampled or not with a candidate resampling factor $\xi_c \triangleq \frac{L_c}{M_c}$, with $L_c, M_c \in \mathbb{N}^+$. Note that, in this case, we will assume that N_z is a multiple of L_c for the sake of simplicity and without loss of generality. To characterize this problem in set-membership terms, we need to define the solution space Ξ, which in this case turns out to

[1] Without loss of generality and for the sake of simplicity, we will assume that $N_x^{(0)}$ is a multiple of M and also that $N_h^{(0)}$ is an odd number.

[2] Note that having the same quantization step size in both cases is not a limiting condition, since the problem can be reformulated if it is not so.

be the Cartesian product of two sets, i.e., $\Xi = \mathcal{X} \times \mathcal{H}$, where the set $\mathcal{X}$ represents the domain of the original signal, and the set $\mathcal{H}$ specifies the domain of the interpolation filter.

Prior knowledge about the problem helps us define these two sets. For the original signal, we know that each sample x_i has been quantized with step size Δ, so we could assume that $x_i \in \Delta\mathbb{Z}$, but this assumption would make the resolution of the subsequent optimization problem notably more complicated. In order to lighten the consequent computational burden, we assume without loss of generality that each sample lies in a real interval $[x_{\text{min}}, x_{\text{max}}]$, thus having

$$\mathcal{X} = \left\{ \mathbf{x} \in \mathbb{R}^{N_x} \, : x_{\text{min}} \leq x_i \leq x_{\text{max}}, \; i = 0, \ldots, N_x - 1 \right\},$$

where N_x represents the dimension of the set and is defined as a function of the number of observations and the candidate resampling factor, i.e., $N_x = N_z \frac{M_c}{L_c}$. Regarding the interpolation filter, we assume that each coefficient falls in a real interval $[h_{\text{min}}, h_{\text{max}}]$, hence

$$\mathcal{H} = \left\{ \mathbf{h} \in \mathbb{R}^{N_h} \, : h_{\text{min}} \leq h_i \leq h_{\text{max}}, \; i = 0, \ldots, N_h - 1 \right\},$$

where the dimension of the set comes from the order of the FIR filter, which is assumed to be $N_h - 1$. The interval $[h_{\text{min}}, h_{\text{max}}]$ can be specified according to any particular filter, for instance, for a linear interpolator we could presume $h_i \in [0, 1]$, $\forall i$.

In order to check if each component z_i of the vector of observations has been generated through the sampling rate conversion of a vector $\mathbf{x} \in \mathcal{X}$ by a candidate resampling factor ξ_c and using an interpolation filter $\mathbf{h} \in \mathcal{H}$, we must rely on the quantization applied to the resampled signal in (2). Since we assume as known the size of the quantization step, i.e., Δ, we have information about the interval where the values of the resampled signal $\mathbf{y} = \mathbf{X}\mathbf{h}$ will lie on.[3] Therefore, any pair $(\mathbf{x}, \mathbf{h})$ from the solution space must generate values of the resampled signal $\mathbf{y}$ with the candidate resampling factor ξ_c inside the interval defined by the quantization error of the scalar quantizer with step size Δ, that can be written as

$$z_i - \frac{\Delta}{2} < y_i \leq z_i + \frac{\Delta}{2}, \quad \text{for} \;\; i = 0, \ldots, N_z - 1.$$

Consequently, we assume that the feasible region imposed by each observation z_i of the signal under analysis is limited by two hyperplanes that yield the following property sets

$$\mathcal{S}_i = \mathcal{X}_i \times \mathcal{H}_i = \left\{ (\mathbf{x}, \mathbf{h}) \in \Xi \, : \, -\frac{\Delta}{2} < \mathbf{x}_i^T \mathbf{h} - z_i \leq \frac{\Delta}{2} \right\}, \tag{3}$$

for $i = 0, \ldots, N_z - 1$, and where $\mathbf{x}_i$ is a column vector built up with the N_h elements of the i-th row of matrix $\mathbf{X}$. Finally, the feasible solution set for our problem will be the intersection of these N_z property sets: $\mathcal{S} = \cap_{i=0}^{N_z - 1} (\mathcal{X}_i \times \mathcal{H}_i)$. If such intersection leads to $\mathcal{S} = \emptyset$, then there exists no $\mathbf{x} \in \mathcal{X}$ and $\mathbf{h} \in \mathcal{H}$ that would generate the vector of observations $\mathbf{z}$ with such candidate resampling factor ξ_c. Otherwise, an estimate of the original signal $\hat{\mathbf{x}}$ together with an estimate of the interpolator $\hat{\mathbf{h}}$ can be obtained by taking any $(\hat{\mathbf{x}}, \hat{\mathbf{h}}) \in \mathcal{S}$.

[3] Note that the matrix $\mathbf{X}$ with size $N_z \times N_h$ is generated according to (1) but with the elements of the vector $\mathbf{x}$.

III. Practical algorithms

One of the widely-known methods for solving feasibility problems in terms of set-membership theory is the Optimal Value Ellipsoid (OVE) algorithm [13]. However, this method can only be applied when constraints are convex and, in our particular case, the modeling of the resampling identification problem requires nonconvex terms. As it can be observed from the definition of the property sets in (3), the constraints of our problem are actually bilinear, due to the product between the variables $\mathbf{x}$ and $\mathbf{h}$. Under these conditions, the feasible solution set is not necessarily convex, leading us to consider nonlinear programming algorithms as a way to solve the problem.

Before explaining the particular strategy we have designed, we formally introduce the feasibility problem (derived from Section II-A) that is addressed for the identification of resampled signals: given a vector of observations $\mathbf{z}$, a candidate resampling factor ξ_c, and a particular length for the interpolation filter N_h, we want to

$$\begin{array}{rll} \text{find} & \mathbf{x}, \mathbf{h}, & \\ \text{subject to} & \mathbf{x} \in \mathbb{R}^{N_x}, \mathbf{h} \in \mathbb{R}^{N_h}, & \\ & x_{\text{min}} \leq x_i \leq x_{\text{max}}, & i = 0, \ldots, N_x - 1, \\ & h_{\text{min}} \leq h_j \leq h_{\text{max}}, & j = 0, \ldots, N_h - 1, \\ & -\frac{\Delta}{2} < \mathbf{x}_k^T \mathbf{h} - z_k \leq \frac{\Delta}{2}, & k = 0, \ldots, N_z - 1. \end{array} \tag{4}$$

If the problem proves to be feasible, then the forensic analyst could also be interested in finding an estimation of both the original signal and interpolation filter that have generated the vector of observations $\mathbf{z}$. This can be done by considering an objective function that measures the squared error between the resampled signal $\mathbf{y} = \mathbf{X}\mathbf{h}$ and the vector of observations $\mathbf{z}$, leading us to the following optimization problem

$$\begin{array}{rll} \text{minimize} & \|\mathbf{X}\mathbf{h} - \mathbf{z}\|_2^2, & \\ \text{subject to} & \mathbf{x} \in \mathbb{R}^{N_x}, \mathbf{h} \in \mathbb{R}^{N_h}, & \\ & x_{\text{min}} \leq x_i \leq x_{\text{max}}, & i = 0, \ldots, N_x - 1, \\ & h_{\text{min}} \leq h_j \leq h_{\text{max}}, & j = 0, \ldots, N_h - 1, \\ & -\frac{\Delta}{2} < \mathbf{x}_k^T \mathbf{h} - z_k \leq \frac{\Delta}{2}, & k = 0, \ldots, N_z - 1, \end{array} \tag{5}$$

where $\| \cdot \|_2^2$ denotes the squared Euclidean norm. We remark that since this is a nonconvex problem, the resulting estimates $\hat{\mathbf{x}}$ and $\hat{\mathbf{h}}$ will probably correspond to local minima. Given this situation, we have first considered global optimization techniques (e.g., branch-and-bound strategies), to solve this optimization problem. However, we have found difficulties handling large-scale problems (with a few hundreds of variables), thus deciding to use a local optimization method as a practical way to solve our problem.

A. Solver based on local optimization

The main goal of local optimization is not the search for a globally optimal solution of the problem, but only the pursuit of a locally optimal point that minimizes the objective function among a feasible region close to it. Local optimization has been deeply studied with the aim of solving nonlinear problems, and many different algorithmic approaches can be found in the literature. In our case, we have selected an interior-point method, that is available through the function `fmincon` of MATLAB.

In general, local solvers are less computationally demanding than global solvers and, consequently, they can handle in a more suitable way large-scale problems. Nevertheless, local solvers require a good starting point for the optimization variable in order to work properly. The selection of the starting point is crucial since it affects the final result provided by the solver. For instance, by choosing a starting point that is far from a feasible region, the solver could wrongly classify a feasible problem as infeasible. This could lead the forensic analyst to wrongly declare that the observed signal was not resampled by a factor ξ when it actually was so.

In the following, we focus on the process we have designed to obtain a starting point near the feasible region of our problem (whenever the problem is actually feasible). Thus, given a vector of observations $\mathbf{z}$, a candidate resampling factor ξ_c, and the length of the filter N_h, the following steps are taken:

1) An approximation $\tilde{\mathbf{x}}$ of the original signal is first obtained. To that end, the vector of observations $\mathbf{z}$ is resampled by a factor equal to the inverse of the candidate resampling factor,[4] i.e., by $\xi_c^{-1} = \frac{M_c}{L_c}$.
2) Since $\mathbf{z} = Q_\Delta(\mathbf{Xh}) \approx \mathbf{Xh}$, an approximation of the interpolation filter can also be obtained if $\mathbf{X}$ is known. For this purpose, an approximation of matrix $\mathbf{X}$, denoted by $\tilde{\mathbf{X}}$, is obtained according to (1) using the components of vector $\tilde{\mathbf{x}}$ (calculated in the previous step) and using the considered values for ξ_c and N_h.
3) After obtaining $\tilde{\mathbf{X}}$, an approximation $\tilde{\mathbf{h}}$ of the interpolation filter is constructed as $\tilde{\mathbf{h}} = \tilde{\mathbf{X}}^{+}\mathbf{z}$, where $\tilde{\mathbf{X}}^{+}$ denotes the Moore-Penrose pseudoinverse of matrix $\tilde{\mathbf{X}}$.

Even though the obtained starting point, composed by $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{h}}$, might not strictly belong to the solution space nor satisfy all the constraints of the problem, it will be sufficiently close to a feasible region of the problem (again, whenever the problem is actually feasible) and the local solver will be able to find a feasible solution after several iterations. Notice that when the candidate resampling factor ξ_c does not match the actual one ξ, the obtained starting point will probably be far from the true feasible region, thus yielding an infeasible solution.

In practice, for solving the feasibility problem in (4), a constant objective function can be considered. As we will show in next section, this practical implementation, i.e., the local solver together with a good starting point, is able to successfully solve the feasibility problem in (4). Moreover, in those cases where the resulting solution set is not empty after solving (4), this practical approach is also able to provide locally optimal solutions by further addressing the optimization problem in (5).

IV. Experimental results

The performance analysis of the proposed technique is twofold. In the first part, synthetic signals are used to quantify the accuracy solving the feasibility problem in (4) and also to measure the Mean Square Error (MSE) of the estimates obtained through the optimization problem in (5). In the second part, a realistic scenario with audio signals is considered.

[4]The low-pass filter used in this particular case is designed to avoid aliasing and it is constructed from a spectral Kaiser window, independently of $\mathbf{h}^{(0)}$.

TABLE I
Details of the interpolation filters for different scenarios.

	Scenario 1	**Scenario 2**
$\xi < 1$	Kaiser, $N_h^{(0)} = 2M+1$	Kaiser, $N_h^{(0)} = 4M+1$
$\xi > 1$	Linear, $N_h^{(0)} = 2L+1$	Cubic, $N_h^{(0)} = 4L+1$

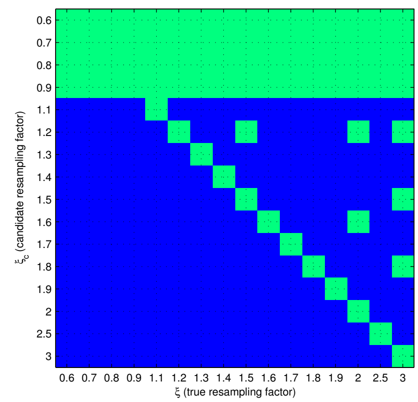


Fig. 1. Illustrative representation of the solutions given by the local solver to the feasibility problem in (4), for the scenario 1 in Table I. Green boxes imply feasibility, whereas blue boxes represent infeasibility.

A. Performance analysis with synthetic signals

For the evaluation of the feasibility problem in (4), we construct the original signal $\mathbf{x}^{(0)}$ using 8-bit precision samples gathered from a discrete uniform distribution in the interval $[0, 255]$, thus having $x_{\min} = 0$, $x_{\max} = 255$ and $\Delta = 1$. We take into consideration a finite discrete set of resampling factors, obtained by sampling the interval $[0.6, 3]$ with step sizes 0.1 (from 0.6 to 2) and 0.5 (from 2 to 3). The same set is used for the true resampling factor ξ and for checking the feasibility problem with ξ_c. Regarding the interpolation procedure, we employ the filters specified under Scenario 1 in Table I: a linear interpolator for $\xi > 1$, and a low-pass FIR filter designed through a spectral Kaiser window when $\xi < 1$. Note that both filters have their coefficients inside the interval $[-1, 1]$, thus we assume $h_{\min} = -1$ and $h_{\max} = 1$. For simplicity, N_h is selected according to Table I, but using ξ_c.

Taking into account all these settings and fixing the number of observations to $N_z^{(0)} = 512$, the study of the feasibility problem is carried out with the proposed local solver providing a starting point (computed as in Section III-A). In Fig. 1, the obtained results are shown in a graphical manner, where the horizontal axis represents the true resampling factor ξ, and the vertical axis contains the tested candidate resampling factor ξ_c. Green boxes mean that the problem has a feasible solution for the pair (ξ, ξ_c), while blue ones symbolize that there exists no solution that satisfies all the constraints of the problem.

There are three important aspects that become apparent from the results shown in Fig. 1:

1) When the feasibility problem is evaluated for a candidate resampling factor $\xi_c < 1$, there is always a feasible solution regardless of the true resampling factor. We must remark that this is not an error due to the set-membership approach; instead, in this case there is not sufficient information (prior or observed) to rule out such ξ_c. In mathematical terms: the number of degrees of freedom of the problem, which is the dimension of the solution space, i.e., $N_x + N_h$, is larger than the number of observations N_z, given that $N_x = N_z \frac{M_c}{L_c}$. This problem could be overcome by adding enough a priori knowledge about the distribution of the original signal.
2) All the cases where the candidate resampling factor ξ_c coincides with the true one ξ have always been categorized as feasible problems. This is an intrinsic property of the set-membership formulation of the problem and perhaps the most valuable feature of this method.
3) For several resampling factors $\xi > 1$ (e.g., $\xi \in \{1.5, 2, 3\}$), when $\xi_c > 1$ the solver is capable of finding a feasible solution, even if the true resampling factor is not equal to the candidate factor (e.g., $\xi = 1.5$ and $\xi_c = 1.2$). This is due to the existence of solutions that are theoretically feasible. However, given that the opposite case (e.g., $\xi = 1.2$ and $\xi_c = 1.5$) will not yield a feasible solution, no ambiguities are possible.

From the last point, we have found that, when an original signal is resampled by a factor $\xi > 1$, then the set of all the possible candidate resampling factors $\xi_c > 1$ that lead to a feasible solution (besides the case $\xi_c = \xi$), are:

$$\xi_c \in \left\{ \frac{L_c}{M_c} : \frac{L_c}{M_c} < \xi, (L_c = kL) \wedge (M_c > kM), k \in \mathbb{N}^+ \right\}, \tag{6}$$

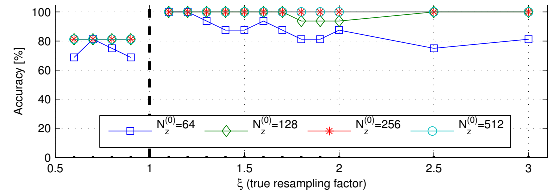
where $L_c \in \mathbb{N}^+$ and $M_c \in \mathbb{N}^+$ must be coprime, and $\wedge$ represents the logical conjunction operation. This property also holds for the second scenario in Table I.

As a conclusion, excepting the cases where the resampling factor $\xi < 1$, if we have a sufficiently large number of observations, then we are able to exactly match the resampling factor applied to the original signal.

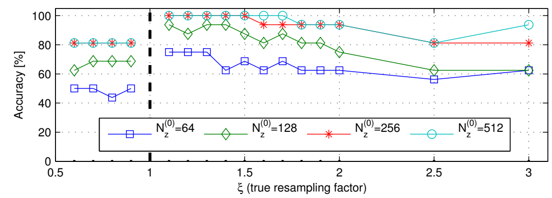
1) Accuracy analysis for different numbers of observations: To quantify the performance of the method solving the problem in (4) we use the accuracy, defined as the following ratio:

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{FP} + \text{TN} + \text{FN}},$$

where TP, TN, FP, FN represent the number of true positives, true negatives, false positives and false negatives, respectively. In our problem, a true positive occurs when a feasible solution is found in (4) and the candidate resampling factor matches the actual one, i.e., $\xi_c = \xi$. On the other hand, a true negative takes place when no feasible solution is achieved in (4) and the candidate resampling factor is indeed different from the true one, i.e., $\xi_c \neq \xi$. Note that for those cases where a feasible solution is theoretically possible even if $\xi_c \neq \xi$ (i.e., for the candidate resampling factors in (6) and for $\xi_c < 1$ when $\xi > 1$), we will consider that a true positive case occurs if such feasible solution is found.



(a) Scenario 1



(b) Scenario 2

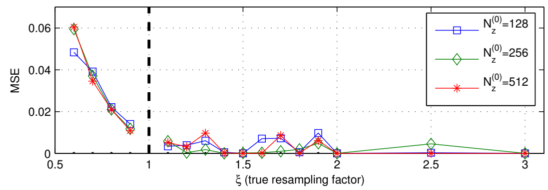
Fig. 2. Accuracy of the proposed approach achieved by the local solver under the two scenarios of Table I, for different numbers of observations.

Fig. 2 shows the accuracy obtained in the two scenarios described in Table I as a function of the true resampling factor ξ and for different numbers of observations $N_z^{(0)} \in \{64, 128, 256, 512\}$. From this plot, we can observe that the accuracy improves as the number of observations increases, which is the expected behavior, since with each new piece of information the feasible set in the solution space generally gets smaller. Furthermore, by comparing the results gathered from the two scenarios, the dependence between the number of observations and the degrees of freedom of the problem becomes evident, obtaining generally worse performance in the second scenario where the order of the interpolation filters is larger. Such dependence also justifies the smaller accuracy when $\xi < 1$ in both scenarios.

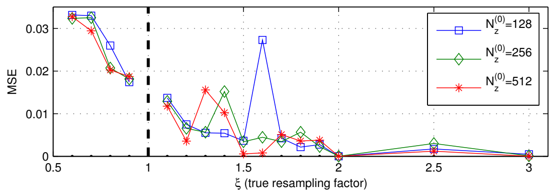
2) MSE analysis for different numbers of observations: Concerning the results obtained when the optimization problem in (5) is solved (after having reached a solution in (4)), we will only show, for the sake of brevity, the empirical MSE of $\hat{\mathbf{h}}$ (i.e., $(1/N_h)\|\mathbf{h}^{(0)} - \hat{\mathbf{h}}\|^2$). Taking into account the two scenarios defined in Table I, the evolution of such empirical MSE as a function of the resampling factor and for different numbers of observations $N_z^{(0)} \in \{128, 256, 512\}$, is depicted in Fig. 3. As we can observe, the MSE of $\hat{\mathbf{h}}$ decreases as the resampling factor increases and, although the differences are not very significative, smaller values are generally attained when the number of observations increases. The important reduction of the estimation error for $\xi > 1$ is mainly due to the higher redundancy that is present on those resampled signals. The noisy shape of the MSE (e.g., $\xi = 1.6$ in Fig. 3(b)) is a consequence of the local optimization performed, which in some cases converges to a local minimum that can be far from the global optimum point, but still yielding a feasible solution.

B. Performance analysis with real audio signals

For the evaluation of the set-membership approach solving the feasibility problem in (4) within a real scenario, we use the "Music Genres" audio database [14], from which we take a subset of 100 uncompressed audio files with 10 different



(a) Scenario 1



(b) Scenario 2

Fig. 3. MSE of $\hat{\mathbf{h}}$ when solving the optimization problem in (5) with the local solver, under the scenarios of Table I, and for different numbers of observations.

music styles. Each original audio signal is quantized to a 16-bit precision per sample, thus having $x_{\min} = 0$ and $x_{\max} = 2^{16} - 1$. For comparison, the same tests are carried out with two state-of-the-art methods: the "EM method" proposed in [4], and the "ML method" in [10]. Given that the ML method has only been defined for linear interpolators and $\xi > 1$, we consider a discrete set of resampling factors in the interval $[1.1, 2]$ (sampled with a step size of 0.1) and a linear interpolation filter as the one specified in scenario 1 from Table I.

In this case, we are interested in comparing the percentage of correct resampling factor estimation for different numbers of observations: $N_z^{(0)} \in \{64, 128, 256, 512\}$. In Fig. 4, we report the obtained results with each method. The best performance is achieved by the ML method, which actually never fails with any of the considered parameters. These optimal results are possible due to the complete knowledge of the original filter used in the resampling process. Interestingly, a similar performance is obtained with the proposed set-membership approach (unless for $N_z^{(0)} = 64$), where limited assumptions are made about the filter, thus increasing the applicability of the method. On the other hand, the EM method clearly exhibits some of the shortcomings mentioned in the Introduction, i.e., a high dependency on the number of observations and a worse performance for those resampling factors close to 1 due to the windowing effect. These limitations are not an issue for the proposed set-membership technique.

V. Conclusions

Set-membership estimation theory has proven to be a useful resource for addressing the problem of resampling factor estimation. The presented technique provides reliable solutions that do not violate any constraint of the problem, and thus are a valuable asset for a forensic analyst, who needs to provide unquestionable proofs of tampering. Moreover, the evaluation of the proposed approach in a real scenario with audio signals has demonstrated its good performance.

As future work, a deeper theoretical analysis of the problem must be carried out to derive, for instance, the average number

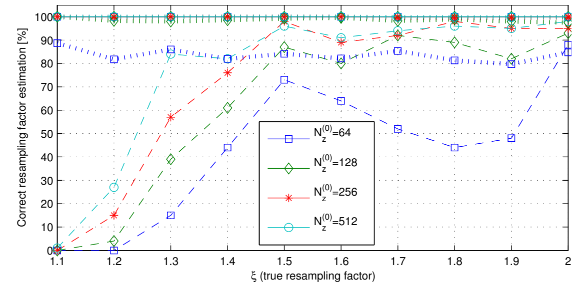


Fig. 4. Comparison of the correct estimation percentage of the proposed set-membership technique (dotted lines) versus the ML method (solid lines) and the EM method (dashed lines).

of observations that are necessary to discard a candidate resampling factor or the minimum number of observations that are necessary to converge to an optimal solution.

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