

# ML Estimation of the Resampling Factor

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**Abstract**—In this work, the problem of resampling factor estimation for tampering detection is addressed following the maximum likelihood criterion. By relying on the rounding operation applied after resampling, an approximation of the likelihood function of the quantized resampled signal is obtained. From the underlying statistical model, the maximum likelihood estimate is derived for one-dimensional signals and a piecewise linear interpolation. The performance of the obtained estimator is evaluated, showing that it outperforms state-of-the-art methods.

## I. INTRODUCTION

In the last few years, an increasing number of passive forensic techniques have emerged with the aim of furnishing information about the authenticity, integrity or processing history of a multimedia content. In the field of digital image forensics, many works have been oriented to the detection and localization of tampered regions. A well-known problem in this research area is the detection of resampling traces as a means to unveil the application of a geometric transformation and the estimation of the resampling factor for specifying the parameters of the applied transformation.

Seminal works addressing this topic [1]-[3], were focused on the detection of the particular correlation introduced between neighboring pixels by the resampling operation inherently present when a spatial transformation (e.g., scaling or rotation) has been performed.

Since the resampling operation can be modeled as a time-varying filtering that induces periodic correlations, links between this problem and the cyclostationarity theory have been established in [4] and [5], providing a theoretical framework for the estimation of the parameters of the transformation. Within this framework, two different approaches have been proposed for finding the optimum prefilter that might be applied to a resampled image for achieving the best performance in the estimation of the resampling factor [6], [7].

At some point, all the mentioned approaches perform an analysis in the frequency domain for the detection or estimation of this periodic behavior, by looking at spectral peaks corresponding to underlying periodicities. Nevertheless, the frequency analysis presents some drawbacks: 1) a considerably large number of samples is needed to obtain reliable results; 2) the presence of periodic patterns in the content of the image

usually misleads the detector and the estimator; and 3) the windowing effect impairs the performance of the mentioned methods when slight spatial transformations are employed (i.e., with a resampling factor near 1).

With these shortcomings in mind, in this work we will address the estimation of the resampling factor following the Maximum Likelihood (ML) criterion. The approximation of the likelihood function of the resampled signal will rely on the rounding operation applied after the resampling. Therefore, by correctly modeling the relationship between the distribution of the quantization noise and the quantized resampled signal, an optimum estimator of the resampling factor will be provided. The proposed approach will only consider one-dimensional (1-D) signals, but the idea can easily be extended to the two-dimensional case, to be applied to images. The three discussed drawbacks of the previous methods will be sorted out with the proposed estimator.

The rest of the paper is organized as follows: In Section II, the formalization of the problem we want to solve is introduced, while the description of the method for estimating the resampling factor based on the ML criterion is considered in Section III. Experimental results with synthetic and real signals are reported in Section IV for evaluating the performance of the estimator. Finally, conclusions and future work are discussed in Section V.

## II. PRELIMINARIES AND PROBLEM FORMULATION

A digital image forgery can be done in many different ways, but it usually involves cropping some region from a particular image and pasting it into a different one. The adjustment of this new content to a specific scene is commonly carried out by applying geometrical transformations (e.g., rotation or scaling) that inherently need to perform a resampling operation. Since the tampering should not introduce visible distortions, only slight transformations will be applied, thus requiring that the resampling estimator should achieve good performance for resampling factors near 1. This work just studies the case where the resampling factor is larger than 1. Of course, the use of resampling factors smaller than 1 are commonly used; however, the analysis is formally quite different, so we leave the study of such case for a future work.

The problem of resampling estimation is addressed for 1-D signals because the derivation of the Maximum Likelihood Estimate (MLE) of the resampling factor is more tractable and

affordable than considering directly the two-dimensional (2-D) case. However, we will see in Section III that the obtained method following the ML criterion can be easily extended to the 2-D case. The same holds for the considered interpolation filter. The use of a piecewise linear interpolation scheme is a clear limitation of our work, which should be considered in this regard as a first attempt to introduce MLE principles in the resampling estimation problem. We notice that the methodology here introduced can be extended to include more general filters.

#### A. Notation

A time-dependent 1-D signal will be represented as  $x(n)$ . Random variables will be denoted by capital letters (e.g.,  $X$ ) and their realizations by lowercase letters (e.g.,  $x$ ). Random vectors will be represented with bold capital letters (e.g.,  $\mathbf{X}$ ), their outcomes with lowercase letters (e.g.,  $\mathbf{x}$ ) and each  $i$ th component will be denoted as  $x_i$ . The length of a vector  $\mathbf{x}$  will be expressed as  $L_x \in \mathbb{N}^+$  and, for convenience, the index  $i$  to identify each component of the vector will satisfy  $i \in \{0, \dots, L_x - 1\}$ . A vector of length  $N$  starting from the  $n$ th component, will be denoted by  $\mathbf{x}_n = (x_n, \dots, x_{n+N-1})^T$ . Floor and ceiling functions will be represented by  $\lfloor \cdot \rfloor$  and  $\lceil \cdot \rceil$ , respectively. To denote the set of all integer numbers multiple of a given integer value  $n$ , we will use the notation  $n\mathbb{Z}$ . For a compact notation, we will use  $\text{mod}(a, b)$  to denote the modulo operation:  $a \text{ mod } b$ .

#### B. Problem formulation

In the following, we will mathematically describe all the steps involved in the change of the sampling rate of a 1-D signal  $x(n)$ , by a resampling factor denoted by  $\xi$ . This description will allow us to set out an approach based on the ML criterion in Section III, for the estimation of the applied resampling factor.

Let us start by defining the resampling factor  $\xi$  as the ratio between the applied upsampling factor  $L$  and downsampling factor  $M$ , i.e.,  $\xi \triangleq \frac{L}{M}$  with  $L \in \mathbb{N}^+$  and  $M \in \mathbb{N}^+$ . To ensure a unique representation of  $\xi$ , we will consider that  $L$  and  $M$  are coprime, but note that this is not a limitation. As it was stated above, the possible range of values for the resampling factor will be  $\xi > 1$ . For this range of resampling factors, the general expression for a resampled signal  $y(n)$  is given by the following relation with the original signal  $x(n)$ :

$$y(n) = \sum_k x(k)h\left(n\frac{M}{L} - k\right),$$

where  $h(t)$  with  $t \in \mathbb{R}$  represents the interpolation filter. As it was previously indicated, the interpolation filter used during the resampling process will be assumed linear, with the following impulse response

$$h(t) = \begin{cases} 1 - |t|, & \text{if } |t| \leq 1 \\ 0, & \text{otherwise} \end{cases}.$$

Therefore, considering this interpolation filter, each component of the resampled vector can be computed as the linear combination of at most two samples from the original signal,

$$y(n) = \begin{cases} x(\lfloor n\frac{M}{L} \rfloor)(1 - \text{mod}(n\frac{M}{L}, 1)) \\ \quad + x(\lfloor n\frac{M}{L} \rfloor + 1)\text{mod}(n\frac{M}{L}, 1), & \text{if } n \notin L\mathbb{Z} \\ x(n\frac{M}{L}), & \text{if } n \in L\mathbb{Z} \end{cases}.$$

Regarding the set of values that the original signal can take, we will consider that all the samples  $x(n)$  have already been quantized by a uniform scalar quantizer with step size  $\Delta$ , in order to fit into a finite precision representation. Even though the interpolated values  $y(n)$  will be generally represented with more bits, a requantization to the original precision is often done prior to saving the resulting signal. This quantized version of the resampled signal, denoted by  $z(n)$ , will be expressed as

$$z(n) = \begin{cases} Q_\Delta(x(\lfloor n\frac{M}{L} \rfloor)(1 - \text{mod}(n\frac{M}{L}, 1)) \\ \quad + x(\lfloor n\frac{M}{L} \rfloor + 1)\text{mod}(n\frac{M}{L}, 1)), & \text{if } n \notin L\mathbb{Z} \\ x(n\frac{M}{L}), & \text{if } n \in L\mathbb{Z} \end{cases}. \quad (1)$$

where  $Q_\Delta(\cdot)$  represents a uniform scalar quantization with step size  $\Delta$  (i.e., the same one used for the original signal).

From the second condition in (1), it is evident that some of the original samples are “visible” in the quantized resampled version. On the other hand, the remaining values of the resampled signal are the combination of “visible” and “non-visible” samples from the original signal that are later quantized. This fact will help to define the likelihood function of the quantized resampled signal.

### III. ML APPROACH TO RESAMPLING ESTIMATION

For the definition of the MLE of  $\xi$ , the original signal will be represented by the vector  $\mathbf{x}$  with  $L_x$  samples and the corresponding quantized resampled signal by the vector  $\mathbf{z}$  with  $L_z$  samples. For convenience, we will assume that the length of the original signal is  $L_x = N + 1$  with  $N$  a multiple of  $M$ , and so, the corresponding length of the resampled signal will be  $L_z = \xi N + 1$ . We will find it convenient to model vectors  $\mathbf{x}$  and  $\mathbf{z}$  as outcomes of random vectors  $\mathbf{X}$  and  $\mathbf{Z}$ , respectively.

Based on the above analysis, the estimation of the resampling factor  $\hat{\xi}$  following the ML criterion relies on finding the value of  $\xi$  that makes the observed values of the quantized resampled vector  $\mathbf{z}$  most likely. Nevertheless, given a vector of observations, their components  $z_i$  could be misaligned with the periodic structure of the resampled signal in (1). Hence, a parameter  $\phi$  must be considered to shift the components of the vector, in order to align the periodic structure of  $z_i$  with  $z(n)$ . The possible values of  $\phi$  lie in the range  $0 \leq \phi \leq L - 1$ . Therefore, the MLE of  $\xi$  becomes

$$\hat{\xi} = \arg \max_{\xi > 1} \max_{0 \leq \phi \leq L - 1} f_{\mathbf{Z}|\Xi, \Phi}(\mathbf{z}|\xi, \phi).$$

Note that we are not considering a set of possible parameters for the interpolation filter because in the case of a piecewise

linear interpolation, once we fix the resampling factor, then the filter is automatically determined (cf. Eq. (1)). On the other hand, given that the shift  $\phi$  is not a determining factor for the derivation of the target function, for the sake of simplicity, we will assume that the vector of observations is correctly aligned and, thus, the MLE can be written as

$$\hat{\xi} = \arg \max_{\xi > 1} f_{\mathbf{Z}|\Xi}(\mathbf{z}|\xi).$$

For the calculation of that joint probability density function (pdf) we will exploit the fact that some samples of the interpolated signal exactly match the original (cf. Eq. (1)), and also the linear relation established between the remaining samples.

#### A. Derivation of $f_{\mathbf{Z}|\Xi}(\mathbf{z}|\xi)$

Along the derivation of the joint pdf  $f_{\mathbf{Z}|\Xi}(\mathbf{z}|\xi)$ , for the sake of notational simplicity, we will refer to this one as  $f_{\mathbf{Z}}(\mathbf{z})$ , considering implicitly that we are assuming a particular resampling factor  $\xi$ . From the dependence between the quantized resampled signal and the original one, the joint pdf can be written in a general way as

$$f_{\mathbf{Z}}(\mathbf{z}) = \int_{\mathbb{R}^{N+1}} f_{\mathbf{Z}|\mathbf{X}}(\mathbf{z}|\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}.$$

We assume that no a priori knowledge on the distribution of the input signal is available. This is equivalent to considering that  $f_{\mathbf{X}}(\mathbf{x})$  is uniform and, consequently, the joint pdf can be approximated by the following relation

$$f_{\mathbf{Z}}(\mathbf{z}) \approx \int_{\mathbb{R}^{N+1}} f_{\mathbf{Z}|\mathbf{X}}(\mathbf{z}|\mathbf{x}) d\mathbf{x}.$$

Equation (1), indicates that every  $L$  samples of the observed vector  $\mathbf{z}$ , we have a visible sample from the original signal. This implies that the random variable  $Z_i$ , given  $X_k$ , is deterministic whenever  $i \in LZ$  and  $k \in MZ$ . For this reason, the previous joint pdf can be obtained by processing  $(L_z - 1)/L = N/M$  distinct and disjoint blocks, i.e.,

$$f_{\mathbf{Z}}(\mathbf{z}) \approx \prod_{j=0}^{N/M-1} \int_{\mathbb{R}^M} f_{\mathbf{Z}_{Lj}|\mathbf{X}_{Mj}}(\mathbf{z}_{Lj}|\mathbf{x}_{Mj}) d\mathbf{x}_{Mj}, \quad (2)$$

where  $\mathbf{Z}_{Lj}$  and  $\mathbf{X}_{Mj}$  (and also their corresponding outcomes) are vectors of size  $L$  and  $M$ , respectively.

The calculation of the contribution of each block of  $L$  samples from the vector of observations  $\mathbf{z}_{Lj}$  in (2), will depend on its relation with the corresponding  $M$  samples of the vector of the original signal, i.e.,  $\mathbf{x}_{Mj}$ . This relation is determined by the assumed resampling factor  $\xi$ .

Therefore, considering an arbitrary sample  $z_i$  that will be linearly related with at most two original samples  $x_k$  and  $x_{k+1}$ , with  $k \triangleq \lfloor i \frac{M}{L} \rfloor$  (cf. Eq. (1)), three cases are possible:

- $z_i$  is a visible sample, thus deterministic. Consequently

$$f_{Z_i|X_k}(z_i|x_k) = \delta(z_i - x_k),$$

where  $\delta(\cdot)$  represents the Dirac delta.

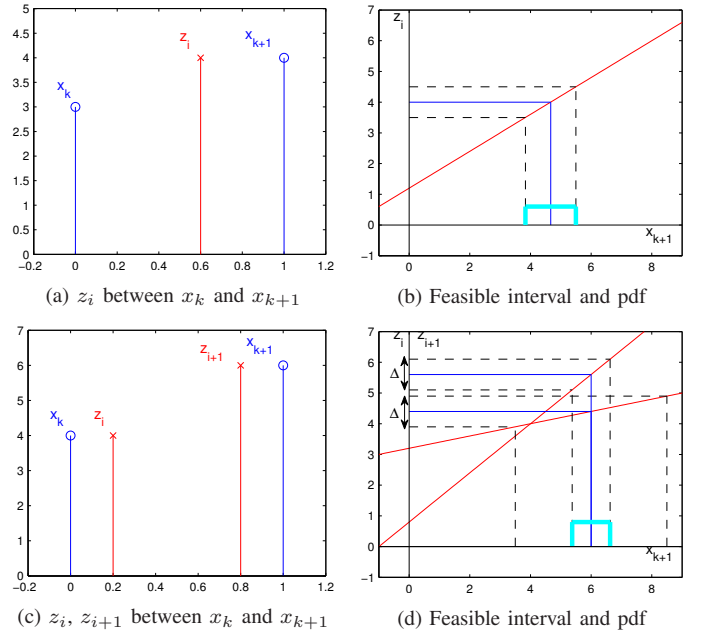


Fig. 1. Illustrative example, showing the last two possible cases for  $z_i$ . Pdfs obtained are shown graphically. Note that  $\Delta = 1$ .

- $z_i$  is the only sample between two original ones as it is shown in Fig. 1(a). In this case, if the variance of the original signal is large enough with respect to the variance of the quantization noise, then the quantization error can be considered uniform (we will call this the “fine-quantization assumption”), and the obtained pdf is

$$f_{Z_i|X_k, X_{k+1}}(z_i|x_k, x_{k+1}) = \Pi\left(\frac{a_i x_k + b_i x_{k+1} - z_i}{\Delta}\right),$$

where  $\Pi(t)$  denotes a rectangular pulse that is 1 if  $t \in [-\frac{1}{2}, \frac{1}{2}]$  and 0 otherwise. In this case, for the sake of clarity, we have used  $a_i \triangleq (1 - \text{mod}(i \frac{M}{L}, 1))$  and  $b_i \triangleq \text{mod}(i \frac{M}{L}, 1)$ , obtained from (1). A graphical representation, depicted in Fig. 1(b), shows how the rectangular pdf is derived from  $z_i$ .

- $z_i$  is one of several resampled values between two original samples, as it is shown in Fig. 1(c). As before, the following pdf is valid if the fine-quantization assumption holds, hence

$$f_{Z_i|X_k, X_{k+1}}(z_i|x_k, x_{k+1}) = \prod_m \Pi\left(\frac{a_m x_k + b_m x_{k+1} - z_m}{\Delta}\right),$$

where  $m$  will increase from  $i$  to the number of resampled values located between the two original samples. Fig. 1(d) shows the resulting pdf for the considered example.

Each time we obtain the pdf for a particular  $z_i$  (or a group of them), the corresponding integral in (2) must be evaluated with respect to the corresponding original sample  $x_k$ . Intuitively, we can observe that the calculation of (2) will finally be the convolution of several rectangular functions,

leading to a feasible and easy implementation. Note that those uniform distributions are obtained only if the fine-quantization assumption holds. Given the importance of this assumption, its effect on the performance of the MLE will be analyzed in Section IV.

### B. Method description

For a better understanding on how the obtained MLE can be easily implemented, we will exemplify the calculation of the target function  $f_{Z|\Xi}(z|\xi)$  when a particular resampling factor  $\xi_t$  is tested. In this illustrative example we will consider a vector of observations  $z$  (already aligned), corresponding to a signal that has been resampled by a factor  $\xi = \frac{5}{3}$ . In Fig. 2(a), an example of this vector of observations is shown, along with the corresponding vector of original samples  $x$ . In the mentioned figure, solid lines are used for representing the resampled values (consequently, also the original samples that are visible), while dashed lines are used for representing the non-visible samples of the original signal.

Since the calculation of the target function  $f_{Z|\Xi}(z|\xi)$  can be split by processing blocks of  $L$  samples of the observed vector, in this example, we will show how to process a single block. For the calculation of the remaining blocks, the same process should be repeated. Assuming that the resampling factor under test is  $\xi_t = \frac{5}{3}$ , these are the followed steps:

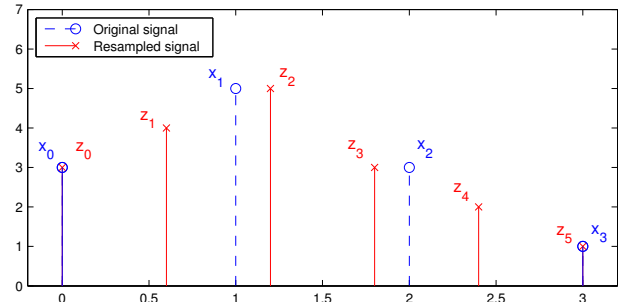
- 1) The first sample  $z_0$  is a visible one, then we know that  $z_0 = x_0$  and, thus,  $f_{Z_0|X_0,\Xi}(z_0|x_0,\xi_t) = \delta(z_0 - x_0)$ .
- 2) The second sample  $z_1$  is located between two original samples, i.e., the visible  $x_0$  and the non-visible  $x_1$ . Hence, we have  $f_{Z_1|X_0,X_1,\Xi}(z_1|x_0,x_1,\xi_t) = \Pi\left(\frac{a_1x_0 + b_1x_1 - z_1}{\Delta}\right)$ .

Fig. 2(b) shows with a red line the linear relation between the interpolated value and the original ones  $y_1 = a_1x_0 + b_1x_1$ , with the value of  $x_0$  fixed, i.e., from the previous step  $x_0 = z_0$ . From the value of  $z_1$  we obtain the feasible interval of  $x_1$  (represented with dashed black lines). Finally, the resulting pdf after the convolution of the rectangular function with the delta obtained in Step 1 is plotted in green.

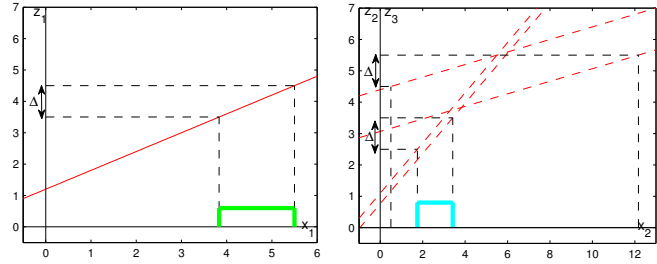
- 3) The third and fourth samples,  $z_2$  and  $z_3$ , are located between the two original samples  $x_1$  and  $x_2$ . In this case, we have seen that  $f_{Z_2|X_1,X_2,\Xi}(z_2|x_1,x_2,\xi_t) = \Pi\left(\frac{a_2x_1 + b_2x_2 - z_2}{\Delta}\right) \Pi\left(\frac{a_2x_1 + b_2x_2 - z_3}{\Delta}\right)$ .

Fig. 2(c) shows in this case the corresponding two linear relations for  $y_2 = a_2x_1 + b_2x_2$  and  $y_3 = a_3x_1 + b_3x_2$ . Be aware that in this case  $x_1$  can take any value in the range obtained in Step 2, and that is the reason why the dashed red lines are plotted. From the product of the two rectangular pdfs, we obtain the feasible interval for  $x_2$  (whose pdf is represented in cyan).

At this point, it is important to note that when the resampling factor under test does not match the true one, the previous product of rectangular pdfs could lead to an empty feasible set for  $x_2$ . If this happened, then we would automatically infer the infeasibility of the tested

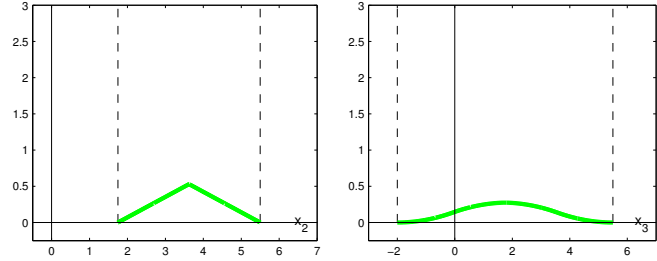


(a) Original and resampled signals



(b) Resulting pdf in Step 2

(c) Feasible interval for  $x_2$



(d) Resulting pdf in Step 3

(e) Resulting pdf in Step 4

Fig. 2. Graphical representation of the method description. Note that  $\Delta = 1$ .

resampling factor, so the estimation algorithm would move to the next resampling factor in the candidate set. If the factor cannot be discarded, then we must compute the convolution of the uniform pdf here obtained with the one resulting from Step 2. The result is plotted in green in Fig. 2(d).

- 4) The fifth sample  $z_4$  is processed in the same way as in Step 2, but considering that now the linear relation  $y_4 = a_4x_2 + b_4x_3$  must be evaluated with the set of possible values of  $x_2$ . Proceeding this way, we obtain the feasible interval for  $x_3$  and the corresponding pdf. Both are shown in Fig. 2(e).

- 5) At this point, we have finished processing the  $L$  samples in the block and we have the resulting pdf as a function of  $x_3$ . Since the next sample is visible, i.e.,  $z_5 = x_3$ , to determine the contribution of these  $L$  samples to the target function  $f_{Z|\Xi}(z|\xi_t)$ , we evaluate the resulting pdf taking into account the actual value of  $z_5$ .

As before, if the value of  $z_5$  falls outside the possible range of  $x_3$ , then the resampling factor under test is discarded.

Following this procedure, the maximization of the target



function  $f_{Z|\Xi}(z|\xi)$  is performed over the set of candidate resampling factors  $\xi > 1$  that have not been discarded, achieving the MLE  $\hat{\xi}$ . After this qualitative explanation, it is clear that the 2-D extension of this method is straightforward.

#### IV. EXPERIMENTAL RESULTS

The experimental validation of the obtained MLE is divided in two parts. In the first one, the performance of the estimator is evaluated by using synthetic signals and its behavior in terms of the fine-quantization assumption is analyzed. In the second part, natural 1-D signals from the audio database in [8] (which contains different music styles) are used to test the estimator in a more realistic scenario. To confirm that the described method is able to sort out the drawbacks pointed out in the Introduction, comparative results with a 1-D version of the resampling detector proposed by Popescu and Farid in [1] are also provided.

##### A. Performance analysis with synthetic signals

In this case, we consider as synthetic signal a first-order autoregressive (AR) process, parameterized by a single correlation coefficient  $\rho$ . The AR(1) model is commonly used for characterizing the correlation between samples of natural signals, where the value of  $\rho$  adjusts the model. Typically, close to 1 values are considered for modeling natural signals, as it is done with images [9]; hence,  $\rho = 0.95$  will be used in the following simulations. The AR(1) process has the following form

$$u(n) = w(n) + \rho u(n-1),$$

where  $w(n)$  is a Gaussian process with zero mean and variance  $\sigma_W^2$ . Note that in this case, the process  $w(n)$  is actually the innovation from one sample to another of the AR(1) process, so results will be drawn as a function of  $\sigma_W^2$  to evaluate the validity of the fine-quantization assumption.

To reproduce the conditions of the considered model, the original signal  $x(n)$  is obtained by quantizing the generated AR(1) process, i.e.,  $x(n) = Q_\Delta(u(n))$  with  $\Delta = 1$ . Regarding the set of considered resampling factors, for the sake of simplicity, we use a finite discrete set, obtained by sampling the interval  $(1, 5]$  with step size 0.05 (from 1.05 to 2) and 0.5 (from 2 to 5). Be aware that we use the same set for the true resampling factor  $\xi$  and the values tested by the ML estimator,  $\xi_t$ . We consider that the estimation of the resampling factor is correct if  $\hat{\xi} = \xi$ , i.e., if the estimated value is indeed the one used for resampling the original signal, up to the precision used when gridding  $\xi$  and  $\xi_t$ . For all the experiments, the length of the vector of observations is  $L_z = 400$ .

Fig. 3 shows the percentage of correct estimation for some of the resampling factors in the set as a function of  $\sigma_W^2$ . From this plot, we can observe that the performance of the estimator strongly depends on the mentioned variance of innovation, as well as on the true resampling factor used. For instance, by resampling the AR process with  $\xi = 5$ , a very small value for the variance of innovation ( $\sigma_W^2 = 0.5$ ), is required to correctly estimate the resampling factor for all the experiments;

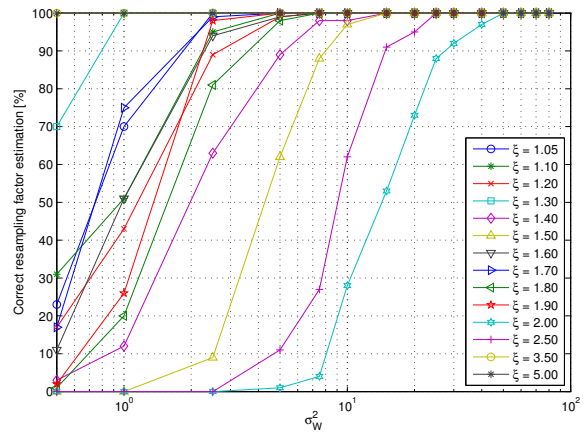


Fig. 3. Correct resampling factor estimation percentage for different resampling factors as a function of  $\sigma_W^2$ .  $\rho = 0.95$ , and 500 Monte Carlo realizations are considered.

nevertheless, for  $\xi = 2$ , almost a value of  $\sigma_W^2 = 50$  will be necessary for getting the same estimation performance. In general, and in accordance with the assumptions backing the analysis introduced in the previous section, the higher  $\sigma_W^2$ , the better the estimation will be.

Although ML-based estimators are frequently thought to be computationally demanding, if the fine-quantization assumption holds, then the estimation proposed in the previous section is very cheap and only a few samples are required for correctly estimating the actual resampling factor. Remember that when a resampling factor under test does not match the true one, then it can be discarded when an empty set is obtained for a non-visible sample or when a visible sample falls outside the obtained interval (cf. Steps 3 and 5 in Section III-B).

This is illustrated at Fig. 4, where the number of samples required for discarding the candidate resampling factor is shown for different values of  $\sigma_W^2$ , when  $\xi = \frac{3}{2}$ . As it can be checked in that figure, whenever the  $\xi_t = \xi$ , the tested resampling factor will not be discarded, even when the full vector of observations is considered, as it should be expected. It is also important to point out that the larger the value of  $\sigma_W^2$ , i.e., the more accurate the fine-quantization assumption is, the smaller number of samples is required for discarding a wrong  $\xi_t$ .

##### B. Performance analysis with real audio signals

For the evaluation of the estimator in a real scenario, we consider the “Music Genres” audio database [8], composed of 1000 uncompressed audio files with 10 different music styles (for instance some of them are blues, country, jazz, pop or rock). The performance of the proposed estimator will be checked by fixing the number of available samples, and looking for inconsistencies in the resampled signal with respect to the tested resampling factor. For comparison, the same tests will be performed with a state-of-the-art resampling detector, i.e., the one proposed by Popescu and Farid in [1].<sup>1</sup>

<sup>1</sup>The neighborhood of the predictor is set to  $N = 3$ , yielding a window of length 7.

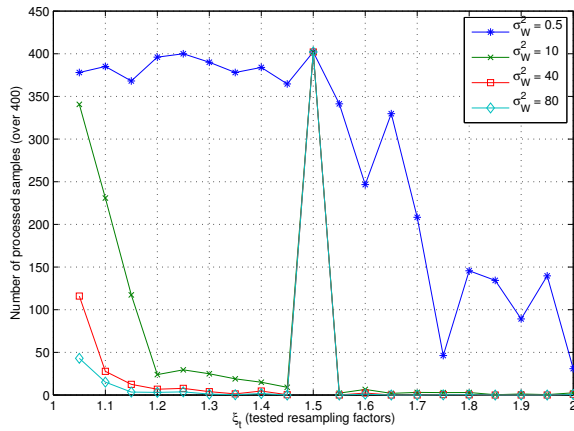


Fig. 4. Number of discarded samples for different values of  $\sigma_W^2$ , as a function of  $\xi_t$ . The true resampling factor is  $\xi = \frac{3}{2}$ . 500 Monte Carlo realizations were performed.

The set of resampling factors that we will consider in this case will be in the interval  $(1, 2]$  (sampled with a step size of 0.05). Since we are interested in comparing the performance with different sizes for the vector of observations, we perform the experiments with the following set of values  $L_z \in \{64, 128, 256, 512\}$ .

The results obtained with both methods are shown in Fig. 5. As we can observe, the method proposed by Popescu and Farid is highly dependent on the number of available samples, whereas our proposed MLE is essentially independent of this parameter. In the same way, the performance achieved by their method is poor when the applied resampling factor is close to 1, which is neither an issue for our estimator. These two limitations of Popescu and Farid's method come from the frequency analysis performed (once the pmap has been computed) for the detection of the resampling factor, as we pointed out in the Introduction. From these results, it is clear that the MLE method becomes very useful for estimating the resampling factor when a small number of samples are available, thus leading to a very practical forensic tool.

Although the performance of the MLE is very good, if we consider a noisy vector of observations then the method of Popescu and Farid is expected to be more robust than the proposed MLE. The reason is that in their model for the EM algorithm, they assume Gaussian noise, and in our case, we are only assuming the presence of uniformly distributed noise, due to the quantization. We note, however, that it is possible to extend our model to the case of Gaussian noise. Such extension is left for future research.

## V. CONCLUSIONS

The problem of resampling factor estimation following the ML criterion has been investigated in this work for the 1-D case. The derived MLE from this analysis has been tested with audio signals showing very good performance. The most distinctive characteristic of the proposed approach is that only a few number of samples of the resampled signal are needed to correctly estimate the used resampling factor.

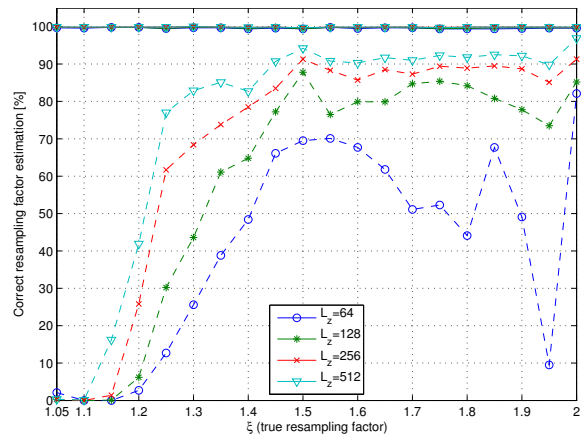


Fig. 5. Comparison of the correct estimation percentage of the proposed MLE versus the method proposed in [1]. Solid lines represent the obtained results with the MLE, while dashed lines are used for the method [1].

Since the scenario where the proposed resampling factor estimator can be employed is quite limited, future work will focus on improving this aspect. As a first step, the 2-D extension of the obtained method will be explicitly derived. Introduction of new parameters in the model such as general interpolation filters, noisy observations or resampling factors smaller than one will be studied. Possible links between this work and set membership algorithms will also be considered.

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