BANDLIMITED OR CONSTANT ENVELOPE? EXPLOITING WAVEFORM PROPERTIES IN WIRELESS MICROPHONE DETECTION

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ABSTRACT

The detection of bandlimited (BL) constant magnitude (CM) signals in white noise of unknown power is analyzed. This is relevant in the context of Dynamic Spectrum Access since Wireless Microphones (WM) typically use analog FM modulation when transmitting in the TV bands, and their bandwidth is much smaller than that of a TV channel. Although detectors exploiting either the BL or the CM properties have been presented in the literature, these two features have not been jointly considered yet. We derive the Generalized Likelihood Ratio test for this setting. Performance is evaluated in the framework for WM simulation developed to assist the IEEE 802.22 Working Group.

Index Terms— Dynamic Spectrum Access, Wireless Microphones, Constant Magnitude, Bandlimited signals.

1. INTRODUCTION

Dynamic Spectrum Sharing [1] has been proposed as a means to alleviate the lack of spectral resources affecting the proliferation of new wireless communications services. Under this scheme, secondary users are allowed to access the spectrum provided that they do not cause detrimental interference to the primary system. According to the Federal Communications Commission (FCC), there are two kinds of primary users operating in the TV band which must be respected, namely the digital TV operators and the Wireless Microphones (WM).

Secondary users need to incorporate spectrum sensing capabilities in order to detect the presence of those primary transmissions, and this has been extensively analyzed in the literature throughout the past decade [2]. Most approaches hinge on Energy Detection, which has been demonstrated to exhibit poor performance under noise power uncertainty [3]. Hence, exploiting additional signal features other than energy becomes desirable.

Whereas the detection of Digital TV signals has spurred many studies exploiting cyclostationarity, pilot tones, sample distribution, etc., the detection of WM signals (typically analog FM waveforms) has been relatively pushed into the background. Previous works include [4] and [5], whose respective test statistics are the maximum of the periodogram and the output of a matched filter in the autocorrelation domain. Both methods require knowledge of the noise power in order to adequately set the threshold, and therefore suffer in the presence of noise power uncertainty, similarly to the Energy Detector.

On the other hand, detectors have been proposed for bandlimited (BL) signals with unknown center frequency [6], and Constant Magnitude (CM) signals [7], both in white noise of unknown power. Such approaches are of practical interest in the context of WM detection in TV bands. As opposed to a TV channel, with typical bandwidth of 6, 7 or 8 MHz, the bandwidth of a WM transmission is never larger than 200 kHz [8]. Intuition suggests that detection performance should improve if this fact is taken into account, since a sizable amount of noise could be filtered out. In addition, FM signals are CM waveforms, and exploiting this feature should help in the detection process as well.

Our main contribution is the derivation of the Generalized Likelihood Ratio (GLR) test exploiting jointly the BL and CM waveform properties. The resulting scheme is seen to outperform previous detectors from [6] and [7] that exploit only one of these features.

We present the system model in Sec. 2 and derive the GLR test in Sec. 3. Simulation results in the context of WM detection in TV bands are shown in Sec. 4. Finally, conclusions are drawn in Sec. 5.

2. SYSTEM MODEL

The spectrum sensor collects K baseband samples gathered in the $K \times 1$ vector $\boldsymbol{y}$. The channel is assumed frequency-flat, and time-invariant during the sensing interval. The hypothesis testing problem can be stated as

$$\begin{aligned} \mathcal{H}_0 &: \quad \boldsymbol{y} = \sigma \boldsymbol{w}, \\ \mathcal{H}_1 &: \quad \boldsymbol{y} = h\boldsymbol{x} + \sigma \boldsymbol{w}, \end{aligned}$$

where the noise vector $\boldsymbol{w}$ is zero-mean circular complex Gaussian with $E\{\boldsymbol{w}\boldsymbol{w}^H\} = \boldsymbol{I}_K$, h is the channel gain, σ^2 is

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the noise power and $\boldsymbol{x}$ is the signal vector. It is assumed that $\boldsymbol{x}$, h and σ^2 are deterministic unknown parameters. The CM property of the signal waveform implies that $\boldsymbol{x}$ is a phase-only vector, i.e. each entry of $\boldsymbol{x}$ has unit magnitude. In addition, it is assumed that $\boldsymbol{x}$ is bandlimited to a frequency interval $\mathcal{B}$ with length $B < 2\pi$ rad and unknown central frequency ω_0. The vector $\boldsymbol{y}$ is conditionally Gaussian under both hypotheses:

$$p(\boldsymbol{y};\sigma^2\,|\,\mathcal{H}_0) \;=\; \frac{1}{(\pi\sigma^2)^K}\exp\left\{-\frac{||\boldsymbol{y}||_2^2}{\sigma^2}\right\}, \qquad (1)$$

$$p(\boldsymbol{y};\sigma^2,\boldsymbol{x},h\,|\,\mathcal{H}_1) \;=\; \frac{1}{(\pi\sigma^2)^K}\exp\left\{-\frac{||\boldsymbol{y}-h\boldsymbol{x}||_2^2}{\sigma^2}\right\}. \qquad (2)$$

3. DETECTION OF CM BANDLIMITED SIGNALS

According to the GLR detection rule [9], the unknown parameters are substituted for their Maximum Likelihood (ML) estimates in the Likelihood Ratio test:

$$\ell(\boldsymbol{y}) \doteq \frac{\max\limits_{\sigma^2,\boldsymbol{x},h} p(\boldsymbol{y};\sigma^2,\boldsymbol{x},h\,|\,\mathcal{H}_1)}{\max\limits_{\sigma^2} p(\boldsymbol{y};\sigma^2\,|\,\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \qquad (3)$$

The ML estimates of σ^2 under $\mathcal{H}_0$ and $\mathcal{H}_1$ are respectively

$$\hat{\sigma}_0^2 \doteq \arg\max_{\sigma^2} p(\boldsymbol{y};\sigma^2\mid\mathcal{H}_0) = \frac{||\boldsymbol{y}||_2^2}{K}, \qquad (4)$$

$$\hat{\sigma}_1^2 \doteq \arg\max_{\sigma^2} p(\boldsymbol{Y};\sigma^2,\boldsymbol{x},h\mid\mathcal{H}_1) = \frac{||\boldsymbol{y}-h\boldsymbol{x}||_2^2}{K}. \qquad (5)$$

In view of (4) and (5), it is clear that (3) can also be written as

$$\ell(\boldsymbol{y}) = \left[\frac{||\boldsymbol{y}||_2^2}{||\boldsymbol{y}-\hat{h}\hat{\boldsymbol{x}}||_2^2}\right]^K \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma. \qquad (6)$$

where $\hat{h}$ and $\hat{\boldsymbol{x}}$ are the ML estimates of h and $\boldsymbol{x}$. Substituting (5) into (2) produces

$$p(\boldsymbol{y};\hat{\sigma_1^2},\boldsymbol{x},h\mid\mathcal{H}_1) = \left[\frac{\pi e}{K}\cdot||\boldsymbol{y}-h\boldsymbol{x}||_2^2\right]^{-K}. \qquad (7)$$

Clearly, the values for $\boldsymbol{x}$ and h maximizing (7) are those minimizing $||\boldsymbol{y}-h\boldsymbol{x}||^2$. Applying the discrete-time version of Parseval's identity enables us to write

$$||\boldsymbol{y}-h\boldsymbol{x}||_2^2 = \frac{1}{2\pi}\int_{2\pi}|Y(e^{j\omega})-hX(e^{j\omega})|^2d\omega,$$

where $Y(e^{j\omega})$ and $X(e^{j\omega})$ are, respectively, the Fourier transforms of $\boldsymbol{y}$ and $\boldsymbol{x}$. If $\hat{\mathcal{B}}$ denotes the ML estimate of the frequency interval $\mathcal{B}$, it is clear that $X(e^{j\omega})$ must vanish outside, so that

$$||\boldsymbol{y}-h\boldsymbol{x}||_2^2 \;=\; \frac{1}{2\pi}\int_{\hat{\mathcal{B}}}|Y(e^{j\omega})-hX(e^{j\omega})|^2d\omega + \frac{1}{2\pi}\int_{2\pi-\hat{\mathcal{B}}}|Y(e^{j\omega})|^2d\omega \;=\; \mathcal{I}+\mathcal{G}. \qquad (8)$$

Let $\hat{\omega}_c$ be the central frequency of $\hat{\mathcal{B}}$, and let $L \doteq \frac{2\pi}{B}$. Let $X_a(e^{j\omega}) \doteq X(e^{j(\omega-\hat{\omega}_c)})$, and let $Y_a(e^{j\omega})$ denote the result obtained after applying an ideal lowpass filter with cutoff frequency $\frac{\pi}{L}$ to $Y(e^{j(\omega-\hat{\omega}_c)})$. Note that both $X_a(e^{j\omega})$, $Y_a(e^{j\omega})$ are baseband spectra, bandlimited to π/L rad. Then the first term in the right-hand side of (8) becomes

$$\mathcal{I} = \frac{1}{2\pi}\int_{-\frac{\pi}{L}}^{\frac{\pi}{L}}|Y_a(e^{j\omega})-h\,X_a(e^{j\omega})|^2d\omega \qquad (9)$$

$$= \frac{1}{2\pi L}\int_{-\pi}^{\pi}|Y_a(e^{j\theta/L})-h\,X_a(e^{j\theta/L})|^2d\theta. \qquad (10)$$

Let $\boldsymbol{x}_a$, $\boldsymbol{y}_a$ be the inverse Fourier transforms of $X_a(e^{j\omega})$, $Y_a(e^{j\omega})$ respectively. Consider the operation of downsampling these vectors by a factor L, and denote the resulting vectors by $\boldsymbol{x}_d$, $\boldsymbol{y}_d$, with respective Fourier transforms $X_d(e^{j\omega})$, $Y_d(e^{j\omega})$. Then (10) can be written as

$$\mathcal{I} = \frac{1}{2\pi L}\int_{-\pi}^{\pi}|L\cdot Y_d(e^{j\theta})-L\cdot h\,X_d(e^{j\theta})|^2d\theta \qquad (11)$$

$$= L||\boldsymbol{y}_d-h\,\boldsymbol{x}_d||_2^2. \qquad (12)$$

Observe that due to the BL property, there is a one-to-one correspondence between $\boldsymbol{x}$ and its frequency-translated and downsampled version $\boldsymbol{x}_d$. Moreover, these operations clearly preserve the CM property. Hence, we must minimize (12) w.r.t. h and $\boldsymbol{x}_d$, subject to $\boldsymbol{x}_d$ being CM. Note that there is no BL constraint on $\boldsymbol{x}_d$.

Minimizing (12) w.r.t. h is a straightforward LS problem, yielding

$$\mathcal{I} = L\left[||\boldsymbol{y}_d||_2^2 - \frac{1}{P}|\boldsymbol{y}_d^H\boldsymbol{x}_d|^2\right], \qquad (13)$$

where $P \doteq \lfloor K/L\rfloor$ is the length of $\boldsymbol{y}_d$ and $\boldsymbol{x}_d$. The minimization of (13) w.r.t. $\boldsymbol{x}_d$ subject to the CM constraint on its entries can be done in the same way as in [7], resulting in

$$\mathcal{I} = L\left[||\boldsymbol{y}_d||_2^2 - \frac{1}{P}||\boldsymbol{y}_d||_1^2\right]. \qquad (14)$$

On the other hand, the second term of the sum in (8) can be rewritten as

$$\mathcal{G} = \frac{1}{2\pi}\int_{2\pi}|Y(e^{j\omega})|^2d\omega - \frac{1}{2\pi}\int_{\hat{\mathcal{B}}}|Y(e^{j\omega})|^2d\omega = ||\boldsymbol{y}||_2^2 - ||\boldsymbol{y}_a||_2^2. \qquad (15)$$

Substituting (14)-(15) in (8) results in

$$||\boldsymbol{y}-\hat{h}\hat{\boldsymbol{x}}||_2^2 = L\left[||\boldsymbol{y}_d||_2^2 - \frac{1}{P}||\boldsymbol{y}_d||_1^2\right] + ||\boldsymbol{y}||_2^2 - ||\boldsymbol{y}_a||_2^2, \qquad (16)$$

which, in turn, can be substituted in (6) to give

$$\ell(\boldsymbol{y}) = \left[\frac{||\boldsymbol{y}||_2^2}{L\left[||\boldsymbol{y}_d||_2^2-\frac{1}{P}||\boldsymbol{y}_d||_1^2\right]+||\boldsymbol{y}||_2^2-||\boldsymbol{y}_a||_2^2}\right]^K \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \qquad (17)$$

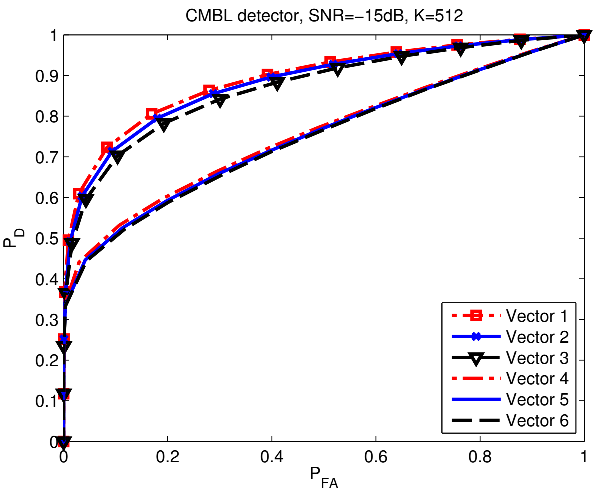

Fig. 1. ROC curves of the proposed detector in the context of Wireless Microphone detection.

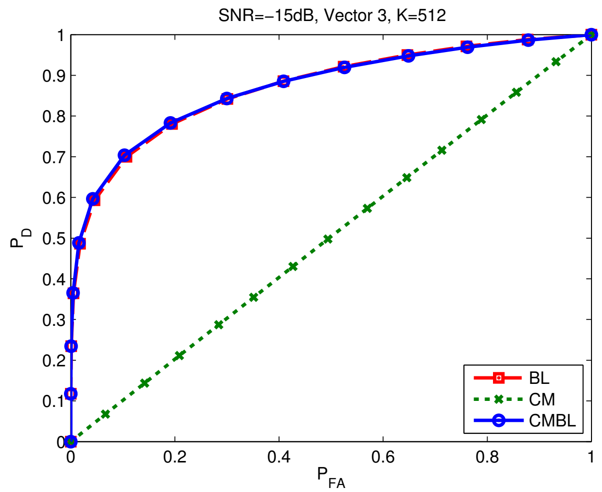

Fig. 2. Comparison of the CM, BL, and CMBL detectors.

#	Description	f_m	Δf	Fading
1	Outdoor, LOS, Silent	32	5	No
2	Outdoor, LOS, Soft Speaker	3.9	15	No
3	Outdoor, LOS, Loud Speaker	13.4	32.6	No
4	Indoor, NLOS, Silent	32	5	Yes
5	Indoor, NLOS, Soft Speaker	3.9	15	Yes
6	Indoor, NLOS, Loud Speaker	13.4	32.6	Yes

Table 1. Test vectors employed in WM simulation [8]. Frequency units are kHz in all cases.

Note that the test (17) is equivalent to the following one:

$$T(\boldsymbol{y}) = \frac{||\boldsymbol{y}_d||^2 - \frac{1}{P}||\boldsymbol{y}_d||_1^2 - \frac{1}{L}||\boldsymbol{y}_a||^2}{||\boldsymbol{y}||^2} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \gamma' \tag{18}$$

where $\gamma' = (1 - \gamma^{-1/K})/L$, since $\ell(\boldsymbol{y})$ is a monotonically decreasing function of $T(\boldsymbol{y})$. An interesting remark ensues from rewriting the statistic in (17) as:

$$\ell(\boldsymbol{y}) = \left[\frac{1}{1 - \frac{||\boldsymbol{y}_a||_2^2}{||\boldsymbol{y}||_2^2}} \cdot \frac{1}{1 + L\frac{||\boldsymbol{y}_d||_2^2 - \frac{1}{P}||\boldsymbol{y}_d||_1^2}{||\boldsymbol{y}||_2^2 - ||\boldsymbol{y}_a||_2^2}} \right]^K. \tag{19}$$

The first factor in (19) indicates how well the energy of the observations is concentrated in the frequency interval $\hat{\mathcal{B}}$, i.e. it is a BL-related measure. The second factor is a CM-related measure, after frequency-translation and decimation.

The final step is to obtain the ML estimate $\hat{\omega}_c$ under $\mathcal{H}_1$, which is the minimizer of (16) (note that both $\boldsymbol{y}_a$, $\boldsymbol{y}_d$ depend on $\hat{\omega}_c$). This minimization is best performed in the frequency domain. Let $\boldsymbol{W}$ be the unitary IDFT matrix $\boldsymbol{W}$, and let the $K \times 1$ vector $\boldsymbol{p}_0 = [\boldsymbol{1}_P^T \; \boldsymbol{0}_{K-P}^T]^T$. Define $\boldsymbol{p}_n$ as the n-th downward circular shift of the vector $\boldsymbol{p}_0$, and let $\boldsymbol{P}_n = \text{diag}\{\boldsymbol{p}_n\}$. Thus, one has $\boldsymbol{y}_a \approx \boldsymbol{W}\boldsymbol{P}_n\boldsymbol{W}^H\boldsymbol{y}$. If L is an integer, then $\boldsymbol{y}_d \approx \boldsymbol{D}\boldsymbol{y}_a$, where $\boldsymbol{D}$ is a $P \times K$ decimation matrix consisting of ones and zeros. When L is not an integer, an interpolation step is required previously to the decimation step. All products involving $\boldsymbol{W}$ can be efficiently computed via the FFT algorithm. For each n, one must evaluate (16) and, finally, choose the n which leads to the smallest result.

4. SIMULATION RESULTS

We analyze the performance of the proposed test in the context of WM signal detection, following the guidelines of the IEEE 802.22 Working Group [8]. Table 1 summarizes the test vectors proposed in [8] for this purpose. The WM signals are generated as

$$x(t) = \exp\left\{ j\left(2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)\right)\right\}$$

with f_c the carrier frequency, Δf the frequency deviation and f_m the modulating frequency. The WM signal is to be detected within a given TV channel of 8 MHz bandwidth, and consequently the sampling rate is set at 8 Msps. Since the maximum bandwidth of a WM signal is 200 kHz, only 2.5% of the bandwidth would be occupied by the WM signal.

All channels considered are frequency flat. In the case of the LOS channel, the magnitude of h is deterministic and the phase uniformly distributed in $[0, 2\pi]$. For the NLOS channel, h is a realization of a complex circular Gaussian random variable (Rayleigh channel), and the SNR is defined as $\eta \doteq E\left\{|h|^2\right\}/\sigma^2$.

The the Receiver Operating Characteristic (ROC) of the proposed CMBL detector, using $L = 40$ and $K = 512$ samples, is shown in Fig. 1 for the six test vectors at SNR $= -15$ dB. Vectors 1-3 appear to be easier to detect than Vectors 4-6; in view of Table 1, we can conclude that this is a consequence

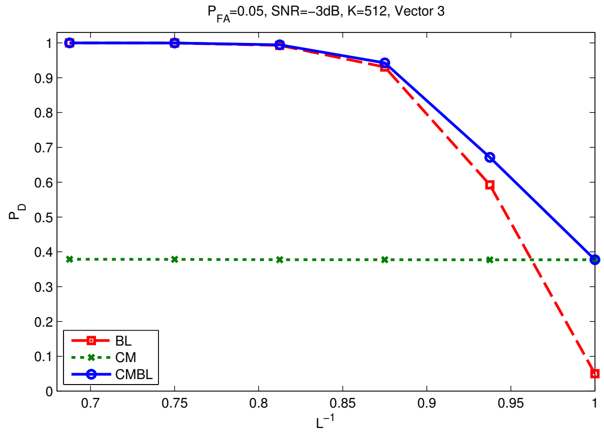

Fig. 3. Detection performance in terms of bandwidth occupancy.

of Rayleigh fading. It is also seen that a larger frequency deviation (and therefore a larger signal bandwidth) leads to a degradation in detection performance.

Using Vector 3, Fig. 2 shows the ROC of the CMBL detector, together with those from [6] and [7], which exploit only the BL and CM properties of the signal, respectively. The CM detector fares badly at this low SNR value. On the other hand, the CMBL detector outperforms the BL detector, but just slightly. The conclusion is that the CM property is a much weaker feature in terms of detection power than the BL property, which is more resilient in the low SNR regime.

In order to further clarify this issue, we present the results obtained by varying the fraction of bandwidth occupancy $1/L$. In particular, the signal was generated using the parameters corresponding to Vector 3 from Table 1, but the sampling rate was reduced in order to achieve different values of $1/L$. Fig. 3 shows the probability of detection *vs.* bandwidth occupancy. It is seen that the performance of the CM detector is independent of bandwidth occupancy. On the other hand, the BL detector performance degrades when bandwidth occupancy is above 80%. In the limit case of full occupancy the BL detector becomes useless (prob. of detection equals the false alarm rate), since it becomes impossible to tell signal from noise without exploiting additional signal features. This is precisely what the CMBL detector does: as expected, its performance is always at least as good as that of the better of the other two detectors (CM and BL).

5. CONCLUSIONS

We have derived the GLR test for bandlimited constant modulus signals in white noise of unknown power. These two waveform features are jointly exploited in order to improve detection performance with respect to previous schemes that only exploit either of these two properties. It has been shown that the bandlimited property is a much more powerful feature (in terms of detection performance) than the constant envelope property, unless the bandwidth occupancy is close to 100%. This observation is particularly relevant in the context of detecting Wireless Microphone signals operating in the TV band, since the bandwidth of WM waveforms is much smaller than that of a TV channel. In particular, the additional complexity that is introduced in order to accommodate the constant envelope constraint within the detector may not be worthwhile at all. The detector for bandlimited signals requires just one FFT, since the energy present within a given band can be estimated in the frequency domain. But if the constant envelope property is to be exploited as well, an additional FFT is required for each candidate carrier frequency that one has to try. Hence, practical usage of the detector proposed is likely to be relegated to the most demanding applications, or to those where the carrier frequency is known *a priori* to be in a finite, small set.

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