Overlay Cognitive Transmission in OFDM Point to Point Systems Exploiting Primary Feedback

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***Abstract*—A secondary user that tries to reuse the spectrum allocated to a primary user can exploit the knowledge of the primary message to perform this task. In particular, the overlay cognitive radio paradigm postulates the use of a fraction of the available power at the secondary transmitter to convey the primary message, so the spectral efficiency of the primary system is increased, and, therefore, some transmission resources (time slots or frequency bands) can be released to the secondary transmission while the primary user rate is kept constant. The fraction of released resources can be incremented if some channel state information is available at the secondary transmitter. In this paper, we present a scenario where the secondary transmitter maximizes the primary link quality (measured in terms of Effective SNR), and obtains its channel state information by exploiting the primary user SNR-based feedback.**

***Index Terms*—Overlay cognitive radio, Power allocation, Channel estimation, Spectrum reuse**

I. Introduction

The Overlay Cognitive Radio paradigm [1] presents a scenario where a secondary user makes use of the knowledge of a primary user waveform in order to reuse the spectrum allocated to licensed users. This knowledge can be possible, for example, if the primary system uses an Automatic Repeat reQuest (ARQ) protocol, so the secondary transmitter could have prior information of a primary retransmission if it was able to decode the first transmission [1], or if the secondary transmitter is located near the primary one, so a high-rate wired or wireless link can be used to convey the primary message.

This knowledge can be exploited by transmitting the primary signal from the secondary transmitter in order to accommodate its own information while preserving the primary user Quality of Service. Depending on the nature of the system, and specially on the existence of feedback channels and adaptive transmission, the primary transmitter can change its behavior in presence of a secondary transmitter that conveys the primary information. For example, in the case of point-to-point communications it is usual to have a feedback channel from the receiver to the transmitter to perform some tasks such as Adaptive Modulation and Coding (AMC), power and bit loading, etc. In this case the secondary transmitter can obtain additional information about the primary link if it is able to demodulate the feedback signal. In these systems, the primary transmitter can operate in different modes, trying to maximize the spectral efficiency for a given channel quality. Thus, the correct metric for the primary user communication is the resulting bit rate. If the bit rate is larger than the one the primary communication needs, the primary transmitter is going to free some transmission resources (in the frequency or time domain) [2]. If this is the case, the secondary information can be transmitted in the released resources due to the primary user reinforcement.

However, it is usually assumed that the secondary transmitter has full Channel State Information (CSI), or, if measured in capacity terms, that the contributions from the primary and secondary transmitters are coherently added at the primary receiver location [2]. As opposed to this, here we will focus on a point-to-point scenario where the secondary transmitter tries to maximize the primary user rate based on the partial CSI obtained by means of the primary feedback concerning the Signal to Noise Ratio (SNR). The paper is organized as follows: in Section II the proposed scenario is presented; in Section III a general optimization problem, based on Effective SNR metrics, is introduced; in Section IV different approximations for the Mutual Information Effective SNR metric are presented, and the optimization problem is solved assuming perfect CSI; in Section V the problem of obtaining CSI by exploiting the primary feedback channel is stated and the effects of imperfect CSI in the previously solved optimization problem are presented; finally, Section VI concludes the paper.

II. Proposed scenario

Let us assume an Orthogonal Frequency Division Multiplexing (OFDM) point-to-point communication system where a Primary Transmitter (PT) is communicating with a single Primary Receiver (PR). A Secondary Transmitter (ST) tries to exploit the knowledge of the primary signal in order to communicate with a Secondary Receiver (SR). The PR conveys Channel State Information (CSI) to the PT, so the latter can use AMC to maximize the link throughput, or minimize the communication time. Let us assume that the communication needs of the primary system can be set to R_p bits per time unit. For the sake of simplicity, we will assume that the OFDM symbols have constant length, and use the OFDM block as the time unit, so R_p denotes the necessary

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bit rate for the primary system, measured in terms of bits per OFDM symbol.

Depending on the quality of the link, the resulting AMC mode will set the transmission rate to R_s bits per OFDM block. It is clear that if $R_s < R_p$ the link does not provide the sufficient quality for the transmission, but if $R_s \geq R_p$ only a fraction $\rho \doteq \frac{R_p}{R_s}$ of the transmission resources will be used, and the remaining $(1-\rho)$ could be used by the secondary transmitter. For an ARQ system, another convenient figure of merit is the throughput or the *goodput*, metrics that include the performance loss due to the presence of message retransmissions. In this case, the fraction of used resources can be written as $\rho = \frac{\mu_p}{\mu_s}$, where μ_p denotes the throughput required by the primary system, and μ_s the total throughput after the insertion of the secondary transmitter.

Thus, the maximization of the secondary user rate is equivalent to the minimization of ρ or, equivalently, the maximization of R_s or μ_s.

In general, choosing the correct AMC mode for a given channel state is not a trivial problem when facing frequency selective channels in OFDM communications, as the mean SNR is not a good indicator of the channel quality. In order to face this problem, different Effective SNR Metrics (ESM) were recently developed [3]. These metrics can be expressed as a generalized mean, parametrized[1] by the function $\Theta(\cdot)$, of the SNR at each carrier

$$\psi_e = \Theta^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\Theta(\psi_i)\right) \tag{1}$$

where ψ_e denotes the effective SNR and ψ_i denotes the SNR of the i-th carrier. The AMC mode (also known as MODCOD) will be selected depending on the value ψ_e from a set $\mathcal{M} = \{m_1, ..., m_M\}$ of M different modes, each one with an associated rate of R_i $i = 1 \dots M$ bits per OFDM block. Without loss of generality, we will order the modes in such a way that $R_1 < R_2 < \dots < R_M$, with associated mode thresholds $0 = t_0 < t_1 < \dots < t_{M-1} < t_M = +\infty$ such that the mode m_i is selected if $t_{i-1} < \psi_e < t_i$. In general, the throughput function is more difficult to approximate, as it must take into account packet retransmissions. In this work, we will approximate the throughput $\mu(\psi_e)$ by the linear interpolation of the rate at the MODCOD threshold values, i.e. $\mu(\psi_e) = \frac{R_i - R_{i-1}}{t_i - t_{i-1}}(\psi_e - t_{i-1}) + R_{i-1}$, with $t_{i-1} \leq \psi_e < t_i$.

Let us denote η_i as the (complex) channel coefficient of the i-th carrier of the PT to PR link, and σ^2 as the noise power at the PR, assumed to be constant at every carrier, without loss of generality. Thus, in absence of the ST, and assuming a unit power primary signal, we can write $\psi_i = \frac{|\eta_i|^2}{\sigma^2}$. With the insertion of the ST, the resulting SNR can be written as $\psi_i = \frac{|\alpha_i\gamma_i + \eta_i|^2}{\sigma^2}$, where α_i is the complex channel coefficient of the i-th carrier of the ST to PR link, and γ_i is a one-tap pre-equalizer at the ST that allows us to change the amplitude and phase of the transmitted symbols. This scenario is depicted in Figure 1.

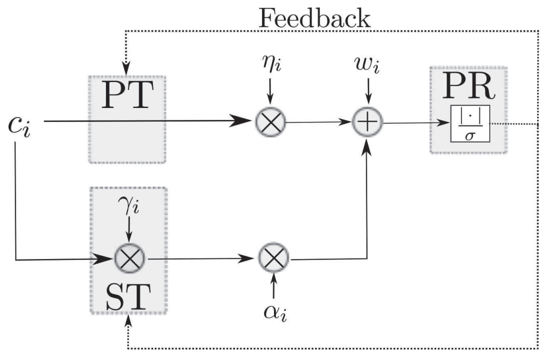

Fig. 1. Diagram of the proposed scenario. The ST uses a one-tap per carrier pre-equalizer to maximize the primary link ESM. The PR feeds back information related to the SNR of each carrier (see Section V). c_i is the symbol to be transmitted in the i-th carrier in a given OFDM block, assumed to have unit variance, and $w_i \sim \mathcal{CN}(0, \sigma^2)$.

III. Problem Statement

The objective of the secondary user is to maximize the quality of the primary link (measured in terms of ESM) in order to obtain a fraction of released resources $1-\rho$ as large as possible. In our first approach, we will assume that the channel coefficients α_i, η_i are perfectly known at the secondary transmitter, and in Section V a method that estimates these parameters exploiting the feedback channel will be described.

The design variables in our optimization problem are the complex values γ_i for a total transmit power below a given value P. It is clear that for a given power allocated to the i-th carrier $|\gamma_i|^2$, the optimum value for γ_i is $\gamma_i = |\gamma_i| e^{j(\angle\eta_i - \angle\alpha_i)}$, so the signals are coherently combined at the receiver, and $\psi_i = \left(|\alpha_i\gamma_i|^2 + |\eta_i|^2 + 2|\alpha_i\gamma_i\eta_i|\right)/\sigma^2$. Therefore, and without loss of generality, we will assume that α_i, γ_i and η_i are real and non-negative values (just by taking the modulus of the complex coefficients), so the optimization problem can be stated as

$$\begin{array}{ll} \text{minimize} & -\Theta^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\Theta(\psi_i)\right) \\ \text{subject to} & \frac{1}{N}\sum_{i=1}^{N}\gamma_i^2 \leq P \\ & -\boldsymbol{\gamma} \preceq 0 \end{array} \tag{2}$$

with $\psi_i = \frac{(\alpha_i\gamma_i + \eta_i)^2}{\sigma^2}$, and $\boldsymbol{\gamma} = [\gamma_1, \dots \gamma_N]^T$. Obviously, the result of the optimization problem is going to vary depending on the function Θ. In the following section, we will study the optimum power allocation corresponding to the Mutual Information Equivalent SNR Metric (MIESM).

[1] For example, following expression (1), the arithmetic mean is parametrized by the function $\Theta(x) = x$, and the geometric mean by $\Theta(x) = \log(x)$.

IV. Power allocation for MIESM metric

The MIESM is based on the mutual information per bit. The expression for Θ, taken from [4], is

$$\Theta(\psi) = \quad (3)$$
$$1-\frac{1}{M\log_2 M}\sum_{m=1}^{M} E_U\left\{\log_2\left(\sum_{k=1}^{M} e^{-\frac{|X_m-X_k+U|-|U|^2}{1/\psi}}\right)\right\}$$

where M is the number of symbols in the constellation, and U is a complex Gaussian random variable of zero mean and variance $1/\psi$ ($1/(2\psi)$ per dimension). We also denote by X_m, $m = 1, ..., M$ the M complex constellation points. Note that this metric does not depend on the code rate being used, but only on the constellation.

As there is not a closed-form expression for (3), we will approximate Θ by two different functions in order to obtain analytical results of interest, although these results will be evaluated using the actual value of Θ, obtained by Monte Carlo integration.

On a first approach, we will approximate Θ by a parametrized exponential function, similarly to [5],

$$\Theta(\psi) = 1-\sum_{l=1}^{L}\phi_l e^{-\beta_l\psi} \quad (4)$$

where $\sum_{l=1}^{L}\phi_l = 1$, and $\phi_l \geq 0$ and $\beta_l \geq 0$ are parameters that have to be properly chosen in order to fit the actual value of (3). Note that the approximation with $L = 1$ makes this metric equivalent to the Exponential ESM (EESM) [3], so we can think of this approximation as a Generalized Exponential Effective SNR Metric of degree L (L-GEESM). Therefore, the results for this approximation can be directly applied to the EESM metric just by setting $L = 1$.

Additionally, we propose to approximate the function Θ by a piecewise linear function (PLF) in the logarithmic domain

$$\Theta(\psi) = \begin{cases} 0 & \psi < \psi_0 \\ \frac{\log_{10}(\psi)-\log_{10}(\psi_0)}{\log_{10}(\psi_1)-\log_{10}(\psi_0)} & \psi_0 \leq \psi \leq \psi_1 \\ 1 & \psi > \psi_1 \end{cases} \quad (5)$$

where ψ_0 and ψ_1 have to be adjusted to approximate (3). Note that this PLF approximation makes the ESM ψ_e equal to the arithmetic mean of the SNR values ψ_i in the logarithmic domain, where extreme values for ψ_i are not taken into account, as values of $\psi_i > \psi_1$ and $\psi_i < \psi_0$ are *clipped* to ψ_1 and ψ_0 respectively.

The results of the fitting, performed with the MATLAB *Curve Fitting* toolbox, are shown in Figure 2. The approximation with the L-GEESM is only shown for values of $L = 1, 2, 3$, as the benefit of using higher order approximation is almost unnoticeable. In the following, we will try to maximize the MIESM by using these two approximations in (2).

A. L-GEESM approximation

The maximization of the L-GEESM can be seen to be equivalent to maximizing $\Theta(\psi_e) = \frac{1}{N}\sum_{i=1}^{N}\Theta(\psi_i)$, as in this case Θ is given by (4), which is a monotonic increasing function. Therefore, by removing constant terms in the objective function and adding a power constraint on $\boldsymbol{\gamma}$, we arrive to

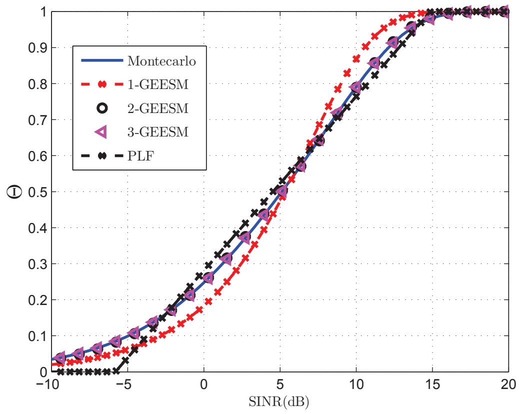

Fig. 2. Approximations for the mutual information Θ for a 16-QAM constellation.

$$\begin{array}{ll} \text{minimize} & f_0(\boldsymbol{\gamma}) \doteq \frac{1}{N}\sum_{i=1}^{N}\sum_{l=1}^{L}\phi_l e^{-\beta_l\frac{(\alpha_i\gamma_i+\eta_i)^2}{\sigma^2}} \\ \text{subject to} & f_1(\boldsymbol{\gamma}) \doteq \frac{1}{N}\sum_{i=1}^{N}\gamma_i^2 - P \leq 0 \\ & \mathbf{f}_2(\boldsymbol{\gamma}) \doteq -\frac{1}{N}\boldsymbol{\gamma} \preceq 0 \end{array} \quad (6)$$

where the factor $1/N$ has been introduced in the last constraint in order to simplify the upcoming expressions, and $\mathbf{f}_2(\boldsymbol{\gamma}) = [f_{2,1}(\boldsymbol{\gamma}), ..., f_{2,N}(\boldsymbol{\gamma})]^T = -\frac{1}{N}[\gamma_1, ..., \gamma_N]^T$.

The Karush-Kuhn-Tucker (KKT) conditions for the problem (6) are

$$-\sum_{l=1}^{L}\phi_l\frac{2\beta_l}{\sigma^2}\alpha_i(\alpha_i\gamma_i+\eta_i)e^{-\frac{\beta_l(\alpha_i\gamma_i+\eta_i)^2}{\sigma^2}}$$
$$+2\lambda_1\gamma_i - \lambda_{2,i} = 0 \quad (7)$$
$$\lambda_1 f_1(\boldsymbol{\gamma}) = 0 \quad (8)$$
$$\lambda_1 \geq 0 \quad (9)$$
$$\boldsymbol{\lambda}_2^T\mathbf{f}_2(\boldsymbol{\gamma}) = 0 \quad (10)$$
$$\boldsymbol{\lambda}_2 \succeq 0 \quad (11)$$

where $\boldsymbol{\gamma}$ is a feasible point of (6). We will study the conditions by making assumptions about the different constraints being active or not.

If f_1 is not active ($f_1(\boldsymbol{\gamma}) < 0$), then we have from (8) that $\lambda_1 = 0$, so (7) reads as

$$-\sum_{l=1}^{L}\phi_l\frac{2\beta_l}{\sigma^2}\alpha_i(\alpha_i\gamma_i+\eta_i)e^{-\frac{\beta_l(\alpha_i\gamma_i+\eta_i)^2}{\sigma^2}} - \lambda_{2,i} = 0\,\forall i = 1..., N. \quad (12)$$

As the values α_i,γ_i, η_i, β_l, σ^2 and $\lambda_{2,i}$ are non-negative, the N equalities in (12) will never be met, except in some degenerate cases, such as $\alpha_i = 0\,\forall i$, which are not of interest. This means that if the point $\boldsymbol{\gamma}$ is optimum, $f_1(\boldsymbol{\gamma}) = 0$. This fact can be easily seen in problem (6), where the terms in the sum of the objective function are decreasing functions of γ_i, so allocating the remaining power to any of the terms will make the objective function decrease and, therefore, a point with non-active f_1 cannot be optimum.

If $f_{2,i}$ and f_1 are active, i.e., $\gamma_i = 0$, (7) reads as $-\sum_{l=1}^{L} \phi_l \frac{2\beta_l}{\sigma^2} \alpha_i \eta_i e^{-\frac{\eta_i \beta_l}{\sigma^2}} = \lambda_{2,i}$, or, as the left part of the equation is non-positive, and condition (11) constraints $\lambda_{2,i}$ to be non-negative, $\sum_{l=1}^{L} \phi_l \frac{2\beta_l}{\sigma^2} \alpha_i \eta_i e^{-\frac{\eta_i \beta_l}{\sigma^2}} = 0$, so the condition is only met if α_i or η_i are equal to zero. Note that in the case $\alpha_i = 0$ it is clear that allocating power to the i-th carrier is not going to change the objective function value, so that power consumption is useless. Since these are again degenerate cases, we can state that for a non-degenerate problem (α_i and η_i being strictly positive), the power constraint f_1 is going to be active ($f_1(\boldsymbol{\gamma}) = 0$), and the N constraints $\mathbf{f}_2$ are going to be inactive ($\gamma_i > 0 \, \forall \, i$).

Therefore, the condition (7) for a non-degenerate problem is

$$\sum_{l=1}^{L} \phi_l \frac{\beta_l}{\sigma^2} \alpha_i \left(\alpha_i + \frac{\eta_i}{\gamma_i} \right) e^{-\frac{\beta_l (\alpha_i \gamma_i + \eta_i)^2}{\sigma^2}} = \lambda_1, \tag{13}$$

with a value of λ_1 such that the power constraint is met with equality. In order to obtain a solution, we define the following function

$$h_i(\gamma_i) = \sum_{l=1}^{L} \phi_l \frac{\beta_l}{\sigma^2} \alpha_i \left(\alpha_i + \frac{\eta_i}{\gamma_i} \right) e^{-\frac{\beta_l (\alpha_i \gamma_i + \eta_i)^2}{\sigma^2}} \tag{14}$$

which is the sum of products of two strictly decreasing functions of γ_i and, therefore, is a strictly decreasing function of γ_i. Taking this fact into account, we can state that the function h is injective, so the inverse h^{-1} is unique.

From all the above, the optimum ST power distribution based on the L-GEESM approximation is computed in two steps:

- Obtain λ_1 as the root for $\frac{1}{N} \sum_{i=1}^{N} h_i^{-2}(\lambda_1) = P$
- Obtain γ_i as $h_i^{-1}(\lambda_1)$.

The inversion of h is a computationally expensive operation, and although its values could be stored in a lookup table in order to speed up the optimization, it is convenient to have an alternative computationally efficient approximation.

B. PLF approximation

In this approximation, we have the problem that the objective function is not differentiable. In a first approach, we will only take into account the logarithmic part of the piecewise function, and afterwards we will add the upper part $\psi > \psi_1$. The lower clipping $\psi < \psi_0$ will be omitted for convenience, as its effect in the final results was found to be negligible. For $\psi_0 < \psi < \psi_1$, the optimum value of $\boldsymbol{\gamma}$ is obtained by solving

$$\begin{array}{ll} \text{minimize} & f_0(\boldsymbol{\gamma}) \doteq -\frac{1}{N} \sum_{l=1}^{N} 2 \log(\eta_i + \alpha_i \gamma_i) \\ \text{subject to} & f_1(\boldsymbol{\gamma}) \doteq \frac{1}{N} \sum_{i=1}^{N} \gamma_i^2 - P \leq 0 \\ & \mathbf{f}_2(\boldsymbol{\gamma}) \doteq -\frac{2}{N} \boldsymbol{\gamma} \preceq 0 \end{array} \tag{15}$$

which is a convex problem. The optimality conditions for this problem read as

$$-\frac{2}{\gamma_i + \eta_i/\alpha_i} + 2\lambda_1 \gamma_i - 2\lambda_{2,i} = 0 \tag{16}$$

$$\lambda_1 f_1(\boldsymbol{\gamma}) = 0 \tag{17}$$

$$\lambda_1 \geq 0 \tag{18}$$

$$\boldsymbol{\lambda}_2^T \mathbf{f}_2(\boldsymbol{\gamma}) = 0 \tag{19}$$

$$\boldsymbol{\lambda}_2 \succeq 0. \tag{20}$$

If the power constraint is not active ($f_1(\boldsymbol{\gamma}) < 0$), following (17) we have that $\lambda_1 = 0$, so we arrive to condition $-\frac{1}{\gamma_i + \eta_i/\alpha_i} - \lambda_{2,i} = 0$, that will only be met in the case $\alpha_i = 0 \, \forall \, i$. In the same way, if $\gamma_i = 0$ for some values of i, we have that $-\frac{\alpha_i}{\eta_i} - \lambda_{2,i} = 0$, a condition that will only be met in the case $\alpha_i = 0$. Therefore, for the non-degenerate cases we have that the power constraint is met with equality, and the non-negativity constraint with inequality.

Thus, we have that

$$\gamma_i = \frac{-\eta_i + \sqrt{4\alpha_i^2 + \eta_i^2 \lambda_1}/\sqrt{\lambda_1}}{2\alpha_i} \tag{21}$$

with a value of λ_1 such that the power constraint is met with equality

$$\frac{1}{N} \sum_{i=1}^{N} \left(\frac{-\eta_i + \sqrt{4\alpha_i^2 + \eta_i^2 \lambda_1}/\sqrt{\lambda_1}}{2\alpha_i} \right)^2 = P. \tag{22}$$

If we add the upper *clipping* to the problem, it is clear that if $(\eta_i + \alpha_i \gamma_i)^2 / \sigma^2 > \psi_1$ some power is being wasted on the i-th carrier, as a value of

$$\gamma_i = \frac{\sqrt{\psi_1} \sigma - \eta_i}{\alpha_i} \tag{23}$$

will lead to the same objective function value with less power consumption. Therefore, we propose to solve the optimization problem iteratively by clipping the values of γ_i with $\psi_i > \psi_1$ according to (23), removing those γ_i from the optimization, and running the algorithm once again. Algorithm 1 describes this iterative approximation.

Algorithm 1 Iterative approximation for the upper-clipped PLF optimization

```
O = {i|ψ_i < ψ_1};
end ← false
while not end do
    Solve Problem (15) over γ_i, i ∈ O
    A ← {i|ψ_i > ψ_1}
    if A = ∅ then
        end ← true
    else
        γ_i = (√ψ_1 σ − η_i)/α_i ∀ i ∈ A
        O ← O \ A
    end if
end while
```

C. Results

We have analyzed the obtained MIESM values for the PLF and 2-GEESM approximations for a 16-QAM constellation. The noise variance was set to $\sigma^2 = 0.2$, and the number of carriers was $N = 128$. The optimization was run for different values of P. In Figure 3 there is a plot of the channel under study (recall that α_i and η_i were assumed, without loss of generality, to be non-negative values) and the resulting γ for different values of P for the 2-GEESM approximation. It can be seen that the secondary power allocation concentrates on those carriers with a weaker primary channel.

In Figure 4 there is a plot of the fraction of released resources $1-\rho$ with respect to the available secondary power P. It can be seen that the PLF approximation offers a performance that is comparable with the one offered by the GEESM approximation, and outperforms a uniform power allocation policy (i.e., $\gamma_i = \sqrt{P}\,\forall\, i$). Thus, this PLF approximation can be of special interest because of its reduced complexity. The fraction of released resources $1-\rho$ is shown to increase with P, reaching near a 40% of released resources for values of P near 1. The thresholds t_i that conform the mapping from effective SNR to MODCOD rate and throughput were taken from the LTE performance study in [6]. In our simulations, the MODCOD evolves from 16-QAM 1/3 to 16-QAM 4/5.

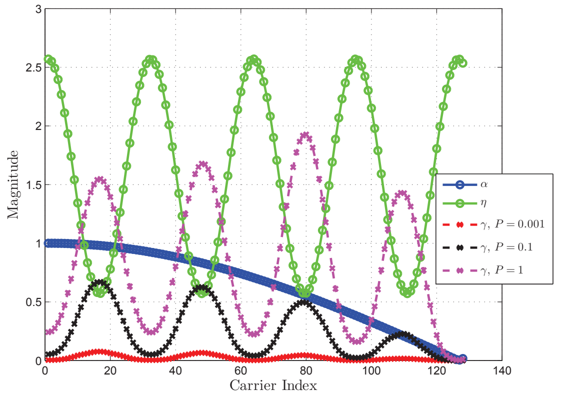

Fig. 3. Channel under study, and optimum power allocation for different values of P. 2-GEESM approximation.

V. Channel estimation

We will assume a simplified scenario where the channel coefficients α_i, η_i are time invariant, and the noise power σ^2 is known at PR and ST. After a group of OFDM symbols, in the n-th feedback message, the PR conveys the square root of the SNR measured at each carrier, that we will model as

$$f_{i,n} = \frac{|\eta_i + \gamma_{i,n}\alpha_i + w_{i,n}|}{\sqrt{(2\sigma^2)}} \tag{24}$$

where the terms $w_{i,n}$ are independent and identically distributed random variables[2] $w_{i,n} \sim \mathcal{CN}\left(0, 2\sigma^2\right)$ that account

[2]In this case, the noise variance was set to $2\sigma^2$ to keep the notation consistent with the usual representation of a Rician random variable, where σ^2 denotes the noise variance per dimension.

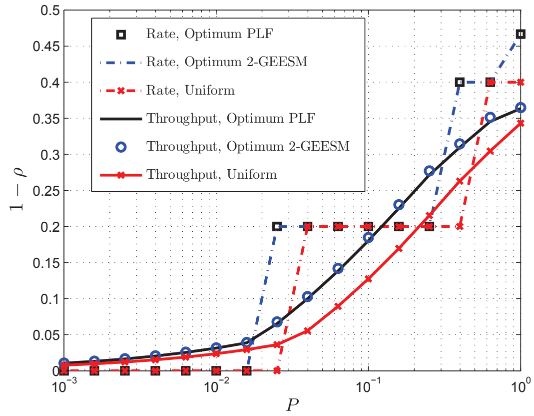

Fig. 4. Fraction of released resources measured in terms of transmit rate and throughput.

for the SNR estimation error. This could be the case of an OFDM system that uses a pilot-based estimation scheme where the pilot symbols have unit power, and the PR just feeds back the modulus of the received pilot. η_i and α_i are modeled as deterministic but unknown parameters, so we will follow a Maximum Likelihood (ML) estimation approach.

As the SNR is fed back separately for each carrier, the estimation can be carried out independently for every carrier, so we will drop the carrier index i in the following expressions. For convenience, we will consider our observations x_n to be

$$x_n = f_n\sqrt{2\sigma^2} = |\eta + \gamma_n\alpha + w_n|\,. \tag{25}$$

Let us denote $\mathbf{x}_J \doteq [x_1, \ldots, x_J]^T$ as the result of stacking J observations into a vector. It can be seen that the observations are independently Rician distributed, so the probability density function of $\mathbf{x}_J$, parametrized by the unknown parameters η and α, is given by

$$p\left(\mathbf{x}_J; \eta, \alpha\right) = \prod_{n=1}^{J} \frac{x_n}{\sigma^2} \exp\left(-\frac{x_n^2 + \nu_n^2}{2\sigma^2}\right) I_0\left(\frac{x_n\nu_n}{\sigma^2}\right) \tag{26}$$

where $\nu_n = |\eta + \gamma_n\alpha|$. In this case, γ_n constitutes a training sequence for the estimation procedure. With this, we can write the log-likelihood function of (η, α) as

$$\mathcal{L}\left(\eta, \alpha\right) = \sum_{n=1}^{J} \frac{-\nu_n^2}{2\sigma^2} + \log\left(I_0\left(\frac{x_n\nu_n}{\sigma^2}\right)\right) \tag{27}$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind, and a constant term that is independent of the parameters (η, α) has been omitted.

It is important to remark that the parameter ν_n is not constant with n, as γ_n can change with time. In fact, if we try to simplify the log-likelihood function by making γ_n constant with n an ambiguity will appear in the estimation procedure, as the likelihood function will have an infinite number of maxima. This fact is illustrated in Figure 5, where the existence of multiple (or even infinite) global and local maxima complicates the problem, even for a simple case with real parameters ($\eta = 3, \alpha = 1, \theta = 0$). In fact, there exists an additional ambiguity that cannot be removed, as the points (η, α) and $\left(\eta e^{j\theta_0}, \alpha e^{j\theta_0}\right)$ lead to the same likelihood value. However, this ambiguity does not affect our optimization procedure, as it only depends on the modulus of the channels ($|\eta_i|$ and $|\alpha_i|$) and the difference in its phase (θ), as presented in Section III. With this, we can rewrite $\mathcal{L}$ as a function of the parameters of interest

$$\mathcal{L}\left(|\eta|, |\alpha|, \theta\right) = \sum_{n=1}^{J} \frac{-\left||\eta| e^{j\theta} + \gamma_n |\alpha|\right|^2}{2\sigma^2} + \log\left(I_0\left(\frac{x_n \left||\eta| e^{j\theta} + \gamma_n |\alpha|\right|}{\sigma^2}\right)\right). \quad (28)$$

A. Results

The log-likelihood function was maximized using a gradient based algorithm with two initial points $(\alpha, \eta, \theta) = (1, 1, \pi/2)$ and $(1, 1, -\pi/2)$, selecting afterwards the one that led to a higher value of $\mathcal{L}$ in order to cope with the presence of local maxima. The training sequence γ was selected randomly following a complex Gaussian distribution, and normalized afterwards to meet the power constraint P. The studied channel is that in Figure 3 with additional phase terms[3] $e^{j\theta_i}$, $\theta_i = 4\pi i/N$, multiplying the coefficents η_i. The estimation procedure was run for different training sequence lengths J, and the PLF-based Algorithm 1 was run taking as input the estimated $(|\alpha_i|, |\eta_i|)$, while the knowledge of θ_i was used to make the primary and secondary contributions to be coherently added at the primary receiver, as explained in Section III. The conditions of the simulation are the same as in Section IV, and the estimation noise was set to $w_k \sim \mathcal{CN}(0, 0.2)$. The obtained results are shown in Figure 6, where it can be seen that even the scenario with a short training sequence ($J = 5$) clearly outperforms the uniform allocation[4], specially for large values of P. For smaller values of P the estimation error is much larger, so the fraction of released resources can be increased by the use of longer training sequences.

VI. Conclusions

We have presented a scenario where a secondary transmitter is aware of the primary message, and exploits this knowledge to free some primary transmission resources to convey a secondary message. Channel knowledge is obtained by exploiting the SNR-based feedback from the primary receiver. This knowledge is shown to dramatically increase the fraction of released resources with respect to a non-CSI aware secondary transmitter.

Future lines of this work include the study of the performance of the proposed method in time-varying channels and quantized CSI values, the extension to multiple secondary transmitters and receivers, the design of the training sequence, and the study of computationally simpler estimation schemes.

[3]The obtained results were similar when a random phase component was applied to each carrier separately.

[4]In Section IV the uniform power allocation assumed phase knowledge, so the primary and secondary contributions were coherently added. In this case, no phase knowledge is assumed, so some of the carriers can experience a lower SNR than that in absence of the ST, so the gain is much smaller.

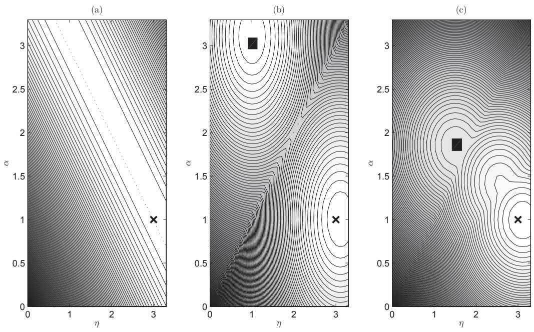

Fig. 5. Level curves of the likelihood function for different training sequences. The point $\times = (3, 1)$ represents the true value of (η, α) In (a) $\boldsymbol{\gamma} = [1,\ 1,\ 1,\ 1]$, and the dashed line represents the points with the same value as $\times$; in (b) ($\boldsymbol{\gamma} = [1,\ -1,\ 1,\ -1]$) the point $\blacksquare = (1, 3)$ has the same value as $\times$; in (c) ($\boldsymbol{\gamma} = [1,\ -1,\ 2,\ -2]$) the only global maximum is $\times$, but a local maximum $\blacksquare$ appears.

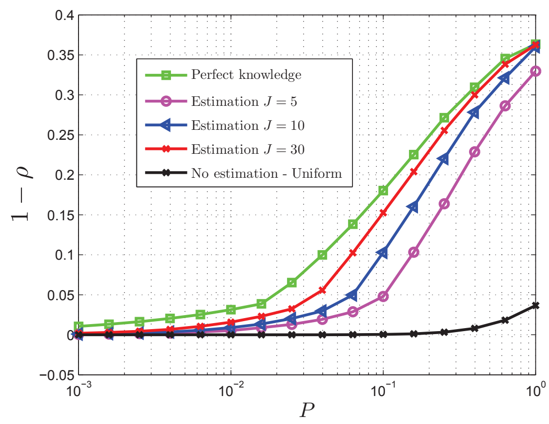

Fig. 6. Fraction of released resources, measured in terms of throughput, for different values of training sequence length. The optimization was run with the PLF approximation.

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