# Subspace-Constrained SINR Optimization in MIMO Full-Duplex Relays under Limited Dynamic Range

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Abstract—In this paper, we propose a method for maximizing the signal-to-interference-plus-noise ratio (SINR) in a wideband full-duplex MIMO regenerative relay that accounts limited dynamic range of the receiver and transmitter impairments. Transmit and receive filters are designed at the relay, by means of an alternating minimization algorithm, to minimize the interference at the decoder input in the destination node. We impose channel shortening and subspace constraints to ensure that the received signal at the destination is not compromised. Simulation results show that the presented method significantly outperforms other constrained approaches.

#### I. INTRODUCTION

Self-interference in full-duplex devices is a direct consequence of simultaneous transmission and reception, wherein part of the transmitted signal leaks back to the receive side. When the full-duplex device is a relay, extreme power imbalance between the near-end and far-end signals may cause an inadmissible interference level [1]-[4], which demands efficient mitigation. Self-interference mitigation techniques are classified according to their domain of application. Analog domain techniques, such as passive physical isolation between antenna arrays or analog cancellation, are applied before digital conversion or baseband demodulation and can provide up to approximately 60-70 dB of self-interference mitigation [1], [5]. This mitigation level is usually insufficient for reliable relay operation. On the other hand, digital domain techniques, such as digital cancellation and interference suppression, mitigate further the residual self-interference by subtracting a replica of the self-interference and exploiting the available degrees of freedom in a multi-antenna scenario [6]-[8].

Besides self-interference, factors limiting the performance are dynamic range and transmission impairments, which introduce noise sources that depend on the statistics of the received and transmitted signals. Those effects can have a significant impact on the performance due to the high gain of the selfinterference channel [2].

We present a mitigation method for SINR maximization in wideband full-duplex regenerative MIMO relays. Similar to our previous method in [9], we take into account transmission impairments and limited dynamic range at the relay under the noise model of [2]. In contrast to [9], new channel shortening constraints and eigenvector subspace transmission, alongside joint design of transmission and reception filters, result in better performance than [9].

## II. SYSTEM MODEL

The baseline system setup, illustrated in Fig. 1, is the same as that of [9] with some minor notation changes. The relay link consists of a source node (S) with  $M_t$  antennas transmitting signal  $\mathbf{s}[n]$ , a destination node ( $\mathcal{D}$ ) with  $M_r$  antennas receiving signal  $\mathbf{d}[n]$ , and a relay ( $\mathcal{R}$ ) with  $N_r$  receive and  $N_t$  transmit antennas receiving signal  $\mathbf{r}[n]$  while simultaneously transmitting signal  $\mathbf{r}_t[n]$ , respectively. The number of independent data streams is m. The received signals at  $\mathcal{R}$  and  $\mathcal{D}$  are

$$\mathbf{r}[n] = \mathbf{H}_{sr}[n] \star \mathbf{s}[n] + \mathbf{H}_{rr}[n] \star \mathbf{r}_t[n] + \mathbf{n}_r[n]$$
(1)

$$\mathbf{d}[n] = \mathbf{H}_{sd}[n] \star \mathbf{s}[n] + \mathbf{H}_{rd}[n] \star \mathbf{r}_t[n] + \mathbf{n}_d[n]$$
(2)

where  $\mathbf{H}_{ij}[n]$ ,  $i \in \{s, r\}$  and  $j \in \{r, d\}$ , is the channel impulse response, of order  $L_{ij}$ , between nodes i and j. Operator  $\star$  denotes convolution whereas vectors  $\mathbf{n}_d[n]$  and  $\mathbf{n}_r[n]$  collect all the noise components at  $\mathcal{D}$  and  $\mathcal{R}$ , respectively. In particular,  $\mathbf{n}_r[n]$  has the following expression:

$$\mathbf{n}_{r}[n] = \mathbf{n}[n] + \mathbf{H}_{rr}[n] \star \mathbf{v}[n] + \mathbf{w}[n]$$
(3)

with  $\mathbf{n}[n]$  denoting temporally white thermal noise. Noise source  $\mathbf{v}[n]$ , see Fig. 1, is statistically independent of  $\mathbf{r}_t[n]$ , temporally white and models imperfections during transmission [2]. Noise source  $\mathbf{w}[n]$ , see Fig. 1, is statistically independent of the signal before digital conversion  $\tilde{\mathbf{r}}[n]$ , temporally white and models limited receiver dynamic range [2]. The statistical distributions of  $\mathbf{n}[n]$ ,  $\mathbf{v}[n]$  and  $\mathbf{w}[n]$  are

$$\mathbf{n}[n] \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}) \tag{4}$$

$$\mathbf{v}[n] \sim \mathcal{CN}(\mathbf{0}, \delta \operatorname{diag}\left(\mathbb{E}\{\mathbf{r}_t[n]\mathbf{r}_t^H[n]\}\right))$$
(5)

$$\mathbf{w}[n] \sim \mathcal{CN}(\mathbf{0}, \gamma \operatorname{diag}\left(\mathbb{E}\{\tilde{\mathbf{r}}[n]\tilde{\mathbf{r}}^{H}[n]\}\right))$$
(6)

where  $0 < \delta, \gamma \ll 1$ . The SINR at the relay input is defined as

$$\operatorname{SINR}_{\mathcal{R}} = \frac{\mathbb{E}\{\|\mathbf{\check{r}}[n]\|^2\}}{\mathbb{E}\{\|\mathbf{i}[n] + \mathbf{n}_r[n]\|^2\}}$$
(7)

with  $\check{\mathbf{r}}[n] = \mathbf{H}_{sr}[n] \star \mathbf{s}[n]$  denoting the information signal arriving at  $\mathcal{R}$  and  $\mathbf{i}[n] = \mathbf{H}_{rr}[n] \star \mathbf{r}_t[n]$  is the self-interference.

Finally, the relay implements a decode-and-forward protocol, which introduces enough processing delay to allow us to make the following assumption:

$$\mathbb{E}\{\check{\mathbf{r}}[n]\mathbf{r}_t^H[n-k]\} = \mathbb{E}\{\check{\mathbf{r}}[n]\mathbf{n}_r^H[n-k]\} = \mathbf{0} \text{ , for } k \geq 0$$

or, in plain words, the samples of the information signal and the transmitted signals are uncorrelated.



Fig. 1. System model of a relay incorporating the cancellation-suppression architecture.

## III. PROPOSED DESIGN

The internal structure of the relay incorporating the mitigation scheme is depicted in Fig. 1, which consists of the  $L_r$ -th order filter  $\mathbf{G}_r[n]$  of size  $m \times N_r$ , the  $L_t$ -th order filter  $\mathbf{G}_t[n]$ of size  $N_t \times m$ , and the  $L_a$ -th order cancellation filter  $\mathbf{A}[n]$ of size  $N_r \times N_t$ . Function  $\mathbf{p}(\cdot)$  represents all the operations related to the decode-and-forward protocol and is responsible for the processing delay within the relay. In general, the relay performance depends on the SINR at the input of  $\mathbf{p}(\cdot)$ , which we aim to maximize by designing  $\mathbf{G}_r[n]$ ,  $\mathbf{G}_t[n]$  and  $\mathbf{A}[n]$ .

Let  $\check{\mathbf{r}}^{(eq)}[n] = \mathbf{G}_r[n] \star \check{\mathbf{r}}[n]$  and  $\mathbf{n}_r^{(eq)}[n] = \mathbf{G}_r[n] \star \mathbf{n}_r[n]$ denote the information signal and the noise after mitigation, respectively. The residual self-interference is given by

$$\mathbf{i}^{(eq)}[n] = \mathbf{G}_r[n] \star (\mathbf{H}_{rr}[n] + \mathbf{A}[n]) \star \mathbf{G}_t[n] \star \hat{\mathbf{r}}_t[n]$$
(8)

whereas the signal at the input of  $\mathbf{p}(\cdot)$  is

$$\hat{\mathbf{r}}[n] = \check{\mathbf{r}}^{(eq)}[n] + \mathbf{i}^{(eq)}[n] + \mathbf{n}_r^{(eq)}[n]$$
(9)

Note that, due to the limitation in the dynamic range,  $\mathbf{n}_{r}^{(eq)}[n]$  depends on  $\mathbf{i}[n]$  through  $\mathbf{w}[n]$ . Therefore, even in the case of perfect cancellation of the self-interference with filter  $\mathbf{A}[n]$ , i.e.  $\mathbf{A}[n] = -\mathbf{H}_{rr}[n]$ , the noise power of  $\mathbf{w}[n]$  is sufficiently high to render low SINR values. The postmitigation SINR at the input of  $\mathbf{p}(\cdot)$  is defined as

$$\operatorname{SINR}_{\mathcal{R}_{eq}} = \frac{\mathbb{E}\{\|\mathbf{\check{r}}^{(eq)}[n]\|^2\}}{\mathbb{E}\{\|\mathbf{i}^{(eq)}[n] + \mathbf{n}_r^{(eq)}[n]\|^2\}}$$
(10)

In view of (10), to maximize the relay performance, we design filters  $\mathbf{G}_r[n]$ ,  $\mathbf{G}_t[n]$  and  $\mathbf{A}[n]$  as the solution to the following optimization problem

$$\begin{array}{ll} \underset{\mathbf{A}[n],\mathbf{G}_{t}[n],\mathbf{G}_{r}[n]}{\text{maximize}} & \text{SINR}_{\mathcal{R}_{eq}} \\ \text{subject to} & \mathbb{E}\{\|\mathbf{r}_{t}[n]\|^{2}\} \leq P_{max} \end{array}$$
(11)

where  $P_{max} > 0$  denotes the maximum transmit power. Note that, from (8), problem (11) has the trivial solution  $\mathbf{G}_t[n] = \mathbf{0}$ , which must be avoided by introducing additional constraints on  $\mathbf{G}_t[n]$  that cause bounded distortion on the  $\mathcal{R}$ - $\mathcal{D}$  channel. Such constraints and the steps required to solve the optimization problem in (11) are explained in the next section.

## IV. Optimal Design of $\mathbf{A}[n]$ , $\mathbf{G}_t[n]$ and $\mathbf{G}_r[n]$

In order to solve problem (11), note that regardless of the value of  $\mathbf{G}_r[n]$  and  $\mathbf{G}_t[n]$  in (8), the optimal  $\mathbf{A}[n]$  is  $\mathbf{A}[n] = -\mathbf{H}_{rr}[n]$ . Estimation of  $\mathbf{H}_{rr}[n]$  using the method in [6] yields a residual self-interference  $\mathbf{i}^{(eq)}[n]$  below the noise level, therefore we can reasonably assume that  $\mathbf{i}^{(eq)}[n] \approx \mathbf{0}$ by fixing  $\mathbf{A}[n]$  to the estimated value of  $\mathbf{H}_{rr}[n]$ .

Nonetheless, noise  $\mathbf{n}_r^{(eq)}[n]$ , which depends on  $\mathbf{i}[n]$  and  $\mathbf{v}[n]$ , cannot be mitigated with cancellation filter  $\mathbf{A}[n]$  and usually has larger power than  $\check{\mathbf{r}}[n]$ , resulting in unacceptable SINR values at  $\mathbf{p}(\cdot)$ . Consequently, filters  $\mathbf{G}_r[n]$  and  $\mathbf{G}_t[n]$  should be designed to mitigate the effect of  $\mathbf{n}_r^{(eq)}[n]$ .

Note that both filters are coupled through  $\mathbf{n}_r^{(eq)}[n]$ , therefore a closed-form solution of (11) might not be available. A solution to the problem can be obtained by means of an alternating optimization algorithm, whose steps are summarized in Algorithm 1. Such an optimization procedure transforms problem (11) into a sequence of optimization problems. Next, we detail how to solve Steps 2 and 3 of Algorithm 1.

Algorithm 1 Alternating optimization procedure	
1:	<b>Initialization point:</b> $\mathbf{G}_r[0]$ , $\mathbf{G}_t[0]$ .
2:	For fixed $\mathbf{G}_r[n]$ , solve (11) with respect to $\mathbf{G}_t[n]$ .
3:	For fixed $\mathbf{G}_t[n]$ , solve (11) with respect to $\mathbf{G}_r[n]$ .
4:	<b>Repeat</b> Steps 2 and 3 until convergence.

## A. Optimal design of $\mathbf{G}_t[n]$

Maximizing (10) with respect to  $\mathbf{G}_t[n]$  amounts to minimizing  $\mathbb{E}\{\|\mathbf{n}_r^{(eq)}[n]\|^2\}$ , because  $\mathbf{i}^{(eq)}[n] \approx \mathbf{0}$  when  $\mathbf{A}[n]$  is fixed to the estimated value of  $\mathbf{H}_{rr}[n]$ . To avoid the trivial solution  $\mathbf{G}_t[n] = \mathbf{0}$ , we constrain  $\mathbf{G}_t[n]$  to belong to a certain subspace that causes bounded distortion on the  $\mathcal{R}$ - $\mathcal{D}$  channel.

A pre-equalization stage at the relay can significantly reduce the complexity of the final equalizer at  $\mathcal{D}$ . We propose the use of channel shortening which is a filter design technique tailored for OFDM-based systems that reduces the overall channel length to simplify the subsequent channel equalization. Channel shortening is commonly performed in time domain before channel compensation takes place in frequency domain [10]–[12]. Let  $\mathbf{H}_{rd}^{(eq)}[n]$  represent the  $L_{eq} = (L_{rd} + L_t)$ -th order equivalent channel between  $\mathcal{R}$  and  $\mathcal{D}$ , i.e.,

$$\mathbf{H}_{rd}^{(eq)}[n] = \mathbf{H}_{rd}[n] \star \mathbf{G}_t[n]$$
(12)

We impose channel shortening constraints on  $\mathbf{G}_t[n]$  that reduce the effective order of  $\mathbf{H}_{rd}^{(eq)}[n]$  to be  $0 \le \tau \le L_{eq}$ , i.e., only the taps of  $\mathbf{H}_{rd}^{(eq)}[n]$  exceeding delay  $\tau \ge 0$  are forced to zero, where constant  $\tau$  is a design parameter. Consequently,  $\mathbf{G}_t[n]$  operates as a channel shortening filter. The channel shortening constraints can be expressed as

$$\mathbf{H}_{rd}\mathbf{g}_t = \mathbf{0} \tag{13}$$

where the  $mM_r(L_{eq} - \tau) \times mN_t(L_t + 1)$  matrix  $\hat{\mathbf{H}}_{rd}$ depends on the convolution matrix of  $\mathbf{H}_{rd}[n]$  and  $\mathbf{g}_t =$  $\operatorname{vec}\{[\mathbf{G}_t[0] \dots \mathbf{G}_t[L_t]]\}$  stacks the columns of  $\mathbf{G}_t[n]$  into a vector of size  $mN_t(L_t + 1)$ .

We require that  $\mathbf{g}_t$  has enough degrees of freedom to allow the minimization of  $\mathbb{E}\{\|\mathbf{n}_r^{(eq)}[n]\|^2\}$  while satisfying (13), i.e.,  $mN_t(L_t+1) > \operatorname{rank}\{\mathbf{H}_{rd}\}$ , or, if  $\mathbf{H}_{rd}$  has full rank,  $N_t(L_t+1) > M_r(L_{eq} - \tau)$ . Denoting the nullspace of  $\mathbf{H}_{rd}$  by  $\mathbf{\tilde{N}}_{rd}$ , any possible  $\mathbf{g}_t$  satisfying (13) can be expressed as

$$\mathbf{g}_t = \mathbf{N}_{rd} \mathbf{t} \tag{14}$$

where t is an arbitrary vector of size  $mN_t(L_t + 1) - \operatorname{rank}{\{\tilde{\mathbf{H}}_{rd}\}}$ , and matrix  $\tilde{\mathbf{N}}_{rd}$  is of size  $mN_t(L_t + 1) \times mN_t(L_t + 1) - \operatorname{rank}{\{\tilde{\mathbf{H}}_{rd}\}}$ . Additionally, the received power at  $\mathcal{D}$  from  $\mathcal{R}$  can be expressed as

$$\mathbb{E}\{\|\mathbf{H}_{rd}[n] \star \mathbf{r}_t[n]\|^2\} = \mathbf{t}^H \mathbf{P}_{rd} \mathbf{t}$$
(15)

where matrix  $\mathbf{P}_{rd}$  depends on  $\mathbf{\tilde{N}}_{rd}$  and the second-order statistics of  $\mathbf{r}_t[n]$ . From (15), the relay delivers maximal power to  $\mathcal{D}$  when t is aligned with the eigenvector associated with the largest eigenvalue of  $\mathbf{P}_{rd}$ , up to a normalization constant yielding  $\mathbb{E}\{\|\mathbf{r}_t[n]\|^2\} = P_{max}$ . At the same time, the delivered power to  $\mathcal{D}$  by  $\mathcal{R}$  is minimal when t is aligned with the eigenvector associated with the smallest eigenvalue of  $\mathbf{P}_{rd}$ . Therefore, if t belongs to the subspace spanned by the eigenvectors of  $\mathbf{P}_{rd}$  associated with the largest  $\kappa$  nonzero eigenvalues, the delivered power at  $\mathcal{D}$  will be a significant fraction of the maximum delivered power.

Let  $\mathbf{V}_{rd}$  represent a matrix whose columns contains the  $\kappa \geq 1$  eigenvectors of  $\mathbf{P}_{rd}$  associated with the  $\kappa$  non-zero largest eigenvalues of  $\mathbf{P}_{rd}$ , which are denoted by  $\lambda_1 \geq \ldots \geq \lambda_{\kappa} > 0$ . Let  $\mathbf{t} = \mathbf{V}_{rd}\mathbf{q}$ , where  $\mathbf{q}$  is an arbitrary vector with constant nonzero norm. With that,  $\mathbf{g}_t$  satisfies the following constraints:

$$\mathbf{P}\mathbf{g}_t = \mathbf{0} \tag{16}$$

$$\|\mathbf{g}_t\|^2 = g_{max} \tag{17}$$

where **P** is a projection matrix onto the subspace orthogonal to that spanned by  $\tilde{\mathbf{N}}_{rd}\mathbf{V}_{rd}$  and  $g_{max}$  is a constant that renders  $\mathbb{E}\{\|\mathbf{r}_t[n]\|^2\} = P_{max}$ . Note that, with constraints (16) and (17),  $\mathbf{g}_t = \mathbf{0}$  is not a valid solution and we guarantee that the minimum delivered power at  $\mathcal{D}$  from  $\mathcal{R}$  is, at least,  $\lambda_{\kappa}/\lambda_1$ times of the maximum achievable one. A practical way of selecting parameter  $\kappa$  is by introducing the constant  $\alpha$  that sets  $\kappa$  based on the ratio between eigenvalues with respect to the largest one,  $\lambda_1$ , i.e.,  $\kappa$  is the number of eigenvalues satisfying

$$\frac{\lambda_j}{\lambda_1} \ge \alpha \tag{18}$$

where  $j = 1, ..., \kappa$ . For example, in the following simulations, we illustrate the cases where  $\alpha = \{75, 50, 25\}\%$  of the largest eigenvalue of  $\mathbf{P}_{rd}$ .

With the previous discussion, the optimization problem for  $g_t$  is equivalent as the following problem with respect to q

minimize 
$$\mathbf{q}^{H} \mathbf{P}_{eq} \mathbf{q}$$
 (19)  
subject to  $\|\mathbf{q}\|^{2} = p_{max}$ 

where matrix  $\mathbf{P}_{eq}$  results from expressing  $\mathbb{E}\{\|\mathbf{n}_{r}^{(eq)}\|^{2}\}\$  as a function of vector  $\mathbf{q}$  (details skipped due to lack of space) and constant  $p_{max}$  ensures that the relay transmits at full power, i.e.,  $\mathbb{E}\{\|\mathbf{r}_{t}[n]\|^{2}\} = P_{max}$ . Vectors  $\mathbf{q}$  and  $\mathbf{g}_{t}$  are related as

$$\mathbf{g}_t = \mathbf{N}_{rd} \mathbf{V}_{rd} \mathbf{q} \tag{20}$$

The solution to problem (19) is given by

$$\mathbf{q} = \sqrt{p_{max}} \mathbf{v}_{min} \tag{21}$$

where  $\mathbf{v}_{min}$  is the eigenvector of  $\mathbf{P}_{eq}$  corresponding to the smallest eigenvalue. Filter  $\mathbf{G}_t[n]$  is recovered from  $\mathbf{q}$  by reshaping the product  $\tilde{\mathbf{N}}_{rd}\mathbf{V}_{rd}\mathbf{q}$ .

Note that the original  $mN_t(L_t + 1)$  degrees of freedom of  $\mathbf{G}_t[n]$  are reduced to  $\kappa$ , which will impact the achievable mitigation. Nevertheless, the solution does not compromise the reception of  $\mathbf{r}_t[n]$  at  $\mathcal{D}$  by establishing a lower bound to the delivered power at  $\mathcal{D}$  from  $\mathcal{R}$ .

## B. Optimal design of $\mathbf{G}_r[n]$

The final step in solving problem (11), as stated in Algorithm 1, involves the design of  $\mathbf{G}_r[n]$  when  $\mathbf{G}_t[n]$  is fixed. The received signal after cancellation consists of  $\check{\mathbf{r}}[n]$  and  $\mathbf{n}_r[n]$ , so solving (11) with respect to  $\mathbf{G}_r[n]$  is equivalent to

$$\underset{\mathbf{g}_{r}}{\text{maximize}} \quad \frac{\mathbf{g}_{r}^{H} \mathbf{P}_{r} \mathbf{g}_{r}}{\mathbf{g}_{r}^{H} \mathbf{P}_{n} \mathbf{g}_{r}} \tag{22}$$

where  $\mathbf{g}_r = \operatorname{vec}\{[\mathbf{G}_r[0] \dots \mathbf{G}_r[L_r]]\}$  stacks the rows of  $\mathbf{G}_r[n]$ into a vector of size  $mN_r(L_r+1)$  and matrices  $\mathbf{P}_r$  and  $\mathbf{P}_n$ stem from expressing  $\check{\mathbf{r}}^{(eq)}[n]$  and  $\mathbf{n}_r^{(eq)}[n]$  in terms of  $\mathbf{g}_r$ , respectively.

The solution to the generalized eigenvalue problem in (22) is given by

$$\mathbf{g}_r = \rho \mathbf{L}^{-1} \mathbf{v}_{max} \tag{23}$$

where  $\mathbf{v}_{max}$  is the eigenvector associated with the maximum eigenvalue of  $\mathbf{L}^{-H}\mathbf{P}_{r}\mathbf{L}^{-1}$ ,  $\mathbf{L}$  is a square root of  $\mathbf{P}_{n}$ , i.e.,  $\mathbf{P}_{n} = \mathbf{L}^{H}\mathbf{L}$ , and  $\rho \neq 0$  is an arbitrary constant.

After  $\mathbf{G}_t[n]$  and  $\mathbf{G}_r[n]$  are computed using expressions (21) and (23), Steps 2 and 3 in Algorithm 1 are iteratively repeated until some stopping criterion is met.

#### V. SIMULATION RESULTS AND DISCUSSION

To evaluate the performance of the SINR optimization method, we consider a relay link with the following characteristics: The source node S has  $M_t = 2$  and transmits m = 2 independent data streams of a 64-QAM modulated OFDM signal with 8192 subcarriers and a normalized cyclic prefix length of 1/4. The source node transmit power is 0 dB, i.e.,  $\mathbb{E}\{\|\mathbf{s}[n]\|^2\} = 1$ . The destination node is equipped with  $M_r = 2$  receive antennas. The relay samples the data signal coming from S using an oversampling factor of 2, such that the received signal covers approximately half of the Nyquist bandwidth. Additionally,  $\mathbb{E}\{\|\hat{\mathbf{r}}_t[n]\|^2\} = 1$  and  $P_{max} = 20$ dB. Filters  $\mathbf{G}_t[n]$ ,  $\mathbf{G}_r[n]$  and  $\mathbf{A}[n]$  have orders  $L_a = L_{rr}$  and  $L_t = L_r = 2$ , respectively. Finally, channels  $\mathbf{H}_{sr}[n]$ ,  $\mathbf{H}_{rd}[n]$ and  $\mathbf{H}_{rr}[n]$  are drawn from a complex Gaussian distribution, with orders  $L_{sr} = L_{rd} = L_{rr} = 2$  and have average gains of 0 dB, 0 dB and 30 dB, respectively. Thermal noise power is set to  $\sigma^2 = -20$ dB.

We compare the method proposed here with the one reported in [9]. For this purpose, the relay is configured with  $N_t = 4$  transmit antennas and  $N_r = 2$  receive antennas,  $\delta = -30$  dB is fixed while  $\gamma$  is variable. The number of iterations used in the optimization procedure is 10. Note that in [9] the equivalent  $\mathcal{R}-\mathcal{D}$  channel is equalized to match the following target channel

$$\mathbf{H}_{rd}^{(eq)}[n] = \begin{cases} \mathbf{I}, & n = 0\\ \mathbf{0}, & n \neq 0 \end{cases}$$
(24)

We define the SINR improvement of the method as the ratio between the SINR resulting from using the proposed method and the SINR resulting from using a reference system that consists of  $\mathbf{G}_t[n] = \mu \mathbf{1}$ . Concretely,  $L_t = 0$  and  $\mathbf{1}$  is an allones matrix of size  $N_t \times m$ , i.e.,  $\mathbf{G}_t[n]$  equally distributes the data streams over the different antennas. Constant  $\mu$  matches the power of both systems.

Figure 2 depicts the SINR improvement as a function of the receiver's dynamic range  $\gamma$ , for different values of  $\alpha$  when  $\tau = 0$ . Recall that  $\kappa$  relates to  $\alpha$  as the number of eigenvalues of  $\mathbf{P}_{rd}$  that are at least  $\alpha$  times the maximum eigenvalue of  $\mathbf{P}_{rd}$ . The lowest dashed line in Fig. 2 represents the results obtained in [9] under the same conditions. Firstly, note that the proposed method outperforms [9] for any value of  $\gamma$ , with larger gap when  $\gamma$  decreases. This is a direct consequence of the joint design of  $\mathbf{G}_t[n]$  and  $\mathbf{G}_r[n]$ , whereas [9] follows a decoupled approach. We also see how the performance is affected by the different values of  $\alpha$ . A lower value of  $\alpha$  results in larger  $\kappa$ , i.e., more degrees of freedom available to  $\mathbf{G}_t[n]$ to further reduce the residual noise. When the receiver noise is dominant,  $\gamma > -30$  dB, the variation in performance for different values of  $\alpha$  reduces to less than 1 dB. In that case, the use of a high value of  $\alpha$  seems reasonable, because the relay performance varies slightly in contrast to the delivered power at  $\mathcal{D}$ , which can be up to three times larger.

Figure 3 depicts the SINR improvement as a function of the receiver's dynamic range  $\gamma$ , for different values of  $\tau$  when



Fig. 2. SINR improvement as a function of the receiver dynamic range and different values of the power delivered at the destination.



Fig. 3. SINR improvement as a function of the receiver dynamic range and different values of the effective relay-destination channel order.

 $\alpha = 75\%$ . Since  $\tau$  represents the largest nonzero tap delay of  $\mathbf{H}_{rd}^{(eq)}[n]$ , a larger value of  $\tau$  results in  $\mathbf{G}_t[n]$  having more degrees of freedom. Similarly to the previous case, we evaluate how  $\tau$  affects the overall performance and we compare it to the results in [9], which is depicted as the lowest dashed line. First, we see that for a receiver with a large dynamic range,  $\gamma < -35$ dB, the difference in performance for different values of  $\tau$  is up to 3–4 dB higher than [9]. Such differences become smaller for a smaller dynamic range, or  $\gamma > -30$  dB. We also see the effect of  $\tau$  is not as pronounced as that of varying parameter  $\alpha$ . This indicates that dedicating more degrees of freedom for  $\mathbf{G}_t[n]$  by increasing  $\tau$  does not result in a significant boost in the performance, but it can be beneficial to the  $\mathcal{R}$ - $\mathcal{D}$  link and consequently, the overall source-destination link.

Figure 4 shows the SINR improvement for different antenna configurations in terms of the receiver's dynamic range  $\gamma$  when  $\alpha = 75\%$ ,  $\tau = 0$ , and  $\delta = -30$  dB. Note that in



Fig. 4. SINR improvement as function of the receiver dynamic range and different relay antenna configurations.

Fig. 4, solid lines depict the results obtained by the proposed method, while dashed lines depict the results in [9]. The proposed method outperforms [9] in all the tested cases, with some noticeable margin, e.g., when  $\gamma = -40$  dB,  $N_t = 4$  and  $N_r = 6$ , the gap between the two methods is approximately 4 dB. The performance behavior in terms of the antenna configuration follows the same conclusions as in [9], where we can roughly distinguish two cases: wide range case when  $\gamma \in (-45, -30)$  dB, and narrow range case when  $\gamma \in (-30, -15)$  dB. In a wide range case, i.e.,  $\gamma$  is low, the best performance is achieved with a large number of receive antennas, see, e.g., the results for case  $N_r = 6$ . For this case, the receiver redundancy,  $N_r/N_t$ , can be used as an indicator of the achievable performance, with larger ratios of  $N_r/N_t$ yielding better results. On the contrary, in a narrow range case, i.e.,  $\gamma$  is large, configurations with a larger number of transmit antennas perform better, see, e.g., the results for case  $N_t = 8$ . In fact, a large number of transmit antennas results in lower self-interference power leaking back into the relay and, consequently, a lower noise power at the receive side of the relay. For this case,  $N_t$  can be used as an indicator of the performance, with better performance for larger  $N_t$ .

In summary, we can conclude that the method presented here yields, overall, better performance than the one in [9], with the gap between them reaching several decibels. Both methods are based on the same cancellation-suppression architecture and require the same side information. Since both methods solve similar convex optimization problems, they are comparable in terms of computational complexity. Additionally, the parametrization of the equivalent  $\mathcal{R}$ - $\mathcal{D}$  channel through constants  $\tau$  and  $\kappa$  allows for a more efficient allocation of the degrees of freedom available in  $\mathbf{G}_t[n]$ , in comparison to [9], where all the coefficients of the equivalent  $\mathcal{R}$ - $\mathcal{D}$  channel are specified beforehand.

## VI. CONCLUSIONS

We have presented a method for the maximization of the SINR in a wideband full-duplex MIMO relay subject to limited dynamic range. The method uses a combined cancellation-suppression architecture and jointly designs the transmit and receive filters of the relay using an alternating optimization procedure, where the individual filters are obtained as the solution to a sequence of two convex optimization problems. In order to avoid excessive distortion on the relay-destination channel, we impose linear constraints on the transmit side that reduce the length of the effective channel, similarly to a channel shortening approach, and ensure that the received power at the destination stays above a minimum quality value. Simulation results show that the presented method outperforms previous results in [9] and can provide SINR improvements of several tens of decibels.

#### VII. ACKNOLEDGEMENT

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