QUANTIZATION LATTICE ESTIMATION FOR MULTIMEDIA FORENSICS

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ABSTRACT

The most widely used information lossy source coding schemes for multimedia signals rely on the quantization of the content samples in a linearly transformed domain. A number of forensic applications (e.g., processing history estimation, tampering detection, software identification) can be posed as the estimation of the equivalent lattice quantizer from the observed samples. We present a new lattice estimation algorithm based on the observation of noisy points of the lattice. Although inspired by the work of Neelamani *et al.*, our scheme uses the so-called "dual lattice" to achieve significant performance improvements with respect to its predecessors as measured by the number of vectors of the lattice basis that can be correctly estimated. Such performance improvement is even more dramatic when small pieces of the contents are considered, which indeed is especially relevant for forensic applications.

***Index Terms*—** Dual lattice, lattice estimation, multimedia forensics, noisy estimation.

1. INTRODUCTION

The most widely used information lossy source coding schemes for multimedia signals are based on the quantization of the content samples in a linearly transformed domain. Typically, such domain is chosen both for perceptual and energy compaction reasons, so that a significant number of coefficients in the transform domain are quantized to zero. Estimation of this transform has a great interest in multimedia forensic applications, such as:

- Estimation of the previous history of a content: for example, given an image in raw format, decide if it was previously compressed in a lossy way, and estimate the compression parameters.
- Detection of quantization inconsistencies between different parts of the content: this might indicate that the content was tampered with. In this practical scenario, the number of altered samples can be very small.
- Identification of post-quantization processing: estimation of the quantization grid can be used as a previous step to determine which processing the content has been subjected to after the quantization (e.g., filtering a JPEG-compressed image).
- Identification of software tools or capturing devices: different software and hardware manufacturers can work in different color spaces, leaving a trace that can be tracked by estimating the color lattice used for image processing or during the capture of the contents.

Due to its interest, this problem has already deserved attention of several researchers. To the best of our knowledge, the first related work is due to Fan and Queiroz [1], where the authors estimate the JPEG quantization steps. This work was significantly improved by Neelamani *et al.* in [2], where the estimation of the quantization lattice is studied for the first time in multimedia forensics. The authors use lattice estimation for determining the color space where the quantization of DCT-coefficients is performed for a JPEG-compressed image without downsampling of color components; this is the problem that we will study in the experimental part of this work, although the proposed methodology can be also extended to the general problem of multidimensional lattice estimation. In fact, our work is inspired by [2] (specifically, by the blind lattice-based algorithm proposed therein), although our estimation method is radically different. Due to our novel approach, the number of vectors of the lattice basis that can be correctly estimated is significantly larger than that in [2]. Even though the scheme proposed in [2] is highly robust to round-off errors, it has also some limitations that have motivated our approach. Those limitations are:

- Neelamani et al.'s method strongly relies on the computation of the multidimensional histogram of the available samples. Therefore, for a given number of bins per dimension, the total number of bins exponentially grows with the dimensionality of the problem, thus hindering the use of efficient solutions.
- In the implementation provided by the authors [3], a minimum number of samples per bin is required. Therefore, valuable information samples are not considered. This is especially relevant when high AC-DCT frequencies are considered, as only few coefficients are quantized to a non-null centroid. This problem also increases with the dimensionality of the lattice, since for a fixed number of available samples an exponentially growing number of bins implies that most bins will be empty.

The lattice estimation problem was also studied in a recent paper by Tagliasacchi *et al.* [4], although for the noiseless case, i.e., when the input samples are exactly the quantizer output. The current work, similarly to [2] deals with the more practical noisy case,

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where the quantized content contains additional noise for instance due to rounding-off to integer values in the pixel domain.

2. NOTATION AND PROBLEM FORMULATION

Random variables will be denoted by capital letters (e.g., X), while small letters will be used for their realizations (e.g., x). For their n-dimensional vector counterparts we will use bold fonts (e.g., $\mathbf{X}$ and $\mathbf{x}$, respectively); in the experiments we will focus on the color space estimation, similarly to [2], so $n = 3$. The probability density function (pdf) of random variable (vector) X ($\mathbf{X}$) evaluated at x ($\mathbf{x}$), will be denoted by $f_X(x)$ ($f_{\mathbf{X}}(\mathbf{x})$).

The problem we want to solve can be posed as follows. Given an input signal $\mathbf{X}$ in some domain (for the experimental results reported in this work, as well as in [2], this domain is the 8×8-DCT, as JPEG images are considered), this signal is quantized using a lattice Λ to produce $\mathbf{Y}$. This quantization depends on some parameter which is not known by the estimator; in the considered use case, that unknown information is the color transformation that is applied to the 8×8-DCT coefficients before quantizing, as well as the corresponding quantization steps. Afterwards, some noise $\mathbf{N}$ is added to $\mathbf{Y}$, yielding the observed signal $\mathbf{Z} = \mathbf{Y} + \mathbf{N}$. As done in Section 6 and [2], noise $\mathbf{N}$ can model the round-off effects introduced when quantizing pixels to integer values, so it is modeled by AWGN independent of $\mathbf{Y}$. Similarly to [2], our aim is to blindly, i.e., without assuming any *a priori* information about the used lattices, estimate Λ given a sequence of L independent observed vectors $\mathbf{z}^i$, $i = 1, \cdots, L$ (i.e., continuing with the parallelism with [2], L stands for the number of blocks of the image). The lattice estimate will be denoted by $\hat{\Lambda}$.

The space of n-dimensional lattices will be denoted by $\mathcal{L}$, the fundamental Voronoi region of lattice Λ by $\mathcal{V}(\Lambda)$, and $r_{\mathcal{V}(\Lambda)} \doteq \min_{\mathbf{x}\notin\mathcal{V}(\Lambda)} ||\mathbf{x}||$, its inner radius. For a certain lattice point $\lambda \in \Lambda$, we denote by P_λ the probability that $\mathbf{X}$ is quantized to λ. If minimum Euclidean distance quantizers are assumed (denoted by $Q_\Lambda(\cdot)$), P_λ can be written as $P_\lambda = \int_{\lambda+\mathcal{V}(\Lambda)} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}$.

In this paper, we make extensive use of the *dual lattice* concept. For a lattice Λ, its dual lattice $\Lambda^\perp$ is defined as $\Lambda^\perp = \left\{\mathbf{x}\in\mathbb{R}^n : \mathbf{x}^T\cdot\mathbf{y}\in\mathbb{Z} \text{ for all } \mathbf{y}\in\Lambda\right\}$ [5]. Let matrix B be a basis of lattice Λ; notice that estimating Λ is equivalent to estimating *any* matrix B generating Λ. If B is full-rank, then it turns out that $D \doteq B^{-1}$ is such that D^T is a basis of $\Lambda^\perp$. Given a minimum distance quantizer $Q_\Lambda(\cdot)$ corresponding to Λ and a vector $\mathbf{x}$, the modulo-Λ reduction of $\mathbf{x}$, is $\mathbf{x} \bmod \Lambda = \mathbf{x} - Q_\Lambda(\mathbf{x})$.

3. DERIVATION OF THE COST FUNCTION

From the independence of the quantized signal $\mathbf{Y}$ and the noise $\mathbf{N}$, the pdf of the estimator input signal, $\mathbf{Z}$, is easily shown to be $f_{\mathbf{Z}}(\mathbf{z}) = \sum_{\lambda\in\Lambda} P_\lambda f_{\mathbf{N}}(\mathbf{z}-\lambda)$. Since *a priori* knowledge about the input signal distribution is not typically available at the estimator, probabilities P_Λ will not be known; therefore, Maximum Likelihood (ML) Estimation based on $\mathbf{z}$ is not generally possible. A strategy widely used in the literature [6, 7] to simplify lattice-based problems is to work with the modulo-$\hat{\Lambda}$ reduced version of vector $\mathbf{z}$, i.e., $\mathbf{v} \doteq \mathbf{z} \bmod \hat{\Lambda}$. Be aware that the mapping from $\mathbf{z}$ to $\mathbf{v}$ will usually be information-lossy. In general, we can write $\mathbf{V} = [\lambda + \mathbf{N}] \bmod \hat{\Lambda}$, where $\lambda \in \Lambda$ is the centroid representing the quantized vector $\mathbf{y}$. Therefore, the distribution of $\mathbf{V}$ is

$$f_{\mathbf{V}}(\mathbf{v}) = \sum_{\lambda'\in\hat{\Lambda}}\sum_{\lambda\in\Lambda} P_\lambda f_{\mathbf{N}}\left(\mathbf{v}-\lambda+\lambda'\right), \tag{1}$$

whenever $\mathbf{v}\in\mathcal{V}(\Lambda')$, and is zero otherwise. Obviously, when $\hat{\Lambda} = \Lambda$, then $\lambda \bmod \hat{\Lambda} = \mathbf{0}$, and consequently $\mathbf{V} = \mathbf{N} \bmod \Lambda$. Notice that, due to the modular reduction, $\mathbf{V}$ is independent of $\mathbf{X}$.

Under the high-SNR assumption, defined as $r^2_{\mathcal{V}(\Lambda)} \gg \sigma_N^2$,[1] the probability of the noise lying outside $\mathcal{V}(\Lambda)$ is negligible. Therefore, if $\hat{\Lambda} = \Lambda$, the pdf of $\mathbf{V}$ can be approximated as $f_{\mathbf{V}}(\mathbf{v}) = f_{\mathbf{N}}(\mathbf{v})$, i.e., the modulo-$\Lambda$ reduction of the observed vector $\mathbf{Z}$ is Gaussian-distributed. On the other hand, when $\hat{\Lambda} \neq \Lambda$, under the high-SNR assumption $f_{\mathbf{V}}(\mathbf{v})$ can be approximated by $\sum_{\lambda\in\Lambda} P_\lambda f_{\mathbf{N}}\left([\mathbf{v}-\lambda] \bmod \hat{\Lambda}\right)$, but now $\lambda \bmod \hat{\Lambda}$ will lie at no specific point in $\mathcal{V}(\hat{\Lambda})$. This effect especially affects the disribution of $\mathbf{V}$ when a large number of P_λ's have significant values.

Therefore, by considering the pdf of $\mathbf{V}$ when $\hat{\Lambda} = \Lambda$ under the high-SNR assumption, we can derive a nearly-ML estimate of Λ when a set of L realizations of $\mathbf{V}$ is available,[2] namely $\mathbf{z}^i$, $1 \le i \le L$. The estimate becomes $\hat{\Lambda} = \arg\min_{\Lambda_c\in\mathcal{L}} \sum_{i=1}^L \left\|\mathbf{z}^i \bmod \Lambda_c\right\|^2$. Be aware that the verification of the high-SNR assumption is critical to avoid the trivial solution in which $\hat{\Lambda} = \mathbf{0}$. In practice (cf. Section 4), constraints on the search space or penalties on the target function must be included.

For the sake of notational simplicity we will denote by $\bar{\mathbf{z}}$ the matrix collecting all the L observations $\mathbf{z}^i$, $1 \le i \le L$; this allows us to write the near-ML target function as

$$g(\bar{\mathbf{z}},\Lambda_c) = \sum_{i=1}^L \|\mathbf{z}^i \bmod \Lambda_c\|^2 = \sum_{i=1}^L \min_{\mathbf{c}^i\in\mathbb{Z}^n} \left\|\mathbf{z}^i - B\mathbf{c}^i\right\|^2 \tag{2}$$

where $\mathbf{c}^i$ are the minimum-distance quantizer coordinates of vector $\mathbf{z}^i$ corresponding to a basis B generating Λ_c. Obviously, the value of $g(\bar{\mathbf{z}},\Lambda_c)$ does not depend on the particular lattice basis B, as all of them will yield the same $g(\bar{\mathbf{z}},\Lambda_c)$ (although using different values of $\mathbf{c}^i$).

A major implementation problem arises when one tries to find $\hat{\Lambda}$ by using (2); all the components of the lattice basis B must be jointly estimated, which introduces a very large computational load, even for moderate values of n. Interestingly, when the true lattice Λ is known to belong to the space of orthogonal lattices, then the cost function in (2) can be decoupled and each basis vector can be independently found. Inspired by this observation, we seek a low-complexity solution in which each basis vector is found at a time. This alternative approach is based on rewriting (2) as

$$g(\bar{\mathbf{z}},\Lambda_c) = \sum_{i=1}^L \min_{\mathbf{c}^i\in\mathbb{Z}^n} \left\|B\left(D\mathbf{z}^i - \mathbf{c}^i\right)\right\|^2. \tag{3}$$

The term $D\mathbf{z}^i$ yields the scalar product between vector $\mathbf{z}^i$ and each vector of the basis of the dual lattice of Λ_c, corresponding to the different rows of D. If we focus on the jth such row, which we denote by $\mathbf{d}^j$, from (2) we can construct the following alternative cost function

$$h(\bar{\mathbf{z}},\mathbf{b}^j,\mathbf{d}^j) = \min_{c^i\in\mathbb{Z}} \sum_{i=1}^L \left\|\mathbf{b}^j\right\|^2 \left\|\mathbf{d}^j\mathbf{z}^i - c^i\right\|^2, \tag{4}$$

[1] Note that the given condition does not strictly apply on the SNR, rather, it is defined for a particular direction; in any case, we have preferred this name for the sake of clarity.

[2] Strictly speaking, the proposed estimator is not ML, since some information is lost in passing from $\mathbf{z}$ to $\mathbf{v}$.

where $\mathbf{b}^j$ is the jth column of B, and we must introduce the constraint that $\mathbf{d}^j \cdot \mathbf{b}^j = 1$, because $D \cdot B = \mathrm{I}_{n\times n}$. Notice that (4) does not depend on the rows of D and columns of B other than the jth, and thus the complexity is greatly reduced at the expense of only approximately solving (2), except for the case of orthogonal bases.

Thus, we can estimate a column of B and of the corresponding row of D by calculating

$$(\hat{\mathbf{b}}, \hat{\mathbf{d}}) = \arg \min_{(\mathbf{b},\mathbf{d})\in\mathbb{R}^{2n}:\mathbf{d}\cdot\mathbf{b}=1} \sum_{i=1}^{L} \|\mathbf{b}\|^2 \left\| \mathbf{d}\mathbf{z}^i \bmod \mathbb{Z} \right\|^2 .$$

It is easy to check that the solution verifies $\mathbf{b} = \frac{\mathbf{d}^T}{\|\mathbf{d}\|^2}$, so

$$\hat{\mathbf{d}} = \arg \min_{\mathbf{d}\in\mathbb{R}^n} \frac{\sum_{i=1}^{L} \left\| \mathbf{d}\mathbf{z}^i \bmod \mathbb{Z} \right\|^2}{\|\mathbf{d}\|^2}, \tag{5}$$

or similarly, due to the relationship between $\mathbf{b}$ and $\mathbf{d}$,

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}\in\mathbb{R}^n} \|\mathbf{b}\|^2 \sum_{i=1}^{L} \left\| \frac{\mathbf{b}^T\mathbf{z}^i}{\|\mathbf{b}\|^2} \bmod \mathbb{Z} \right\|^2 . \tag{6}$$

Therefore, the consideration of the dual lattice properties enables us to work with a cubic lattice in (5) or, equivalently, solve for a basis vector at a time.

3.1. Additional considerations

From (5) it is clear that as $\mathbf{d}\mathbf{z}^i \bmod \mathbb{Z}$ is always bounded, an unwanted solution would be to make $\mathbf{d}$ as large as possible (similarly, $\mathbf{b}$ would be as small as possible). Indeed, special attention should be paid to the following two related situations:

- Estimation of nested lattices: for any lattice Λ_1, such that $\Lambda \subset \Lambda_1$ (i.e., Λ is nested into Λ_1 [8]), if one takes arbitrary vectors $\mathbf{x} \in \Lambda$ and $\mathbf{y} \in \Lambda_1^{\perp}$, then $\mathbf{x}^T\mathbf{y} \in \mathbb{Z}$. The estimation of a lattice finer than the real lattice but containing the latter is in fact a correct solution to the lattice estimation problem.
- Degenerated solutions: a solution to (5) is $\|\mathbf{d}\| \to \infty$ (alternatively, $\|\mathbf{b}\| \to 0$). Nevertheless, this degenerated solution contravenes the high-SNR assumption introduced at the beginning of this section. To avoid the assumption being violated, an upper bound on $\|\mathbf{d}\|$ or a related penalty on the target function can be established in practice.

4. ALGORITHM DISCUSSION

In this section we give an outline of the algorithm that implements the solution presented in the previous section. We also briefly discuss some relevant implementation decisions. The algorithm comprises the following steps:

- Smallest quantization step estimation (only performed if *a priori* information about such step is not available). Most of the subsequent stages need knowledge of the quantization steps. Assuming that there are not large differences between them, we will focus on the estimate of the smallest. The proposed estimation strategy considers those input observations with Euclidean norm larger than a threshold, and orders them by increasing Euclidean norm. Let those norms be denoted by $t_1, \ldots, t_L$. The ratio between successive norms ($R_i \doteq t_i/t_{i-1}$) is computed. For samples corresponding to the same centroid, the resulting value will be close to 1; in contrast, for samples $\mathbf{z}^i$ and $\mathbf{z}^{i-1}$ coming from different centroids, R_i will be very large, more so when $\mathbf{z}^{i-1}$ comes from the $\mathbf{0}$ centroid. Therefore, we compute $i^* = \arg\max_i R_i$ and the quatization step is estimated as $\sqrt{t_{i^*}}$. This value is used for defining thresholds that help us to decide if an input sample corresponds to the $\mathbf{0}$ centroid, and if a vector is small enough to be a good candidate for being in the lattice basis.
- Lattice quantization implementation. Minimum distance quantizers and the corresponding modulo lattice reduction were implemented following [9].
- Low-SNR penalty. As mentioned in Section 3, some constraint or penalty should be introduced in order to ensure the verification of the high-SNR assumption. In the current implementation a penalty should be introduced in (6). Specifically, very low values of the candidate norm (orders of magnitude smaller than the estimated smallest quantization step) will increase the target function.
- Local minima. The main problem when dealing with (6), or equivalently with (5), is that the target function has lots of local minima due to the modulo reduction; therefore, off-the-shelf optimization algorithms do not guarantee the convergence to the global minimum (or even to a good enough solution). In the proposed algorithm this issue is solved by working with a set of candidate points; if the candidate point minimizing (6) provides a small (with respect to a threshold) value of that function, then it is used as starting point of a local optimization. In the current implementation, this is done by means of `fminsearch`, from the optimization toolbox of Matlab). The optimization result is included in the basis estimation (unless it can be written as linear combination of vectors already included in the basis estimation).
- Candidate set definition. Two different strategies are considered for defining the candidate set:
  1. We include both the input samples and their modulo-lattice reduction using the lattices spanned by other samples.
  2. We consider n input samples corresponding to different lattice centroids that are linearly independent. The matrix containing those n samples is a noisy basis of a sublattice of Λ, the original lattice we wish to estimate. Therefore, its inverse transpose is a noisy basis of a lattice containing $\Lambda^{\perp}$. From the latter basis, and using the target function introduced above, we can estimate $\Lambda^{\perp}$, and consequently Λ.

  We have experimentally observed that no choice is superior than the other for all cases. Therefore, the results provided by both are combined to produce an improved estimate.
- Candidate set update. Once we have estimated one vector in the lattice basis, we remove its contribution on the remaining vectors in the candidate set. In order to do that, we modulo reduce the vectors in the candidate set using the lattice spanned by the current estimate of the basis.
- Nested lattice. The final lattice basis estimate is processed in order to avoid the possible estimation of a finer lattice containing the true one.

5. FINAL STEP AND BIAS ANALYSIS

Once the vector-wise estimation of matrix B, that we will denote by B_v, is performed, a refinement of those estimated vectors can be

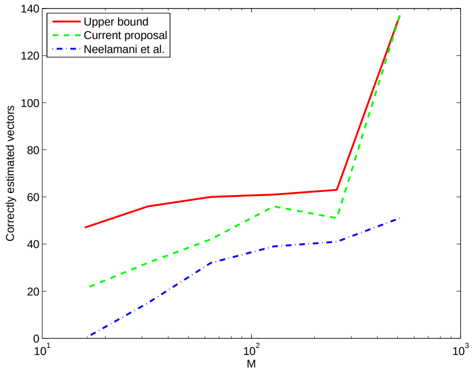

Fig. 1. Number of lattice basis vectors correctly estimated by the method proposed in [2] and the current proposal, following the experimental setup proposed in [2]: Lena image and the same color space and quantization table (corresponding to QF = 70). Subimages of $M \times M$ pixels are considered. $M \in \{16, 32, 64, 128, 256, 512\}$.

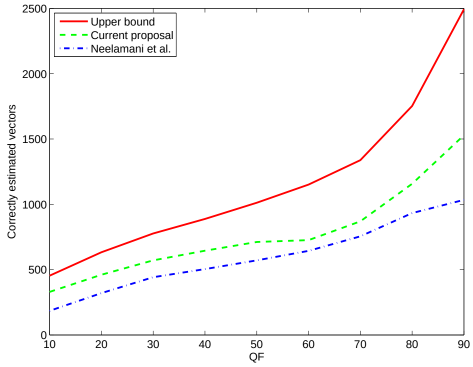

Fig. 2. Number of lattice basis vectors correctly estimated by the method proposed in [2] and the current proposal, as a function of the QF. The 16 color images in USC-SIPI Miscellaneous Image Database [10] are tested.

carried out by exploiting the structure of the resulting basis. We will assume that this estimate is good enough for correctly decoding the coordinates of the quantized signal $\bar{\mathbf{y}}$, i.e., for a basis B of the correct lattice Λ, this assumption implies that

$$\arg\min_{\mathbf{c}_1 \in \mathbb{Z}^n} \left\| \mathbf{z}^i - B_v \mathbf{c}_1 \right\|^2 = \arg\min_{\mathbf{c}_2 \in \mathbb{Z}^n} \left\| \mathbf{z}^i - B\mathbf{c}_2 \right\|^2 \doteq \mathbf{c}^i, \quad (7)$$

for all $1 \leq i \leq L$. Using these decided centroids $\mathbf{c}^i$, $i = 1, \cdots, L$, one can further improve the estimate by solving (2) as

$$\hat{B} = \arg\min_{B_c \in \mathbb{R}_{n \times n}} \sum_{i=1}^{L} \left\| \mathbf{z}^i - B_c \mathbf{c}^i \right\|^2. \quad (8)$$

A similar refinement is considered by Neelamani *et al.* [2], although therein it is carried out in each step of the algorithm after including a new histogram bin in the estimation (for updating the basis estimated so far); in contrast, our algorithm resorts to (8) just once. As discussed in [2], the solution to (8) is $\hat{B} = \left(\bar{\mathbf{z}}\bar{\mathbf{c}}^T\right)\left(\bar{\mathbf{c}}\bar{\mathbf{c}}^T\right)^{-1}$, where $\bar{\mathbf{c}}$ is a matrix whose columns are $\mathbf{c}^i$, $1 \leq i \leq L$.

In fact, the latter expression can be used to determine whether the resulting estimator is biased. Under the assumption that (7) holds, and taking into account that $\bar{\mathbf{Z}} = \bar{\mathbf{N}} + B\bar{\mathbf{C}}$ the mean of $\hat{B}$ is

$$\mathrm{E}\{\hat{B}\} = \mathrm{E}\left\{ (\bar{\mathbf{N}} + B\bar{\mathbf{C}})\, \bar{\mathbf{C}}^T \left(\bar{\mathbf{C}}\bar{\mathbf{C}}^T\right)^{-1} \right\} = B;$$

consequently, the proposed estimator is unbiased and constitutes a basis of Λ.

6. EXPERIMENTAL RESULTS

The main target of this section is to compare the results achieved by our method with those obtained by Neelamai *et al.* [2, 3]. In order to do so, a first experiment considers the framework used in [2], i.e., the quantization of image Lena 512×512 with the quantization table corresponding to Quality Factor (QF) 70, and the color space indicated therein. Furthermore, the number of independent centroids observed in our input samples is also computed, in order to provide an upper bound to the number of basis vectors that can be estimated; getting close to this bound is difficult, especially when jonly a few samples of a finely quantized signal are observed. Fig. 1 compares the number of lattice basis vectors correctly estimated by the method in [2] and ours, for different sub-images of Lena with the specified sizes. This serves to illustrate the variation of the estimated vectors versus the number of available samples. Remarkably, our scheme estimates a significantly larger number of basis vectors for all the sub-image sizes. When the full image is considered, the upper bound (137 vectors) is achieved, in contrast to the 51 vectors estimated in [2].

In the second experiment the 16 color images in [10] are considered, and we try to estimate the 63 3-dimensional lattices for each of them (therefore, the maximum number of estimated vectors would be 3024); since the sizes of some of those images are 512×512 while others are 256×256, for the former we have kept the upper left corner of size 256×256 to avoid the dependence of the results with the number of observations. The resulting images are JPEG compressed with different QFs, and then the basis vectors are estimated. Fig. 2 shows the overall number of correctly estimated basis vectors as a function of QF; again, the performance improvement achieved by our method is evident in all the QF range. Specifically, when low QFs are considered, our proposal is more suitable for extracting the information from the few available non-null centroids. Alternatively, for very high QFs the number of non-null centroids with just a few samples is very high; as discussed in Section 1, unlike our method, [3] is sensitive to this spread of the samples.

7. CONCLUSIONS AND FURTHER WORK

In this work a blind lattice estimation scheme, whose input are the noisy versions of the quantizer outputs, is proposed. The general problem is inspired in [2]. Nevertheless, the approach we follow, based on the use of the dual lattice, is substantially different, and entails a reduction of the computational complexity. The results of some of our experiments, run on the same use case in [2], show a dramatic increase in the number of estimated basis vectors that can be correctly estimated. Finally, we want to remark that our method can be applied to estimate higher dimensional lattices; preliminary results, not reported here, show its feasibility. In fact, the performance gains increase with the number of dimensions. This extension to higher-dimensional lattices will be discussed in a future work.

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