

Multuser Detection Performance in Multibeam Satellite Links under Imperfect CSI

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Abstract—In multibeam satellite systems, there is a growing need for signal processing techniques able to mitigate the interference among beams, since they could enable a much more aggressive spectrum reuse. In this paper, we investigate the effect of the absence of perfect Channel State Information (CSI) at the receiver end of the multibeam satellite return link. Under the assumption of a large number of beams, random matrix theory tools are used to obtain closed-form expressions of the performance for a given channel matrix and different profiles of estimation errors.

Index Terms—Multuser detection; on-ground beamforming; rain fading; multibeam satellites.

I. INTRODUCTION

In the last few years, the use of multiple spot beams in modern broadband satellites has increased, in an effort to serve higher throughput demands with a scalable cost. For this task, frequency reuse among users beams is required and, if total spectrum reuse is the goal, then it is necessary to somehow counteract the interference among beams that appears due to the side lobes in the satellite's radiation pattern. Many studies, such as [1], [2], have been conducted in order to evaluate the performance of interference mitigation techniques both in the forward link and return link.

However, it is important to support the practical application of these techniques with adequate performance prediction tools which consider commonly found impairments. One of the impairments to be taken into account is the non-perfect nature of the Channel State Information (CSI) available at the receiver, which will degrade the performance of the above mentioned interference mitigation schemes. In such a case, what the receiver has is an estimate of the channel matrix of the form $\mathbf{H} + \mathbf{E}$, where \mathbf{H} is the actual channel matrix and \mathbf{E} is a random matrix modeling the estimation errors.

In general, such a problem is difficult to solve analytically, because it requires manipulating the eigenvalue distribution of complicated matrices. The available literature reports solutions for some cases; for instance, [3] studies the performance

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of both the Zero-Forcing (ZF) and Linear Minimum Mean-Squared Error (LMMSE) receivers under imperfect CSI, but under the assumptions of \mathbf{H} consisting of i.i.d. entries (which is common in terrestrial scenarios).

In this work, we propose to study the mean squared error (MSE) obtained by a LMMSE receiver. Differently from other studies, we will allow \mathbf{H} to be any fixed matrix with finite entries; this allows to accommodate any antenna radiation pattern, provided in numerical or analytical form, at the cost of focusing over a time interval during which \mathbf{H} can be assumed not to change (that is, we focus on a coherence interval of the channel). Also, we will be assuming that the user terminals are fixed.

The proposed method relies on a tight approximation for the error covariance matrix for medium and high signal to interference plus noise ratio (SINR), and then exploits the large dimensionality of the system by resorting to asymptotic random matrix theory (RMT). As a result, the MSE will be proven to converge to a non-random value as the matrix dimensions grow large while preserving their proportion, and this value will be computed efficiently by solving a system of equations which has a unique solution. This procedure will be valid for any error matrix \mathbf{E} with zero-mean and independent entries of finite variance, which leaves room for a number of estimation error profiles.

The structure of the paper is the following: Section II describes the system model and the metric of study, Section III explains the proposed method, Section IV illustrates its performance by computer simulations and, finally, conclusions are drawn on Section V.

II. SYSTEM MODEL

A. Channel Model

The system under study consists of a coverage area formed by K beam spots in which a single user is active at a given time and carrier block. The satellite uses a fed reflector antenna array with N ($N \geq K$) feeds and relays the impinging signals to a gateway station on Earth through a transparent link.

The signal model for a given time instant will thus read as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (1)$$

where $\mathbf{y} \in \mathcal{C}^{N \times 1}$ is the set of received symbols, $\mathbf{s} \in \mathcal{C}^{K \times 1}$ is the set of unit-power transmitted symbols, $\mathbf{H} \in \mathcal{C}^{N \times K}$ is the

channel matrix and $\mathbf{n} \in \mathcal{C}^{N \times 1}$ contains zero-mean Gaussian samples with covariance matrix $\mathbb{E}[\mathbf{nn}^H] \doteq \Sigma$; for notational convenience, we will also define $\tilde{\Sigma}$ such that $\Sigma = N_0 \tilde{\Sigma}$, with N_0 the noise power.

Upon reception of \mathbf{y} , the gateway will use a LMMSE receiver to recover \mathbf{s} . However, we will assume that it has perfect knowledge of the noise covariance, but only an estimate of the channel matrix, $\hat{\mathbf{H}}$, so that the receiver operation will read as

$$\hat{\mathbf{s}} = \bar{\mathbf{W}}^H \mathbf{y} \quad (2)$$

where $\bar{\mathbf{W}}^H$ is the LMMSE combining matrix built from channel estimates [4]

$$\bar{\mathbf{W}}^H = \left(\mathbf{I} + \hat{\mathbf{H}}^H \Sigma^{-1} \hat{\mathbf{H}} \right)^{-1} \hat{\mathbf{H}}^H \Sigma^{-1}. \quad (3)$$

B. Estimation errors

We will assume data aided channel estimation, where each user employs a different training sequence of length L , known as its unique word. As in [5] and [6], we will neglect the impairments caused by imperfect synchronization, and assume that the different unique words can be considered orthogonal. Following this considerations, we come up with the following model for the estimation errors

$$\begin{aligned} \hat{\mathbf{H}} &= \mathbf{H} + \frac{N_0}{L} \mathbf{E} \\ &= \mathbf{H} + \alpha \mathbf{E} \end{aligned} \quad (4)$$

where \mathbf{H} is a perfectly known deterministic matrix, \mathbf{E} is assumed to be a matrix of independent, zero-mean Gaussian entries, and we have defined $\alpha \doteq N_0/L$ for simplicity.

From (4), we can see that the variance of the error at each entry will be proportional to the noise power and inversely proportional to the training sequence length. However, note that so far we have imposed no further constraints on the variance of each element in \mathbf{E} . As we will discuss in Section III-B, we will allow each element to have a different variance as long as some mild constraints are satisfied.

C. Mean squared error as performance metric

A common performance metric is the mean-squared error, defined as

$$\text{MSE} = \mathbb{E}[\|\mathbf{s} - \hat{\mathbf{s}}\|^2]. \quad (5)$$

The error covariance matrix, denoted as \mathbf{Q} , would read as

$$\mathbf{Q} = \mathbb{E}[(\mathbf{s} - \hat{\mathbf{s}})(\mathbf{s} - \hat{\mathbf{s}})^H] \quad (6)$$

and the error for the i -th user would therefore be

$$\epsilon_i^2 = \mathbf{Q}_{ii} \quad (7)$$

so that, averaging the error over all the users, we can define:

$$\epsilon^2 \doteq \frac{1}{K} \text{trace } \mathbf{Q}. \quad (8)$$

One of the reasons for the popularity of this metric is its immediate relation with the SINR after combining, namely [4, Eq. (6.32)]

$$\rho_i = \frac{1}{\epsilon_i^2} - 1. \quad (9)$$

Other reason is that, when stated as in (8), it can be analytically tackled for a number of cases of interest. For example, under the assumption of perfect CSI, the matrix \mathbf{Q} would have the following expression [4]

$$\mathbf{Q} = (\mathbf{I} + \mathbf{H}^H \Sigma^{-1} \mathbf{H})^{-1}. \quad (10)$$

Unfortunately, obtaining \mathbf{Q} in our case would lead to a much more complex expression, so that deriving closed-form formulas for ϵ^2 or any ρ_i seems analytically intractable. To overcome this problem, in the next section we will explore the use of an approximation of \mathbf{Q} which holds whenever the SINR is not very low. From this expression, we will make use of an existing result in the field of RMT to compute ϵ^2 ; we will also discuss on the conditions the system must meet for the validity of this result.

III. APPROXIMATION OF ϵ^2

A. Approximation of \mathbf{Q}

One of the problems for computing ϵ^2 is that, with imperfect CSI, the covariance matrix has a much more involved expression. As a consequence, we will firstly derive a much more tractable approximation which will hold for medium and high SINR values.

Lemma 1. *Assume that the receiver uses a LMMSE receiver, but has only an imperfect estimation of the channel matrix \mathbf{H} given by $\hat{\mathbf{H}} = \mathbf{H} + \alpha \mathbf{E}$. Then, the error covariance matrix can be approximated by*

$$\mathbf{Q} \approx \left(1 + \frac{K}{L} \right) \left(\mathbf{I} + \hat{\mathbf{H}}^H \Sigma^{-1} \hat{\mathbf{H}} \right)^{-1}. \quad (11)$$

Proof: Expanding the channel model, and taking into account that $\hat{\mathbf{H}} = \mathbf{H} + \alpha \mathbf{E}$, we have

$$\mathbf{y} = \hat{\mathbf{s}} - \alpha \mathbf{E} \mathbf{s} + \mathbf{n}. \quad (12)$$

Let us define $\mathbf{u} \doteq \alpha \mathbf{E} \mathbf{s} + \mathbf{n}$ with $\mathbb{E}[\mathbf{uu}^H] = (1 + K/L)\Sigma$. After applying the LMMSE receiver, we will have

$$\hat{\mathbf{s}} = \bar{\mathbf{W}}^H \mathbf{H} \mathbf{s} - \bar{\mathbf{W}}^H \mathbf{u}. \quad (13)$$

To obtain an approximation, we will start by focusing on the SINR interval in which the contribution to the error is much greater in $\bar{\mathbf{W}}^H \mathbf{u}$ than in $\bar{\mathbf{W}}^H \mathbf{H} \mathbf{s}$; to this end, recall that the transmit power is included into \mathbf{H} . Assuming this is so, we can approximate the covariance matrix by

$$\begin{aligned} \mathbf{Q} &\approx \mathbb{E}[\bar{\mathbf{W}}^H \mathbf{u} \mathbf{u}^H \bar{\mathbf{W}}] \\ &= \left(1 + \frac{K}{L} \right) \bar{\mathbf{W}}^H \Sigma \bar{\mathbf{W}}. \end{aligned} \quad (14)$$

Plugging the expression of $\bar{\mathbf{W}}$ we get

$$\begin{aligned} \mathbf{Q} &\approx \left(1 + \frac{K}{L} \right) \\ &\times \left(\mathbf{I} + \hat{\mathbf{H}}^H \Sigma^{-1} \hat{\mathbf{H}} \right)^{-1} \hat{\mathbf{H}}^H \Sigma^{-1} \hat{\mathbf{H}} \left(\mathbf{I} + \hat{\mathbf{H}}^H \Sigma^{-1} \hat{\mathbf{H}} \right)^{-1} \\ &\approx \left(1 + \frac{K}{L} \right) \left(\mathbf{I} + \hat{\mathbf{H}}^H \Sigma^{-1} \hat{\mathbf{H}} \right)^{-1} \end{aligned} \quad (15)$$

where in the last equality we have used the approximation $(\mathbf{I} + \hat{\mathbf{H}}^H \boldsymbol{\Sigma}^{-1} \hat{\mathbf{H}})^{-1} \hat{\mathbf{H}}^H \boldsymbol{\Sigma}^{-1} \hat{\mathbf{H}} \approx \mathbf{I}$ for simplifying the first two factors. ■

The main advantage of the expression above is that it allows a simple computation of ϵ^2 . Recalling that $\boldsymbol{\Sigma} = N_0 \tilde{\boldsymbol{\Sigma}}$, then

$$\begin{aligned} \epsilon^2 &= \frac{1}{K} \text{trace} \left[\left(\mathbf{I} + \hat{\mathbf{H}}^H \boldsymbol{\Sigma}^{-1} \hat{\mathbf{H}} \right)^{-1} \right] \\ &= \frac{1}{K} \sum_{i=1}^K \frac{1}{1 + \lambda_i \left\{ \hat{\mathbf{H}}^H \boldsymbol{\Sigma}^{-1} \hat{\mathbf{H}} \right\}} \\ &= \frac{N_0}{K} \sum_{i=1}^K \frac{1}{N_0 + \lambda_i \left\{ \hat{\mathbf{H}}^H \tilde{\boldsymbol{\Sigma}}^{-1} \hat{\mathbf{H}} \right\}} \end{aligned} \quad (16)$$

where $\lambda_i \{ \}$ denotes the i -th largest eigenvalue of the matrix between brackets. In the limit, we have that

$$\begin{aligned} \lim_{N, K \rightarrow \infty} \epsilon^2 &= \\ \lim_{N, K \rightarrow \infty} \frac{N_0}{K} \sum_{i=1}^K \frac{1}{N_0 + \lambda_i \left\{ \hat{\mathbf{H}}^H \tilde{\boldsymbol{\Sigma}}^{-1} \hat{\mathbf{H}} \right\}} & \\ = N_0 \mathbb{E} \left[\frac{1}{N_0 + \lambda \left\{ \hat{\mathbf{H}}^H \tilde{\boldsymbol{\Sigma}}^{-1} \hat{\mathbf{H}} \right\}} \right] & \\ = N_0 \cdot \mathcal{S}_{\hat{\mathbf{H}}^H \tilde{\boldsymbol{\Sigma}}^{-1} \hat{\mathbf{H}}}(-N_0). & \end{aligned} \quad (17)$$

Here, $\mathcal{S}_{\mathbf{A}}(z)$ denotes the *Stieltjes transform* [7] of the (empirical eigenvalue distribution of) matrix \mathbf{A} , hence the last equality follows directly from its definition

$$\mathcal{S}_{\mathbf{X}}(z) = \mathbb{E} \left[\frac{1}{X - z} \right]. \quad (18)$$

B. Computing the Stieltjes transform

Linking ϵ^2 with the Stieltjes transform of $\hat{\mathbf{H}}^H \tilde{\boldsymbol{\Sigma}}^{-1} \hat{\mathbf{H}}$ will prove very useful for our purposes, since this transform has been derived for a wide range of matrices, many times without requiring the actual empirical eigenvalue distribution. In our case, we will make use of the following result from [8]:

Lemma 2. *Let $\boldsymbol{\Delta}$ be a matrix of the form*

$$\boldsymbol{\Delta} = \frac{1}{\sqrt{K}} \boldsymbol{\Xi} \odot \mathbf{X} \quad (19)$$

where \odot denotes the *Hadamard (or entry-wise) product*, \mathbf{X} has zero-mean i.i.d elements with some finite moment of order higher than four and $\boldsymbol{\Xi}$, the so-called *variance profile*, has only real finite entries; consider also a deterministic matrix \mathbf{A} whose columns have finite Euclidean norm, and denote $\boldsymbol{\Theta} = \mathbf{A} + \boldsymbol{\Delta}$.

Then, there exists a deterministic matrix $\tilde{\mathbf{T}}(N_0)$ such that, while the ratio K/N is kept constant,

$$\lim_{N, K \rightarrow \infty} \frac{1}{K} \text{trace} \tilde{\mathbf{T}}(N_0) = N_0 \cdot \mathcal{S}_{\boldsymbol{\Theta}^H \boldsymbol{\Theta}}(-N_0). \quad (20)$$

In [8] they also prove that $1/K \text{trace} \tilde{\mathbf{T}}(N_0)$ can be computed by solving a system of $N + K$ equations which has a unique solution.

Applying this result to our problem, we immediately have that

$$\mathbf{A} = \hat{\mathbf{H}} \boldsymbol{\Sigma}^{H/2} \quad (21)$$

and

$$\boldsymbol{\Delta} = \alpha \boldsymbol{\Sigma}^{1/2} \mathbf{E}. \quad (22)$$

From the expression above, we can see that the assumptions on \mathbf{A} always hold. In what refers to $\boldsymbol{\Delta}$, the variance of its elements will be finite, and thus the condition of some finite moment of order higher than four –which is needed for the proof of Lemma 2 given in [8]– is met. Still, we need to check under which conditions the elements of $\boldsymbol{\Delta}$ are independent¹; we will do this in the following paragraphs, and we will also point out a common particular case of this problem which leads to a simple solution.

1) *Conditions on $\boldsymbol{\Sigma}$:* Now that $\boldsymbol{\Delta} = \alpha \boldsymbol{\Sigma}^{1/2} \mathbf{E}$, it is worth discussing which conditions must $\boldsymbol{\Sigma}$ meet in order to fulfill the assumptions of Lemma 2, and in particular the assumption of independence among its entries. To do this, let us define $\mathbf{e} = \text{vec}(\boldsymbol{\Delta})$ so that $\mathbf{e} \in \mathcal{C}^{KN \times 1}$ is the stack of all the columns of $\boldsymbol{\Delta}$.

For the entries of $\boldsymbol{\Delta}$ to be independent, we must have that

$$\mathbb{E}[e_i e_j] = \mathbb{E}[e_i] \mathbb{E}[e_j] = 0 \quad i \neq j, \quad (23)$$

given that all the elements have zero mean and that we have assumed \mathbf{E} to be formed by Gaussian elements; this is equivalent to saying that the covariance matrix of \mathbf{e} , $\mathbb{E}[\mathbf{e} \mathbf{e}^H]$ must be diagonal. Operating, we have that

$$\begin{aligned} \mathbb{E}[\mathbf{e} \mathbf{e}^H] &= \alpha^2 \mathbb{E} \left[\text{vec} \left(\boldsymbol{\Sigma}^{1/2} \mathbf{E} \right) \text{vec} \left(\boldsymbol{\Sigma}^{1/2} \mathbf{E} \right)^H \right] \\ &= \alpha^2 \mathbb{E} \left[\left(\mathbf{I} \otimes \boldsymbol{\Sigma}^{1/2} \right) \text{vec}(\mathbf{E}) \text{vec}(\mathbf{E})^H \left(\mathbf{I} \otimes \boldsymbol{\Sigma}^{H/2} \right) \right] \\ &= \alpha^2 \left(\mathbf{I} \otimes \boldsymbol{\Sigma} \right) \end{aligned} \quad (24)$$

where \otimes denotes the *Kronecker product* and we have used the identity $\text{vec}(\mathbf{A} \mathbf{B}) = (\mathbf{I} \otimes \mathbf{A}) \text{vec}(\mathbf{B})$.

From this equation, it is clear that $\boldsymbol{\Sigma}$ must be diagonal for (23) to hold, even though its elements can be different. However, and as we will show in Section IV, it is still possible to obtain good results as long as the off diagonal elements of $\boldsymbol{\Sigma}$ are very small.

2) *The particular case $\boldsymbol{\Sigma} = N_0 \mathbf{I}$, $\boldsymbol{\Xi}_{ij} = 1$:* Assume now that $\boldsymbol{\Sigma} = N_0 \mathbf{I}$, and that the elements of \mathbf{E} have unit variance. These conditions lead to a very simple, illustrative solution since, in this case, the system of equations $N + K$ to be solved

¹It is worth noticing that, even though the elements of \mathbf{X} must be identically distributed, those of $\boldsymbol{\Delta}$ are allowed to have different variances; it suffices to comprise them in matrix $\boldsymbol{\Xi}$.

Table I
SIMULATION PARAMETERS

Simulation parameters	
Frequency band	Ka band (30 GHz)
User rate	4 Mbaud/s
Receiver noise figure	2.5 dB
Total receiver noise temperature	517 K
Feed gain patterns	Provided by ESA
Number of beams	$K = 100$
Number of feeds	$N = 155$
UTs location distribution	Fixed, one per beam
Training sequence length	256 symbols
Monte Carlo iterations	10,000

reads as

$$\begin{aligned}
 x &= \text{trace}(\mathbf{T}) \\
 y &= \text{trace}(\tilde{\mathbf{T}}) \\
 \mathbf{T} &= \left(N_0(1 + \alpha x)\mathbf{I} + \frac{1}{1 + \alpha y} \hat{\mathbf{H}}\hat{\mathbf{H}}^H \right)^{-1} \\
 \tilde{\mathbf{T}} &= \left(N_0(1 + \alpha y)\mathbf{I} + \frac{1}{1 + \alpha x} \hat{\mathbf{H}}^H\hat{\mathbf{H}} \right)^{-1}
 \end{aligned} \quad (25)$$

with $\alpha = N_0/L$; as proven in [8], the solution is unique.

The above system can be straightforwardly extended to the case $\Xi_{ij} = C$, with C some real, finite constant different from 1, by a simple scaling of α . However, when the assumptions above are not met, we must resort to a more involved formulation of the system which requires the definition of some auxiliary matrices; the expression for the general case can be checked on Appendix A.

Before going further and showing the simulation results, let us summarize the procedure we have described so far: in order to compute ϵ^2 , we start by obtaining an approximation of the error covariance matrix \mathbf{Q} (Lemma 1); then, we use a result from RMT (Lemma 2) to compute its trace –and thus compute ϵ^2 – by solving a system of $N + K$ equations.

IV. SIMULATION RESULTS

In this section, we illustrate the performance of the derived approximation by computer simulations. The simulator setup is summarized on Table I; we used a 155×100 beam pattern provided by the European Space Agency (ESA) –aimed at covering the whole European continent– where each feed contributes to forming about six beams.

Additionally, we will also test the case in which Σ is not diagonal, but has small off-diagonal elements. We will do so by assuming a fixed beamforming matrix $\mathbf{B} \in \mathcal{C}^{100 \times 155}$, with $\mathbf{B}\mathbf{B}^H$ a matrix with small off-diagonal elements², to be applied on the received vector \mathbf{y} before any processing; this matrix also comes from ESA, and is aimed at minimizing the average interference for a uniform distribution of the users

²This is a consequence of \mathbf{B} having to force zeros on off-diagonal elements of the product $\mathbf{B}\mathbf{H}$ in order to minimize interference.

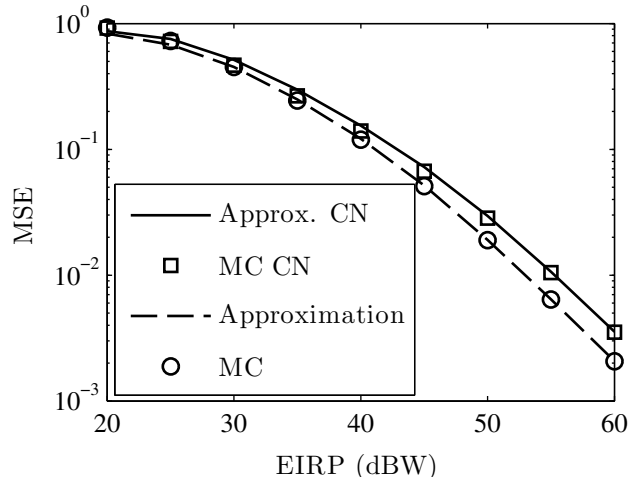


Figure 1. Comparison between the MSE and its approximation, both for uncorrelated noise and correlated noise (CN). MC stands for Monte Carlo, and Approx. is the derived approximation.

within their beam spots. With this fixed beamforming, the deterministic matrix \mathbf{A} would be given by $\mathbf{B}\mathbf{H}\Sigma^{H/2}$, whereas the random matrix is $\alpha\mathbf{B}\Sigma^{1/2}\mathbf{E}$ and $\Sigma = N_0\mathbf{B}\mathbf{B}^H$.

The system of equations (25) was solved by Matlab’s function *fminsearch*. With uncorrelated noise, the convergence of the algorithm is very fast –almost instantaneous– but, with correlated noise, the new variance profile of matrix Δ and the existence of some non-zero off-diagonal elements will affect the accuracy and convergence speed; still, results can be obtained fastly by feeding the algorithm with a good starting point.

Focusing on the results, Figure 1 compares the derived approximation with the actual Monte Carlo results as a function of the terminals’ equivalent isotropic radiated power (EIRP); the accuracy is quite remarkable whenever the EIRP is not very low, since that would be the area where the approximation of \mathbf{Q} in (14) does not hold. Additionally, Figure 2 depicts the accuracy by means of the *absolute relative error*. We can see that that the proposed method is more accurate when $\tilde{\Sigma} = \mathbf{I}$, and that its accuracy increases as the SINR increases.

V. CONCLUSIONS

In this paper, we have obtained a tight approximation for the mean-squared error of a multi-user communications system with interference mitigation. In particular, we have tackled the problem of a satellite return link equipped with an LMMSE receiver that has access to partial CSI only. Our method relies on an approximation of the error covariance matrix, for which existing results in the field of random matrix theory can be applied. Simulation results have shown a remarkable tightness at medium and high SINR.

APPENDIX A

FINDING \mathbf{T} IN THE GENERAL CASE

When the variance profile of matrix Δ is not constant, then we must resort to a more general formulation of the system, as

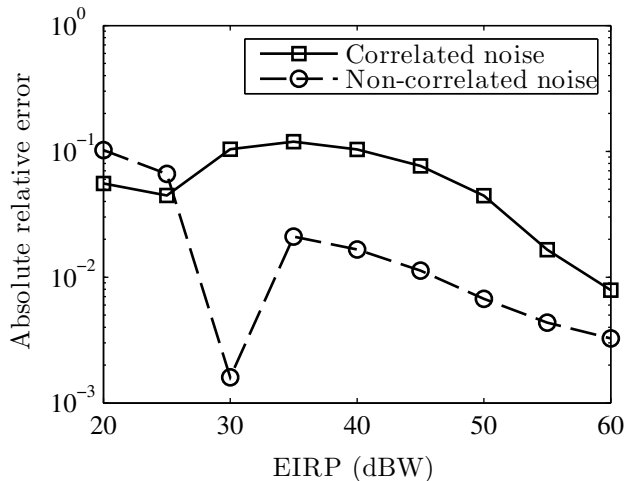


Figure 2. Accuracy of the approximation measured by the absolute relative error.

described in [8]. In this case, before describing the expressions of \mathbf{T} and $\tilde{\mathbf{T}}$, we need to define

$$\psi_i(z) = -\frac{1}{z \left(1 + \frac{1}{K} \text{trace}(\tilde{\mathbf{D}}_i \tilde{\mathbf{T}}(z))\right)} \quad 1 \leq i \leq N \quad (26)$$

$$\tilde{\psi}_j(z) = -\frac{1}{z \left(1 + \frac{1}{K} \text{trace}(\mathbf{D}_j \mathbf{T}(z))\right)} \quad 1 \leq j \leq K$$

where

$$\mathbf{D}_j = \text{diag}(\text{col}_j(\mathbf{\Xi})) \quad (27)$$

and

$$\tilde{\mathbf{D}}_i = \text{diag}(\text{row}_i(\mathbf{\Xi})). \quad (28)$$

Here, $\text{col}_i(\mathbf{M})$ and $\text{row}_i(\mathbf{M})$ denote the i -th column and the i -th row of matrix \mathbf{M} , respectively, and $\text{diag}(\mathbf{m})$ denotes a diagonal matrix whose diagonal is constituted by the elements in vector \mathbf{m} .

From (26), we also build

$$\Psi(z) = \text{diag}(\psi_i(z)) \quad (29)$$

$$\tilde{\Psi}(z) = \text{diag}(\tilde{\psi}_j(z))$$

and finally

$$\mathbf{T}(z) = \left(\Psi^{-1}(z) - z \mathbf{A} \tilde{\Psi}(z) \mathbf{A}^H \right)^{-1} \quad (30)$$

$$\tilde{\mathbf{T}}(z) = \left(\tilde{\Psi}^{-1}(z) - z \mathbf{A}^H \Psi(z) \mathbf{A} \right)^{-1}.$$

REFERENCES

[1] G. Caire, M. Debbah, L. Cottatellucci, R. De Gaudenzi, R. Rinaldo, R. Mueller, and G. Gallinaro, "Perspectives of adopting interference mitigation techniques in the context of broadband multimedia satellite systems," in *ICSSC 2005, 23rd AIAA International Communications Satellite Systems Conference, 25-28 September, 2005, Rome, Italy*, 09 2005.

[2] J. Arnau-Yanez, M. Bergmann, E. Candrea, G. Corazza, R. de Gaudenzi, B. Devillers, W. Gappmair, F. Lombardo, C. Mosquera, A. Perez-Neira, I. Thibault, and A. Vanelli-Coralli, "Hybrid space-ground processing for high-capacity multi-beam satellite systems," in *Global Telecommunications Conference (GLOBECOM 2011), 2011 IEEE*, dec. 2011, pp. 1–6.

[3] J. Hoydis, M. Kobayashi, and M. Debbah, "Asymptotic performance of linear receivers in network mimo," in *Signals, Systems and Computers (ASILOMAR), 2010 Conference Record of the Forty Fourth Asilomar Conference on*, nov. 2010, pp. 942–948.

[4] J. Choi, *Optimal Combining and Detection: Statistical Signal Processing for Communications*, 1st ed. New York, NY, USA: Cambridge University Press, 2010.

[5] J. Arnau, B. Devillers, C. Mosquera, and A. Perez-Neira, "Performance study of multiuser interference mitigation schemes for hybrid broadband multibeam satellite architectures," *EURASIP Journal on Wireless Communications and Networking*, vol. 2012, no. 1, p. 132, 2012. [Online]. Available: <http://jwcn.eurasipjournals.com/content/2012/1/132>

[6] G. Gallinaro, M. Debbah, R. Müller, R. Rinaldo, and A. Vernucci, "Interference Mitigation for the Reverse-Link of Interactive Satellite Networks," in *9th International Workshop on Signal Processing for Space Communications*, Sep. 2006.

[7] A. M. Tulino and S. Verdú, "Random matrix theory and wireless communications," *Commun. Inf. Theory*, vol. 1, no. 1, pp. 1–182, Jun. 2004. [Online]. Available: <http://dx.doi.org/10.1516/0100000001>

[8] W. Hachem, P. Loubaton, and J. Najim, "Deterministic equivalents for certain functionals of large random matrices," *The Annals of Applied Probability*, vol. 17, no. 3, pp. 875–930, 2007. [Online]. Available: <http://www.jstor.org/stable/25442865>