OVERLAY SPECTRUM REUSE IN A MULTICARRIER BROADCAST NETWORK: COVERAGE ANALYSIS

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ABSTRACT

A secondary cognitive user overlaying its message in a broadcast multicarrier network is studied. The secondary user exploits the primary message knowledge to convey its own information while preserving the primary user coverage area, determined by a bound on the BER, and taking into account the degradation due to the insertion of an echo in a dominant line of sight environment. The results are compared with those obtained when the coverage area is defined in capacity terms, which do not consider the practical degradation caused by the secondary replica of the primary message.

1. INTRODUCTION

The overlay approach to a secondary user accessing a licensed band has arisen as an alternative to the interweave paradigm: while in the latter the cognitive user senses the spectrum, looking for unused frequency bands (the so-called *white spaces*) to use for the secondary transmission, the former tries to exploit the knowledge of the primary message to transmit using the same frequency resources as the primary system while preserving the primary user Quality of Service (QoS) [1].

One of the scenarios where a secondary user could gain access to the primary signal is in broadcast Single Frequency Networks (SFN) [2], where the signal to be transmitted is delivered to the different primary transmitters via a distribution network. Therefore, the secondary transmitter could join the primary network, acquire the primary signal and exploit this knowledge to increase its spectral efficiency while preserving the primary user QoS.

Thus, the insertion of a secondary signal is similar to the case of a transmitter inserting local content in a broadcast SFN using hierarchical constellations: the secondary (or local) transmitter allocates some of its available power to the primary (or global) message in order to meet an interference constraint, while the remaining power is used to convey the secondary (or local) information.

Although this scenario has been previously studied in the literature [3, 4] for the Additive White Gaussian Noise (AWGN) channel, the insertion of a secondary user conveying the primary message creates a multipath channel with a potential performance degradation, especially in those systems with a strong Line Of Sight (LOS) component, with the AWGN channel the worst case scenario, as shown in [5].

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In [2] it was shown that the prefiltering of the primary message at a cooperative secondary transmitter (a secondary transmitter that acts as a pure relay, without inserting its own information) dramatically improves the performance of the primary user. In the companion paper [6] the insertion of a secondary message with a constraint on the Bit Error Rate (BER) of a single receiver is studied. In this paper we study the problem of inserting a secondary message while keeping the original coverage area of the primary transmitter, and compare the results with those obtained when the degradation due to the secondary replica is not taken into account.

The paper is structured as follows: in Section 2 we present a capacity-based analysis of the primary coverage, while in Section 3 we propose an alternative analysis, based on a bound on the BER, and taking into account the possible degradation due to the insertion of a secondary replica in an AWGN channel. Finally, Section 4 presents the conclusions.

2. SIMPLIFIED SCENARIO

In this section, we present a simple scenario where a secondary transmitter, who wants to maximize its own rate subject to a QoS constraint regarding the primary service, is placed within the coverage area of a primary user.

Let us assume that the primary user transmits at a fixed rate R_p with power P_p, and is located at $\mathbf{x}_p \in \mathbb{R}^2$. The capacity of the channel from the primary transmitter to a primary receiver located at $\mathbf{x}_r$ is $C(\mathbf{x}_r) = \log_2\left(1 + \frac{P_p l(\mathbf{x}_p, \mathbf{x}_r)}{\sigma^2}\right)$ bits per channel use, where $l(\mathbf{x}_1, \mathbf{x}_2)$ denotes the propagation loss from a transmitter located at $\mathbf{x}_1$ to a receiver located at $\mathbf{x}_2$, and σ^2 is the noise power, assumed constant for all receivers. Thus, we define the initial coverage zone as $\mathcal{C}_0 = \{\mathbf{x} \,|\, C(\mathbf{x}) \geq R_p\}$, or, equivalently, $\mathcal{C}_0 = \left\{\mathbf{x} \,\middle|\, \frac{P_p l(\mathbf{x}_p, \mathbf{x})}{\sigma^2} \geq \Upsilon_0\right\}$ where $\Upsilon_0 = 2^{R_p} - 1$ is the required Signal to Noise Ratio (SNR) for a correct reception.

We will assume that the secondary user has a fixed location inside the coverage zone, $\mathbf{x}_s \in \mathcal{C}_0$, and has a total transmit power P_s. This power has a twofold purpose, the reinforcement of the primary signal, for which γ^2 units of power are used, and the transmission of the secondary message, with a power consumption of ρ^2, such that $\rho^2 + \gamma^2 \leq P_s$. With the insertion of the secondary transmitter, the capacity of the channel of a primary receiver located at $\mathbf{x}_r$ with the primary transmitter will be

$$C\left(\mathbf{x}_r, \gamma^2, \rho^2\right) = \log_2\left(1 + \frac{P_p l(\mathbf{x}_p, \mathbf{x}_r) + \gamma^2 l(\mathbf{x}_s, \mathbf{x}_r)}{\rho^2 l(\mathbf{x}_s, \mathbf{x}_r) + \sigma^2}\right) \quad (1)$$

where it has been assumed that the transmission of the primary signal from the secondary transmitter results in the addition of the primary and secondary contributions, similarly to [3].

In order to maximize its own transmission rate, the secondary user can apply interference cancellation techniques such as Dirty Paper Coding (DPC) at the transmitter, exploiting the available side information, or Sequential Interference Canceling (SIC) at the receiver, as the secondary receiver is located in the primary user coverage area and, therefore, is able to decode the primary message and subtract it from the original signal. Thus, the capacity of the channel from the secondary transmitter to a secondary receiver located at $\mathbf{x}_c$ is

$$C_s\left(\mathbf{x}_c,\gamma^2,\rho^2\right)=\log_2\left(1+\frac{\rho^2 l\left(\mathbf{x}_s,\mathbf{x}_c\right)}{\sigma^2}\right), \quad (2)$$

so the maximization of the secondary capacity is equivalent to the maximization of the power allocated to the secondary message. Under the restriction of preserving the original coverage area, the optimization problem is stated as

$$\begin{aligned} \text{maximize} \quad & \rho^2 \\ \text{subject to} \quad & \Upsilon\left(\mathbf{x},\gamma^2,\rho^2\right)\geq\Upsilon_0\,\forall\mathbf{x}\in\mathcal{C}_0 \\ & \gamma^2+\rho^2\leq P_s \end{aligned} \quad (3)$$

with

$$\Upsilon\left(\mathbf{x},\gamma^2,\rho^2\right)=\frac{P_p l\left(\mathbf{x}_p,\mathbf{x}\right)+\gamma^2 l\left(\mathbf{x}_s,\mathbf{x}\right)}{\rho^2 l\left(\mathbf{x}_s,\mathbf{x}\right)+\sigma^2}. \quad (4)$$

Despite having an infinite number of constraints, this problem is analytically tractable. In the neighborhood of the secondary transmitter, the power coming from the primary user and the noise power are expected to be negligible with respect to the power received from the secondary transmitter, so the condition $\lim_{\mathbf{x}\to\mathbf{x}_s}\Upsilon\left(\mathbf{x},\gamma^2,\rho^2\right)=\gamma^2/\rho^2\geq\Upsilon_0$ has to be met. However, if we force this inequality, we have that for $\mathbf{x}\in\mathcal{C}_0$

$$\begin{aligned} \Upsilon\left(\mathbf{x},\gamma^2,\rho^2\right) &= \frac{P_p l\left(\mathbf{x}_p,\mathbf{x}\right)+\gamma^2 l\left(\mathbf{x}_s,\mathbf{x}\right)}{\rho^2 l\left(\mathbf{x}_s,\mathbf{x}\right)+\sigma^2} & (5)\\ &\geq \frac{P_p l\left(\mathbf{x}_p,\mathbf{x}\right)+\Upsilon_0\rho^2 l\left(\mathbf{x}_s,\mathbf{x}\right)}{\rho^2 l\left(\mathbf{x}_s,\mathbf{x}\right)+\sigma^2} & (6)\\ &\geq \frac{\Upsilon_0\sigma^2+\Upsilon_0\rho^2 l\left(\mathbf{x}_s,\mathbf{x}\right)}{\rho^2 l\left(\mathbf{x}_s,\mathbf{x}\right)+\sigma^2}=\Upsilon_0 & (7)\end{aligned}$$

where the second inequality is obtained from the definition of the coverage area $P_p l\left(\mathbf{x}_p,\mathbf{x}\right)/\sigma^2\geq\Upsilon_0$. Note that this approach is more conservative that the one in [3], as we are adding a *worst case* receiver, located near the secondary transmitter.

Thus, the problem is solved just by setting the ratio between the power allocated to the primary and secondary messages over the reception threshold Υ_0, which allows the secondary user to set

$$\rho^2=\frac{P_s}{1+\Upsilon_0},\ \gamma^2=\frac{\Upsilon_0 P_s}{1+\Upsilon_0} \quad (8)$$

and meet both coverage and power constraints. It is remarkable that this result does not depend neither on the location of the secondary user, provided it is inside the primary coverage zone, nor on the concrete propagation loss model. This conclusion is a direct consequence of the unrealistic assumption of the two power contributions of the primary signal being directly added, without taking into account the possible performance degradation caused by the secondary replica of the primary message.

A more realistic system model will be studied in the remaining of the paper, taking into account the possible SFN loss due to the presence of echoes in strong LOS reception, and focusing on a bound of the BER as the measure of quality of the primary user.

3. COVERAGE ANALYSIS

A BER analysis will be carried out for all receivers within the initial coverage area, thus showing the limitations of the previous capacity approach. Bounds will be obtained following similar principles to those in [6], where Effective SNR metrics were validated for the single receiver case.

We will assume that the primary transmitter uses Orthogonal Frequency Division Multiplexing (OFDM) with N carriers, and that the links from both primary and secondary transmitters to a given primary receiver located in $\mathbf{x}$ can be modeled as AWGN channels, so the equivalent baseband received signal in the Discrete Fourier Transform (DFT) domain after the Cyclic Prefix (CP) removal can be written as

$$Y_k(\mathbf{x})=\left(1+\gamma(\mathbf{x})e^{-j(2\pi k n_0(\mathbf{x})/N+\theta(\mathbf{x}))}F_k\right)C_k+\rho(\mathbf{x})S_k+W_k(\mathbf{x}) \quad (9)$$

where the equivalent channel was normalized to set the channel from the primary transmitter to 1, while $\gamma(\mathbf{x})$, $\theta(\mathbf{x})$ and $n_0(\mathbf{x})$ are the relative amplitude, phase and delay of the primary signal contribution sent from the secondary transmitter, C_k denotes the primary symbol on the k-th carrier (normalized to have unit power), $\rho(\mathbf{x})$ denotes the relative amplitude of the secondary signal $S_k\sim\mathcal{CN}(0,1)$, assumed to be Gaussian, sent from the secondary transmitter, and $W_k(\mathbf{x})\sim\mathcal{CN}\left(0,\sigma^2(\mathbf{x})\right)$ is a sample of white Gaussian noise. The filter F_k (assumed to be real, without loss of generality) is used to shape the primary signal spectrum at the secondary transmitter to reduce the effect of the SFN loss.

If we assume the use of a Quadrature Phase Shift Keying (QPSK) constellation, we can write the Chernoff Bound (CB) of the Uncoded BER as

$$\eta(\mathbf{x})=\frac{1}{N}\sum_{k=1}^{N}e^{-\Upsilon|H_k(\mathbf{x})|^2/2}=\frac{1}{N}\sum_{k=1}^{N}e^{-\frac{|H_k(\mathbf{x})|^2}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} \quad (10)$$

with $H_k(\mathbf{x})=1+\gamma e^{-j(2\pi k n_0(\mathbf{x})/N+\theta(\mathbf{x}))}F_k$ the equivalent channel seen by the k-th carrier at a given receiver, so

$$|H_k(\mathbf{x})|^2=1+\gamma_k(\mathbf{x})^2+2\gamma_k(\mathbf{x})\cos\left(\theta(\mathbf{x})+2\pi k n_0(\mathbf{x})/N\right) \quad (11)$$

where $\gamma_k(\mathbf{x})\doteq\gamma(\mathbf{x})F_k$, and $\Upsilon(\mathbf{x})\doteq\frac{1}{\sigma^2(\mathbf{x})+\rho^2(\mathbf{x})}$ denotes the SNR if the secondary transmitter is not reinforcing the primary communication, which is constant along all the carriers due to the AWGN assumption. Moreover, we have defined $\psi(\mathbf{x})\doteq2\sigma^2(\mathbf{x})$ for the sake of simplicity.

In order to express the received signal amplitudes $\boldsymbol{\gamma}(\mathbf{x})=[\gamma_1,\dots\gamma_N]$ and $\rho(\mathbf{x})$ as a function of some transmit parameters, we define the *transmit mask* $\tilde{\boldsymbol{\gamma}}=[\tilde{\gamma}_1,\tilde{\gamma}_1,\dots\tilde{\gamma}_N]$, and the *secondary ratio* $\tilde{\rho}$ as the transmit parameters such that we can write $\boldsymbol{\gamma}(\mathbf{x})=\sqrt{P(\mathbf{x})}\tilde{\boldsymbol{\gamma}}$ and $\rho(\mathbf{x})=\sqrt{P(\mathbf{x})}\tilde{\rho}$, where $P(\mathbf{x})$ is the total received power from the secondary transmitter (normalized by the primary one). Thus, a power constraint at the transmitter can be stated as $\frac{1}{N}||\tilde{\boldsymbol{\gamma}}||_2^2+\tilde{\rho}^2\leq1$.

In the following, we will assume that the values of $\gamma_k(\mathbf{x})$ and $\rho(\mathbf{x})$ are deterministic, as the value of $P(\mathbf{x})$ can be obtained by means of a propagation model or by measurements, and model $\theta(\mathbf{x})$ as a uniform Random Variable (RV) $\theta(\mathbf{x})\sim U\left(0,2\pi\right]$ as it is impossible to determine the exact phase difference between echoes $\theta(\mathbf{x})$, so the CB defined in (10) is a RV. In order to obtain a deterministic value for the CB, our figure of merit for a primary receiver is

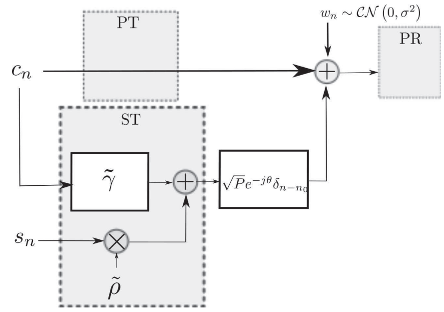

Fig. 1. System model: the Secondary Transmitter (ST) knows the message c_n of the Primary Transmitter (PT). The ST filters the primary signal with a filter with frequency response $\tilde{\boldsymbol{\gamma}}$, and scales the secondary message s_n with $\tilde{\rho}$. The noise power σ^2 and relative secondary power P will depend on the position of the Secondary Receiver (SR).

obtained after substituting (11) in (10), and averaging over θ

$$\eta(\mathbf{x},\tilde{\boldsymbol{\gamma}},\tilde{\rho}) = \frac{1}{N}\sum_{k=1}^{N} E_{\theta(\mathbf{x})}\left\{\frac{1}{N}\sum_{k=1}^{N} e^{-\frac{|H_k(\mathbf{x})|^2}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}}\right\} \quad (12)$$
$$= \frac{1}{N}\sum_{k=1}^{N} e^{-\frac{1+\gamma_k^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_k(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right)$$

where $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind and $E_X\{\cdot\}$ denotes the expectation operator over the RV X.

In order to properly define the coverage area, that is usually defined using the coded BER, the performance of the primary system can be evaluated by using the following analytical bound for the BER after Viterbi decoding for DVB-T, taken from [7]:

$$BER(\mathbf{x}) \leq \frac{1}{4}\sum_{d=d_{min}}^{\infty} c_d \eta(\mathbf{x},\tilde{\boldsymbol{\gamma}},\tilde{\rho})^d \quad (13)$$

which is a function of the CB. The usefulness of this bound was demonstrated in [6] by means of software simulations and hardware measurements.

Finally, we can express the coverage area as a function of (12) as follows:

$$\mathcal{C}_0 = \{\mathbf{x} = (r,\theta) \,|\, \eta(\mathbf{x},\mathbf{0},0) \leq \eta_0\}, \quad (14)$$

where η_0 is the limit CB value for correct reception. In the same way, we define $\Upsilon_0 = 2\log(\eta_0)$ as the SNR limit for the coverage zone in an AWGN channel, and $\psi_0 = \frac{2}{\Upsilon_0}$.

For the sake of simplicity, we will only consider those points in the coverage zone that are aligned with the primary and secondary transmitters, and have the two transmitters at the same side. This is equivalent to assuming receivers with perfectly aimed antennas with a gain of $-\infty$ dB for all angular directions (except 0º). Thus, the points that are affected by the secondary user and, therefore, the points we must take into account in the coverage constraint can be written as

$$\mathcal{C}_0 = \{(r,\theta) \,|\, r \in [r_s, r_0],\, \theta = \theta_0\} \quad (15)$$

where r_0 is the radius of the coverage zone, assumed to be a circle centered on the primary transmitter, and the secondary transmitter is placed at $\mathbf{x}_s = (r_s,\theta_0)$.

With this, and considering again as in Section 2 that the maximization of the secondary user rate is equivalent to the maximization of the power allocated to the secondary message, we can formulate the following optimization problem:

$$\begin{array}{ll} \text{minimize} & -\tilde{\rho} \\ \text{subject to} & \eta(\mathbf{x},\tilde{\boldsymbol{\gamma}},\tilde{\rho}) \leq \eta_0 \,\forall \mathbf{x} \in \mathcal{C}_0 \\ & \tilde{\rho}^2 + \sum_{i=1}^{N}\tilde{\gamma}_i^2 \leq 1 \end{array} \quad (16)$$

which is a Semi-Infinite Program (SIP), i.e., a problem with infinite constraints (the infinite number of points within the coverage area) and a finite number of design variables (the N values of $\tilde{\boldsymbol{\gamma}}$ plus the value of $\tilde{\rho}$).

With the purpose of gaining insight on the implications of having several primary receivers under different reception states, we will first analyze a simplified two-user scenario before presenting a numerical approach based on discretization and dimensionality reduction in order to solve the optimization problem.

3.1. Two different receivers

In the proposed scenario it is likely to find two receivers that are in extremely different reception situations. For instance, if the secondary transmitter is located far from the coverage edge and its transmit power is much smaller than that of the primary transmitter, those receivers in the limit of the coverage zone, i.e., those receivers with a position $\mathbf{x}_f = (r_0,\theta_0)$, will have a value of the CB $\eta(\mathbf{x_f},\mathbf{0},0) = \eta_0$, and a value of $P(\mathbf{x}_f) \to 0$, whereas the receivers near the secondary transmitter, located in $\mathbf{x}_n = (r_s,\theta_0)$ will have a value of $P(\mathbf{x}_n) \to \infty$. We will study this case as a simplification of the general case covering receivers under many different values of P.

As stated in [6], the power allocation for the primary message $\tilde{\boldsymbol{\gamma}}$ that maximizes the value of $\tilde{\rho}$ when taking into account just one single receiver consists in uniformly concentrating the power in a fraction ϕ of carriers, leaving the remaining fraction $1-\phi$ set to zero. If we were only taking into account one primary receiver, the value of ϕ for the nearby receiver would be set to $\phi(\mathbf{x}_n) = 1$, and for the far-off receiver $\phi(\mathbf{x}_f) = ||\boldsymbol{\gamma}(\mathbf{x}_f)||_2^2/4 < P(\mathbf{x}_f)/4 \approx 0$, as stated in [6]. Therefore, even for this seemingly straightforward case, obtaining a solution is more involved than just studying a *worst case* receiver.

As the design of a generic transmit mask $\tilde{\boldsymbol{\gamma}}$ is not analytically tractable, we will restrict our analysis to two-level solutions for the primary power weighting, not necessarily zero one of them (i.e., solutions of the form $\tilde{\boldsymbol{\gamma}} = [\tilde{\gamma}_1\mathbf{1}_{N\phi}\ \tilde{\gamma}_2\mathbf{1}_{N(1-\phi)}]$). In Appendix A it is shown that a fraction of power

$$\tilde{\rho}^2 = \frac{\psi_0^2\left(e^{4/\psi_0} - I_0\left(\frac{4}{\psi_0}\right)\right)}{e^{4/\psi_0}\left(\Upsilon_0(\psi_0-2)^2 + \psi_0^2 + 8\right) - (\Upsilon_0+1)\psi_0^2 I_0\left(\frac{4}{\psi_0}\right)} \quad (17)$$

can be assigned to the secondary message in this scenario. As we will see in the following section, even this simplified case is a good approximation to the general one, where all the receivers in the coverage zone are taken into account.

3.2. Numerical approach and results

In addition to the infinite number of constraints, the dimension of the design set in (16) is a problem itself. For instance, this problem for a

DVB-T system operating in the 8K-Mode[1] will have 8193 variables: the 8192 elements in $\tilde{\boldsymbol{\gamma}}$ plus $\tilde{\rho}$.

We can reduce this dimensionality by grouping the N parameters of amplitude $\tilde{\gamma}_1$, ..., $\tilde{\gamma}_N$ in M groups $\mathcal{G}_1$, ... $\mathcal{G}_M$, such that the power allocation is constant in each group, this is, $\tilde{\gamma}_j = \tilde{\gamma}_k \forall \tilde{\gamma}_j$, $\tilde{\gamma}_k \in \mathcal{G}_i$. Thus, we can rewrite the optimization problem as

$$\begin{aligned} \text{minimize} \quad & -\tilde{\rho} \\ \text{subject to} \quad & \tilde{\eta}(\mathbf{x}, \tilde{\boldsymbol{\gamma}}, \tilde{\rho}, \boldsymbol{\phi}) \leq \eta_0 \, \forall \mathbf{x} \in \mathcal{C}_0 \\ & \tilde{\rho}^2 + \textstyle\sum_{i=1}^{M} \tilde{\gamma}_i^2 \phi_i \leq 1 \\ & \textstyle\sum_{i=1}^{M} \phi_i = 1 \\ & \phi_i \geq 0 \end{aligned} \tag{18}$$

where ϕ_i is the fraction of carriers in the i-th group, $\phi_i = \frac{|\mathcal{G}_i|}{N} \leq 1$, $|\mathcal{X}|$ denotes the cardinality of set $\mathcal{X}$, $\boldsymbol{\phi} = [\phi_1, \ldots, \phi_M]$ and

$$\tilde{\eta}(\mathbf{x}, \tilde{\boldsymbol{\gamma}}, \tilde{\rho}, \boldsymbol{\phi}) = \sum_{k=1}^{M} \phi_i e^{-\frac{1+\tilde{\gamma}_k^2 P(\mathbf{x})}{\psi(\mathbf{x})+2\tilde{\rho}^2 P(\mathbf{x})}} I_0 \left(\frac{2\tilde{\gamma}_k \sqrt{P(\mathbf{x})}}{\psi(\mathbf{x}) + 2\tilde{\rho}^2 P(\mathbf{x})} \right). \tag{19}$$

In this problem, the number of variables is $2M$: the amplitudes for the groups $\tilde{\gamma}_1$, ..., $\tilde{\gamma}_M$, the secondary power $\tilde{\rho}$ and the fractions of carriers ϕ_1, ..., ϕ_{M-1}. The remaining fraction can be computed as $\phi_M = 1 - \sum_{i=1}^{M-1} \phi_i$. We will also assume that there is a large enough number of carriers in every group, so we can approximate $\phi_i \in [0, 1]$.

We have used `MATLAB` function `fseminf` in order to solve the optimization problem. This algorithm is of the discretization type [8], and uses a quasi-Newton Sequential Quadratic Programming (SQP) algorithm applied to a finite number of restrictions, as a result of the discretization of the semi infinite constraint[2]. This optimization method will return a local minimum, but as the problem is not convex we cannot guarantee global optimality. In order to overcome this problem, the optimization algorithm was run for each value of M (and, therefore, different degrees of complexity) and $\mathbf{x}_s$, the position of the secondary transmitter, for 2,000 different initial random points, selecting afterwards the solution that provided the lowest value of the objective function.

The other parameters used in the optimization are described in Table 1, where the propagation model is taken from [9]. In Figure 2 the obtained results are compared with those corresponding to the study of a single user on the border of the coverage zone (taken from [6]) which might be thought to be a *worst case*, but as previously seen, the solution is more involved. The achievable $\tilde{\rho}^2$ for the two user scenario (17) and the solution for the reference scenario (8), that does not include the degradation due to the SFN operation, are also shown.

It can be seen that the lower values of r_s suffer from a higher degradation with respect to the single user case, while for higher values this difference does not exist. Not surprisingly, the degradation is much lower if we compare the actual result with the simplified two-user scenario, as it is closer to the studied case. Moreover, the fraction of power allocated to the secondary message can be seen to be highly dependent on the position of the secondary transmitter.

For higher values of r_s, all the affected receivers have large values of P, so the optimum fraction of active carriers is one for all of

[1] Here, we are not taking into account the carriers used for signaling, training and guard bands.

[2] The used discretization step was 200m (the coverage radius is 26.29Km), and the point on the border of the coverage zone was always included on the finite set.

Table 1. Parameters of the proposed scenario.

Parameter	Value
Height of primary transmitter	324m
EIRP of primary transmitter	70dBm
Position of primary transmitter	r=0Km, $\theta = 0$
Height of secondary transmitter	40m
EIRP of secondary transmitter	36dBm
Position of secondary transmitter	$r = r_s$ (Variable), $\theta = 0$
Height of receivers	30m
Thermal Noise Power	-105dBm
Propagation model	Okumura-Hata Urban Model

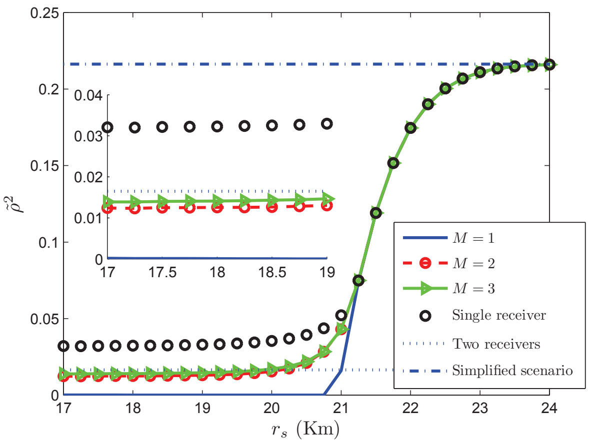

Fig. 2. Fraction of transmit power of the secondary transmitter allocated to the secondary message as a function of the secondary transmitter position.

them, and the obtained solution is equivalent to the single-receiver solution, placed at the coverage edge. In this region the proposed approximation with two users is not valid, as even the users on the border receive a much higher power contribution from the secondary transmitter than from the primary one.

It is also remarkable that the result of the simplified scenario (8) acts as an upper bound on $\tilde{\rho}^2$, and is only achieved when the secondary user is placed near the coverage edge. For lower values of r_s, the importance of the proposed analysis is clear, as the predicted degradation due to the SFN operation is remarkable. In fact, it can be seen that if we apply the power allocation resulting from the simplified analysis (8) to the scenario under study, those receivers near the original coverage edge will not meet the CB constraint in (16).

With respect to the complexity of the problem (the number M of groups), for the lower values of r_s, similarly to the single receiver case in [6], $M = 1$ results in a null power allocated to the secondary message, whereas for values of $M > 3$ no additional gain is attained. Moreover, the solution $M = 2$ (which was shown in to be optimum for the single user case) suffers a slight degradation with respect to $M = 3$. For higher values of r_s, the solution is to perform a uniform power allocation for the primary message, so the optimum number of groups is $M = 1$ and, therefore, further gain is not achieved by incrementing the order of the problem.

4. CONCLUSIONS

A scenario where a secondary cognitive user is inserted in a multicarrier broadcast network has been studied. The objective of the secondary user is to maximize its own transmission rate while preserving the original coverage area of the primary user, defined by means of the Chernoff bound for the bit error rate. As the secondary user also transmits the primary signal, the original AWGN channel is transformed into a frequency selective one due to the insertion of the secondary replica. This fact has been shown to be of special importance, as a simpler analysis could overestimate the coverage area and, therefore, compromise the primary service. The available power for the secondary service has been obtained numerically for the general case, with an infinite number of primary receivers, and simple asymptotic expressions have been derived.

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A. TWO DIFFERENT RECEIVERS

In this appendix we will prove that the fraction of power in (17) can be allocated to the secondary user message in the simplified two-receivers scenario. If we constrain the frequency power weighting to have only two different levels, we can write the Chernoff bound as

$$\eta(\phi,\tilde{\gamma}_1,\tilde{\gamma}_2,\tilde{\rho},\mathbf{x}) = \phi e^{-\frac{1+\gamma_1^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_1(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right) + (1-\phi)e^{-\frac{1+\gamma_2^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right), \quad (20)$$

where $\phi \in [0,1] \subset \mathbb{R}$ since we are assuming a large enough number of carriers, and $\gamma_i(\mathbf{x}) = \sqrt{P(\mathbf{x})}\tilde{\gamma}_i$, $\rho(\mathbf{x}) = \sqrt{P(\mathbf{x})}\tilde{\rho}$. We will try to find a solution $(\phi,\gamma_1,\gamma_2,\rho)$ that fulfills the BER constraint at both receivers even with the insertion of a secondary signal, i.e., $\eta(\phi,\tilde{\gamma}_1,\tilde{\gamma}_2,\tilde{\rho},\mathbf{x}) \le \eta_0$, $\forall \mathbf{x} \in \{\mathbf{x}_n, \mathbf{x}_f\}$. For the nearby receiver, the signal coming from the primary transmitter will be negligible with respect to the secondary transmission, so we have that

$$\eta(\phi,\tilde{\gamma}_1,\tilde{\gamma}_2,\tilde{\rho},\mathbf{x}_n) = \phi e^{-\frac{\tilde{\gamma}_1^2}{2\tilde{\rho}^2}} + (1-\phi)e^{-\frac{\tilde{\gamma}_2^2}{2\tilde{\rho}^2}} \quad (21)$$

just by taking the limit $P(\mathbf{x}) \to \infty$ in (20).

Let us define $\tilde{\gamma}_m = \min\{\tilde{\gamma}_1,\tilde{\gamma}_2\}$. Then $\eta(\phi,\tilde{\gamma}_1,\tilde{\gamma}_2,\tilde{\rho},\mathbf{x}_n) < e^{-\frac{\tilde{\gamma}_m^2}{2\tilde{\rho}^2}}$, so if we set $e^{-\frac{\tilde{\gamma}_m^2}{2\tilde{\rho}^2}} = \eta_0$, then $\tilde{\gamma}_m^2/\tilde{\rho}^2 = \Upsilon_0$, the CB constraint will be met.

With this restriction, we can write a simplified CB constraint for the distant receiver using[3] $\phi = \gamma^2/4$ and $\gamma_2 = \sqrt{\Upsilon_0}\rho$, with $\gamma^2 = \phi\gamma_1^2$ the total power spent in the carriers with amplitude $\gamma_1 = 2$, as

$$\eta(\gamma,\rho,\mathbf{x}_f) = \frac{\gamma^2}{4}e^{-\frac{5}{\psi_0+2\rho^2}} I_0\left(\frac{4}{\psi_0+2\rho^2}\right) + \left(1-\frac{\gamma^2}{4}\right)e^{-\frac{1+\Upsilon_0\rho^2}{\psi_0+2\rho^2}} I_0\left(\frac{2\rho\sqrt{\Upsilon_0}}{\psi_0+2\rho^2}\right). \quad (22)$$

Let us define $f(\gamma,\rho) = \eta(\gamma,\rho,\mathbf{x}_f) - \eta_0$. As $P \to 0$, we have that $\gamma \to 0$ and $\rho \to 0$, so we can write $f(\gamma,\rho) = \frac{1}{2}[\gamma\ \rho]\nabla^2_{\gamma,\rho}f(0,0)[\gamma\ \rho]^T$, being $\nabla^2_{\gamma,\rho}f(0,0)$ a diagonal matrix with entries

$$\frac{\partial^2 f}{\partial\gamma^2}(0,0) = \frac{1}{2}e^{-5/\psi_0}\left(I_0\left(\frac{4}{\psi_0}\right) - e^{4/\psi_0}\right) \quad (23)$$

$$\frac{\partial^2 f}{\partial\rho^2}(0,0) = \frac{(4-2\Upsilon_0(\psi_0-1))e^{-1/\psi_0}}{\psi_0^2}. \quad (24)$$

The maximum value of ρ will be obtained when the power constraint is met with equality. In this case $\gamma^2 + \rho^2 + \left(1-\frac{\gamma^2}{4}\right)\Upsilon_0\rho^2 = P$, so $\gamma^2 = \frac{4\left((\Upsilon_0+1)\rho^2-P\right)}{\Upsilon_0\rho^2-4} \approx P - (\Upsilon_0+1)\rho^2$, where the last approximation holds provided $\Upsilon_0\rho^2$ is small enough with respect to 4. With these expressions, we get to the desired equation

$$\tilde{\rho}^2 = \frac{\rho^2}{P} = \frac{\frac{\partial^2 f}{\partial\gamma^2}(0,0)}{(\Upsilon_0+1)\frac{\partial^2 f}{\partial\gamma^2}(0,0) - \frac{\partial^2 f}{\partial\rho^2}(0,0)} = \frac{\psi_0^2\left(e^{4/\psi_0} - I_0\left(\frac{4}{\psi_0}\right)\right)}{e^{4/\psi_0}\left(\Upsilon_0(\psi_0-2)^2+\psi_0^2+8\right) - (\Upsilon_0+1)\psi_0^2 I_0\left(\frac{4}{\psi_0}\right)}. \quad (25)$$

[3] With this simplification, $\gamma_1 = 2$, and as $\gamma_2 \approx 0$, $\gamma_m = \gamma_2$. In the following, we will omit the position indexing $(\mathbf{x})$, as we are only taking into account the far-off receiver.