Overlay Cognitive Transmission in a Multicarrier Broadcast Network with Dominant Line of Sight Reception

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***Abstract*—The insertion of a secondary transmitter in a multicarrier broadcast single frequency network is studied. The secondary information is overlaid on top of the primary waveform, which is also reinforced by the secondary transmitter. The degradation of the primary service due to the presence of echoes in a strong line of sight environment is taken into account, and mitigated with an appropriate filtering at the secondary transmitter. The transmit rate of the secondary system is maximized while keeping the original primary coverage area, defined as a function of a BER bound. The analytical results are verified by means of software simulations and hardware tests, where the importance of the proposed filtering is clearly shown.**

***Index Terms*—Overlay Cognitive Radio, Single Frequency Network, Broadcast, OFDM**

I. Introduction

Recently, there has been an increased interest for learning the potential of those Cognitive Radio (CR) systems where the secondary transmitter has knowledge of the primary message, in what is known as the *overlay paradigm* [1]. This prior knowledge of the primary transmission can be exploited by the secondary users to convey their own information when accessing primary user spectrum in an efficient way, while preserving the primary user's Quality of Service (QoS). Hence, the usefulness of the knowledge of the primary message is twofold: on the one hand, the degradation of the primary user link due to the insertion of a secondary signal can be compensated by the secondary transmitter by using a fraction of its available power to transmit the primary message, keeping the SNR at the primary receivers above a given threshold; on the other hand, since the secondary transmitter knows the primary message, some kind of interference cancellation scheme can be applied, like Dirty Paper Coding (DPC) [2].

However, the knowledge of the primary signal by the secondary transmitter is hard to justify, and, therefore, limited to a small quantity of practical cases [1]. In this paper, similarly to [3], we introduce another practical scenario where the knowledge of the primary signal is possible: in broadcasting systems working as a Single Frequency Network (SFN), e.g., the European Digital Video Broadcasting - Terrestrial (DVB-T) based service, deployed in many countries worldwide, the primary signal is sent via satellite (or other kind of distribution network) to some major transmitters, which need to apply the corresponding delay to keep the synchronization required by the SFN mode. Thus, a potential secondary transmitter might also gain access to the primary signal, keeping time and frequency synchronization with the primary transmitters and, therefore, join the primary network. The ultimate goal is to overlay the secondary information on the primary signal which can be decoded by secondary receivers, while preserving and possibly reinforcing the quality of service of the primary network (see Figure 1) without any modification on the primary receivers. Thus, the present work is focused on the cognitive spectrum reuse of the frequency bands used by any broadcast system working as a Single Frequency Network, as they are specially interesting due to the high amount of bandwidth that these services are allocated, the possibility of accessing the primary message, and also due to the good propagation conditions of these frequency bands.

Although the case of secondary transmitters with knowledge of the primary signal has been addressed from an information theoretic point of view, see e.g., [4], [5], [6] among others, there is still an important gap between *capacity-achieving* models and practical implementations where successful spectrum reuse is expected to be achieved. In short, some of the main issues to address are:

Metric for primary QoS. A usual metric for the QoS of the primary user is the capacity of a transmitter-receiver pair: if this capacity is greater than the transmission rate of the primary user, then the primary communication is not compromised [5]. If the channel is of time-varying nature, the QoS is measured in terms of probability of outage for a given transmission rate [7]. However, in broadcasting scenarios, *coverage areas* become the relevant metric as a result of the achieved *bit error rates*.

Primary user reinforcement. Even in the absence of

Manuscript received 21 Feb 2012; revised 17 Jul 2012; accepted 4 Sep 2012. Research supported by the European Regional Development Fund (ERDF) and the Spanish Government under projects DYNACS (TEC2010-21245-C02-02/TCM) and COMONSENS (CONSOLIDER-INGENIO 2010 CSD2008-00010), and the Galician Regional Government under projects *Consolidation of Research Units* 2009/62 and 2010/85.

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Preliminary versions of this work were presented at CogART'11 (Barcelona, Spain) and SPAWC'12 (Çesme, Turkey).

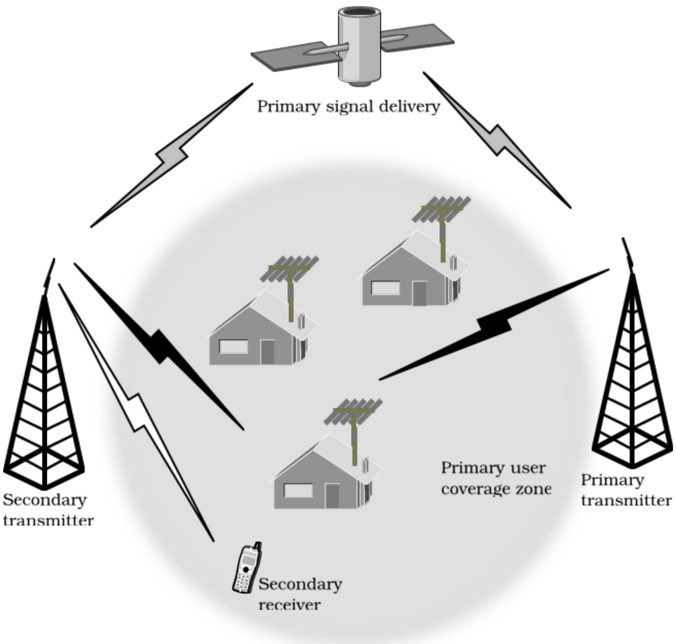

Fig. 1. The secondary transmitter conveys the primary signal (black rays), which is delivered via a distribution network (gray rays). The secondary transmitter overlays the secondary message (white ray) on top of the primary one.

a secondary information signal, the simple transmission of the primary signal from a secondary transmitter will not necessarily improve the primary service quality, since echoes can degrade performance as it is well-known in current SFN deployments [8], unless proper countermeasures can be taken. This effect is especially noticeable in those systems with a dominant Line of Sight (LoS) component, and almost negligible in high scattering environments. In practical cases, the degradation coming from the secondary echo could be higher than the power gain due to the extra contribution of the secondary transmitter. This type of problems is expected to be mitigated in the future with new standards such as DVB-T2 [9], which include some precoding schemes such as Alamouti space-time coding or constellation rotation. On the other side, some specific channels, such as Rayleigh fading channels, benefit from the diversity created by SFN deployments, as illustrated in [8]. In this paper, we will model both the primary and secondary channels as a pure LoS component, which is indeed the case for which a higher degradation is expected, according to [8].

Interference cancellation techniques. In many cases practical interference mitigation techniques at the transmitter exploiting side information cannot be directly applied, as they require knowledge of the channel state. In [10] it was shown that the uncertainty in the channel phase suffices to decrease the achievable capacity of the secondary link dramatically. Interference cancellation can be also performed at the secondary receiver, provided the interfering power is strong enough [11], as proposed in [12].

Given the widespread current use of DVB-T, we will focus on this multicarrier technology as support for the primary signal, and show how *an appropriate secondary transmission of the primary signal can reinforce the original QoS, as a first step towards a cognitive secondary transmitter which additionally includes a secondary information signal while preserving the primary user coverage area.*

The remaining of the paper is organized as follows: in Section II we will introduce the notation and the analytical expressions to be used afterwards. In the next sections, the problem is treated in an incremental way, using the aforementioned analytical expressions as quality metrics for the primary system: in Section III, a pure cooperative secondary user that tries to maximize a primary receiver QoS is studied, and practical transmission strategies are derived; in Section IV the case of a secondary user maximizing its own transmission rate in presence of a single secondary receiver is presented; Section V completes the study, introducing the restriction of preserving the original coverage zone of the primary user. In Section VI the analytical expressions are verified by means of software simulations and hardware measurements. Finally, the conclusions are presented in Section VII.

II. Problem statement

Throughout the paper we will assume that the links from both primary and secondary transmitters to a given primary receiver can be modeled[1] as Additive White Gaussian Noise (AWGN) channels, so the equivalent baseband received signal after the Cyclic Prefix (CP) removal can be written as

$$y_n = \left(\delta_n + \gamma e^{-j\theta} f_{n-n_0}\right) \circledast x_n + \rho e^{-j\theta} s_{n-n_0} + w_n \quad (1)$$

where the equivalent channel was normalized to set the channel from the primary transmitter to δ_n, while γ, θ and n_0 are the relative amplitude, phase and delay of the primary signal contribution sent from the secondary transmitter, $\circledast$ denotes the circular convolution operator, x_n denotes the n-th sample of the primary signal (normalized to have unit power), ρ denotes the relative amplitude of the secondary signal $s_n \sim \mathcal{CN}(0,1)$, assumed to be white Gaussian[2], sent from the secondary transmitter, and $w_n \sim \mathcal{CN}\left(0, \sigma^2\right)$ is a sample of white Gaussian noise. As an additional degree of freedom, the secondary transmitter is allowed to (circularly) filter the primary signal with a transmit filter f_n. The convenience of this filtering will be illustrated in the remaining of the paper. In the Discrete Fourier Transform (DFT) domain, the previous relation reads for a given carrier k as

$$\begin{aligned} Y_k &= \left(1 + \gamma e^{-j(2\pi k n_0/N + \theta)} F_k\right) X_k + \\ &\quad + \rho e^{-j(2\pi k n_0/N+\theta)} S_k + W_k \;\; k = 1, ..., N \end{aligned} \quad (2)$$

where X_k, S_k, F_k and W_k denote the N-DFT of x_n, s_n, f_n and w_n, respectively, with N the number of carriers. Figure 2 summarizes the system model.

[1]The simple AWGN channel can be a good approximation, specially for those receivers with rooftop antennas (very common in terrestrial television broadcasting), which allow the existence of a strong LOS propagation path.

[2]This can be a good approximation, for example, in the case of a secondary transmitter using an OFDM waveform, where the time-domain signal is generated by combining a relatively large number of independent random variables (the symbols on the different carriers). The gaussianity is mantained in the DFT domain provided both primary and secondary waveforms are not identical (for example, by using different FFT sizes or CP lengths).

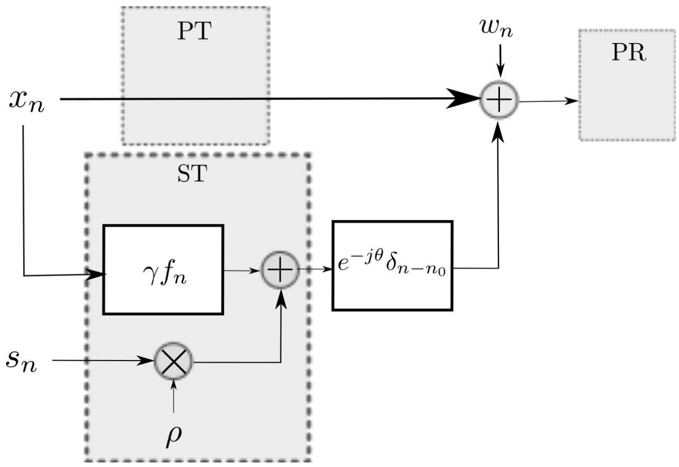

Fig. 2. System model: the Secondary Transmitter (ST) knows the message x_n of the Primary Transmitter (PT). The ST filters the primary signal with the filter γf_n, and scales the secondary message s_n with ρ. The signal received by the Primary Receiver (PR) is described by equations (1) and (2).

For the sake of simplicity, we will assume perfect channel estimation[3] and frequency synchronization in the analytical derivations, and an overall channel length shorter than the CP. Moreover, we will consider a Quadrature Phase Shift Keying (QPSK) constellation in the primary system, as the derived analytical bounds are easier to deal with. However, these results will be extended to higher order constellations and practical synchronization schemes by means of hardware measurements.

Unlike previous approaches to similar problems that use a capacity-based quality metric for the primary system [3], [7], we propose to analyze the performance of the primary system by means of the Chernoff Bound (CB) for the uncoded Bit Error Rate (BER) or, equivalently, by the Exponential Effective Signal to noise ratio Metric (EESM) [13], one of the potential metrics to be used in next generation Orthogonal Frequency Division Multiplexing (OFDM) systems with Adaptive Coding and Modulation (ACM) [14], and one of the Physical Layer abstraction methods proposed in IEEE 802.16 [15]. The expression for the effective Signal to Noise Ratio (SNR) using the EESM metric is[4] $\Upsilon_{eff} = -2\log(\eta)$, where

$$\eta = \frac{1}{N}\sum_{k=1}^{N} e^{-\Upsilon|H_k|^2/2} = \frac{1}{N}\sum_{k=1}^{N} e^{-\beta|H_k|^2} \tag{3}$$

is the expression for the CB. From (2) we have that $H_k = 1 + \gamma e^{-j(2\pi k n_0/N+\theta)}F_k$ is the equivalent channel seen by the k-th carrier at a given receiver, so

$$|H_k|^2 = 1 + \gamma_k^2 + 2\gamma_k \cos(\theta + 2\pi k n_0/N) \tag{4}$$

[3]We are assuming that the primary waveform carries some pilot symbols (which are also transmitted by the secondary transmitter) to perform the channel estimation, so the *equivalent channel* $\left(1+\gamma e^{-j(2\pi k n_0/N+\theta)}F_k\right)$ can be accurately estimated at the primary receivers.

[4]The general expression for the EESM is $\Upsilon_{eff} = -\lambda\log\left(\frac{1}{N}\sum_{k=1}^{N} e^{-\Upsilon|H_k|^2/\lambda}\right)$, being λ a degree of freedom that depends on the particular modulation and coding scheme [13]. In this paper we will set $\lambda = 2$, as it is the value for the CB of the BER of a QPSK, although results can be easily extended to other values of λ.

where $\gamma_k \doteq \gamma F_k$ is assumed to be real, without loss of generality, and $\Upsilon \doteq \frac{1}{\sigma^2+\rho^2}$ denotes the Signal to Noise Ratio of the system in absence of the secondary transmitter conveying the primary message, which is constant along all the carriers due to the AWGN assumption. Moreover, we have defined $\beta = \Upsilon/2$ for the sake of simplicity.

In the following, we will assume that the value of the relative amplitude γ is deterministic, as it can be obtained by means of a propagation model or by measurements, and model θ as a uniform Random Variable (RV) $\theta \sim U(0, 2\pi]$ as it is not possible to determine the exact phase difference between echoes θ. Note that the metric η as defined in (3) is a RV, so a deterministic figure of merit for a primary receiver is obtained after substituting (4) in (3) and averaging[5] over θ:

$$\begin{aligned}\eta(\boldsymbol{\gamma},\rho) &= \frac{1}{N}\sum_{k=1}^{N} E_\theta\left\{e^{-\beta\left(1+\gamma_k^2+2\gamma_k\cos(\theta+2\pi k n_0/N)\right)}\right\} \\ &= \frac{e^{-\beta}}{N}\sum_{k=1}^{N} e^{-\beta\gamma_k^2}\frac{1}{2\pi}\int_0^{2\pi} e^{-2\beta\gamma_k cos(\theta)}\mathrm{d}\theta \\ &= \frac{e^{-\beta}}{N}\sum_{k=1}^{N} e^{-\beta\gamma_k^2} I_0(2\beta\gamma_k)\end{aligned} \tag{5}$$

where $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind, $E_X\{\cdot\}$ denotes the expectation operator over the RV X, and $\boldsymbol{\gamma} = [\gamma_1, .., \gamma_N]^T$. As the obtained expression does not depend on the time difference n_0, there is no need to make any assumption about this value. This CB-based metric η will be recurrent throughout the paper, and will appear as the optimization objective in Section III, and as a design constraint in Sections IV and V.

In order to obtain a relationship between the CB and the definition of the coverage zone, which is determined by the coded BER, we introduce the following analytical bound for the BER after Viterbi for DVB-T, taken from [17]:

$$BER \leq \frac{1}{4}\sum_{d=d_{min}}^{\infty} c_d \eta(\boldsymbol{\gamma},\rho)^d \tag{6}$$

with d_{min} the minimum Hamming distance of the convolutional code, and c_d the total input weight due to an error event at distance d from the all-zero path.

III. Optimum power allocation for a purely cooperative secondary user

In this section, we will obtain the optimum carrier power allocation (with respect to the metric η in (5)) for a secondary transmitter that cooperates by minimizing the BER of a single primary receiver as a first approach, without inserting a secondary message[6]. For a primary receiver location where

[5]If we assume a static channel model, the value θ will not change for different OFDM blocks in given receiver, but only change among different receivers. Thus, in order to make the quality metric process ergodic in every receiver, a different random phase component could be applied to every OFDM block at the secondary transmitter (similarly to [16]), so the long-term average η seen by a single receiver is the expected value of η, even for a static channel scenario.

[6]This case is of special interest, as it provides the solution to the optimum power weighting $\boldsymbol{\gamma}$ given a total power γ^2 allocated to the primary waveform at the secondary transmitter.

the ratio between the powers coming from the secondary and primary transmitters is γ^2, the minimization of the CB (5) reads as

$$\text{minimize} \quad \sum_{k=1}^{N} e^{-\beta\gamma_k^2} I_0(2\beta\gamma_k) \quad \text{subject to} \quad \frac{1}{N}\sum_{k=1}^{N}\gamma_k^2 \leq \gamma^2. \tag{7}$$

This is a non-convex problem over N variables, which makes numerical methods difficult to apply. However, as shown in Appendix A, those points of the form $\boldsymbol{\gamma} = [\mathbf{0}_{N(1-\phi)} \ K\mathbf{1}_{N\phi}]$, (where $\mathbf{1}_p$ and $\mathbf{0}_q$ denote the all-ones row vector of p elements and the all-zeros row vector of q elements, respectively) with K such that the power constraint is met with equality, and with a fraction of active carriers ϕ such that $N\phi$ is an integer, are critical points of the Lagrangian of the proposed optimization problem. For this type of solutions, the optimization problem (7) can be recast as[7]

$$\begin{aligned} &\text{minimize} && (1-\phi) + \phi e^{-\beta\gamma^2/\phi} I_0\left(\frac{2\beta\gamma}{\sqrt{\phi}}\right) \\ &\text{subject to} && 0 \leq \phi \leq 1. \end{aligned} \tag{8}$$

As shown in Appendix B, the asymptotic solution of (8) for large SNR values is $\phi = \frac{\gamma^2}{4}$, which forces to allocate $\gamma_k^2 = 4$ to the corresponding fraction of carriers. Following (4), in this case we have that $|H_k| \geq 1$, so no carrier suffers from an SNR loss with respect to the scenario without a secondary transmitter. Note that the optimum solution is only dependent on the fraction of active carriers, and not on their specific locations, due to the symmetry of the problem. In any case, the unidimensional problem (8) is computationally tractable as opposed to (7).

We have evaluated the analytical bound for the BER in (6) for those solutions found in (8): Figure 3 shows that the proposed method always decreases the BER bound, even when the *unfiltered* approach leads to a huge degradation, thus showing the importance of the proposed filtering, which intends to reduce the degradation due to the presence of SFN echoes. Interestingly, the solution $\phi = \frac{\gamma^2}{4}$ is quite a good approximation to the optimum value of the fraction of active carriers, especially for the higher SNR case.

IV. Optimum Power Distribution for a Single Primary Receiver

In this section, we will focus on the strategy the secondary transmitter must follow in order to maximize its own capacity subject to a controlled degradation of the primary service at a given receiver. We will assume that the secondary users are able to use some kind of interference mitigation techniques so the capacity of the secondary link is equivalent to that in absence of the primary transmitter. As we explained previously, the use of DPC techniques [2] would require channel knowledge at the transmitter [10] and, therefore, a feedback channel to convey that information, whereas the use of Successive Interference Cancellation (SIC) at the receivers is more likely to be performed. A similar idea was developed in [12], [7], where the use of Opportunistic Interference Cancellation (OIC) was shown to dramatically increase the secondary user rate. In our case, we will assume that interference cancellation can be always performed, as the secondary user is expected to be in the primary user coverage area. Therefore, our channel model will be an interference Z channel [1], where the secondary message is treated as noise by the primary receivers, and the primary interference can be completely cancelled out by the secondary receivers.

[7]Here, we are assuming that the number of carriers is large enough to approximate the fraction of active carriers by any real number in the interval $[0, 1]$. If the resulting optimum value of ϕ is such that $N\phi$ is not an integer, the loss of performance taking $\lfloor N\phi \rfloor$ as the number of active carriers will be negligible.

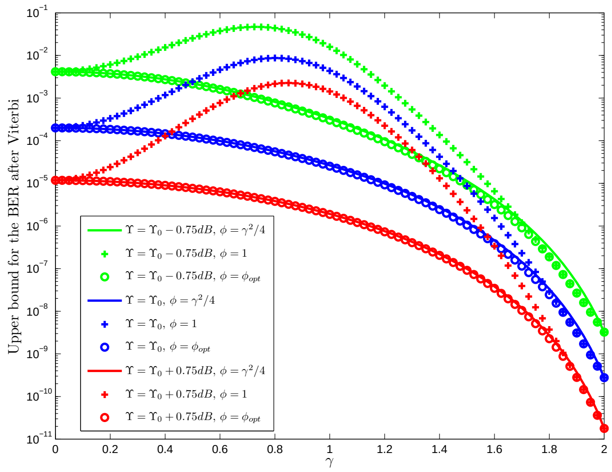

Fig. 3. Analytical bound for the BER for QPSK, convolutional rate 2/3, with different secondary transmission approaches: no filtering ($\phi = 1$), filtered with $\phi = \gamma^2/4$ and with $\phi = \phi_{opt}$ as found by `fminbnd`. Υ_0 denotes the SNR that the bound predicts for the QEF threshold for the system under analysis, which is $\Upsilon_0 \approx 5.6dB$.

In the design of practical multicarrier receivers it is sometimes assumed that the noise power is constant for all the carriers. If this is the case, the fact of transmitting with high power in a few carriers will be a source of narrowband interference, which is very harmful to OFDM transmission [18]. In consequence, we will restrict the design of secondary signals to those with constant power along the carriers, although the proposed methodology can be extended to the general case.

Let us denote by P the secondary received power (normalized by the primary one) at a given location, that has to be split between the primary $\left(\frac{1}{N}\sum_{k=1}^{N}\gamma_k^2 = \gamma^2\right)$ and secondary (ρ^2) signals. Note that the flat spectrum constraint for the secondary message turns the maximization of the capacity equivalent to the maximization of the power allocated to the secondary message ρ^2, so introducing a power constraint and a constraint on the primary user CB η in (5), we can formulate the optimization problem as

$$\begin{aligned} &\text{minimize} && -\rho \\ &\text{subject to} && \frac{1}{N}\sum_{k=1}^{N} e^{-\frac{1+\gamma_k^2}{\psi+2\rho^2}} I_0\left(\frac{2\gamma_k}{\psi+2\rho^2}\right) \leq \eta_0 \\ & && \rho^2 + \frac{1}{N}\sum_{k=1}^{N}\gamma_k^2 \leq P \end{aligned} \tag{9}$$

where η_0 is the constraint on the CB η, and $\psi \doteq 2\sigma^2$, leading to $\beta = \frac{1}{2(\sigma^2+\rho^2)} = \frac{1}{\psi+2\rho^2}$.

In Section III it was shown that, for a given allocated average power of a purely cooperative secondary user to the primary user message $\gamma^2 = \frac{1}{N}\sum_{k=1}^{N}\gamma_k^2$, the optimum power distribution consisted on concentrating the power in a fraction ϕ of carriers, leaving the remaining fraction $1-\phi$ set to zero. Again, for a sufficiently large number of carriers, we can approximate the fraction ϕ by a real number in the interval $[0,1]$, so problem (9) can be rephrased as

$$\begin{array}{ll} \text{minimize} & -\rho \\ \text{subject to} & e^{-\frac{1}{\psi+2\rho^2}} \times \\ & \times \left((1-\phi) + \phi e^{-\frac{\gamma^2/\phi}{\psi+2\rho^2}} I_0\left(\frac{2\gamma}{\sqrt{\phi}(\psi+2\rho^2)}\right)\right) \leq \eta_0 \\ & \rho^2+\gamma^2 \leq P \\ & 0 \leq \phi \leq 1. \end{array} \quad (10)$$

With this simplification we have reduced the number of variables from $N+1$ (the N variables γ_k to perform the power weighting, and ρ) to three. Furthermore, we can reduce the number of variables to two by approximating ϕ by its asymptotic optimum (and heuristic) value $\left(\phi = \frac{\gamma^2}{4}\right)$ for the sake of analytical tractability. With this last simplification, the CB constraint in (10) can be rewritten as

$$\begin{array}{l} f(\gamma,\rho) = e^{-\frac{1}{\psi+2\rho^2}} \times \\ \times \left(\left(1-\frac{\gamma^2}{4}\right) + \frac{\gamma^2}{4} e^{-\frac{4}{\psi+2\rho^2}} I_0\left(\frac{4}{(\psi+2\rho^2)}\right)\right) - \eta_0 \leq 0 \end{array} \quad (11)$$

or, equivalently,

$$\gamma^2 \geq 4\frac{1-\eta_0 e^{\frac{1}{\psi+2\rho^2}}}{1-e^{-\frac{4}{\psi+2\rho^2}} I_0\left(\frac{4}{\psi+2\rho^2}\right)}. \quad (12)$$

The solution to this problem presents a different behavior depending on the values of the SNR in absence of the secondary transmitter, $\Upsilon_{NS} \doteq 2/\psi$, and the received power from the secondary transmitter P, as detailed next.

A. *Moderate values of* P

For non-extreme values of P, if $\gamma^2 \leq 4$, the approximate optimum value of γ is the one that maximizes ρ while meeting constraint (12) and, therefore, is the value obtained from (12) with equality, so the BER restriction is active and the remaining power is used to transmit the secondary information. By substituting $\gamma^2 = P-\rho^2$ in (12) we can obtain the value of ρ as the root of the following equation:

$$\rho^2 = P - 4\frac{1-\eta_0 e^{\frac{1}{\psi+2\rho^2}}}{1-e^{-\frac{4}{\psi+2\rho^2}} I_0\left(\frac{4}{\psi+2\rho^2}\right)}. \quad (13)$$

If $\gamma^2 = P-\rho^2 > 4$, then the obtained solution is not valid, as $\phi > 1$. In such a case the solution would be obtained by forcing $\phi = 1$ and $\rho^2+\gamma^2 = P$ in problem (10), so the desired value of ρ^2 would be the root of

$$e^{-\frac{1+P-\rho^2}{\psi+2\rho^2}} I_0\left(\frac{2\sqrt{P-\rho^2}}{\psi+2\rho^2}\right) - \eta_0 = 0. \quad (14)$$

B. $P \to 0$

For small values of P the solution will be strongly dependent on the SNR in absence of the secondary transmitter Υ_{NS}. Let us define Υ_0 as the value of SNR such that the BER constraint is met with equality in absence of the secondary transmitter, i.e., $e^{-\frac{\Upsilon_0}{2}} = \eta_0$. Equivalently, we define $\psi_0 \doteq \frac{2}{\Upsilon_0} = \frac{-1}{\log(\eta_0)}$. We will restrict our analysis to those receivers in the original coverage region, i.e., $\Upsilon_{NS} \geq \Upsilon_0$.

1) $\Upsilon_{NS} > \Upsilon_0$*:* In this case, as $\psi < \psi_0$, we have that $e^{-\frac{1}{\psi+2P}} \leq \eta_0$ for sufficiently small values of P. Therefore, the secondary transmitter can allocate all the available power to the secondary message without violating the BER constraint, i.e, its optimum allocated power to the secondary message is $\rho^2 = P$. This could be the case of a primary receiver operating at a very high SNR, or a low-power secondary user.

2) $\Upsilon_{NS} = \Upsilon_0$*:* In this case, as the BER constraint is met with equality, we have that $e^{-\frac{1}{\psi_0+2P}} > \eta_0$, so the CB constraint is not fulfilled if all the power P is allocated to the secondary message. Following expression (11) and from the definition of ψ_0, we have $f(0,0) = 0$. For $\rho \approx 0$, $\gamma \approx 0$ and as $\nabla_{\gamma,\rho} f(0,0) = \mathbf{0}$, we can approximate the CB constraint (11) by its second order Taylor polynomial:

$$f(\gamma,\rho) = \frac{1}{2}[\gamma\ \rho]\nabla^2_{\gamma,\rho} f(0,0)\,[\gamma\ \rho]^T \quad (15)$$

where $\nabla_{\gamma,\rho} f(\gamma_0,\rho_0)$ denotes the gradient of the function f evaluated in (γ_0,ρ_0), and $\nabla^2_{\gamma,\rho} f(\gamma_0,\rho_0)$ denotes the Hessian matrix evaluated in the same point. In this case, the Hessian evaluated in $(0,0)$ is a diagonal matrix with entries

$$\frac{\partial^2 f}{\partial\gamma^2}(0,0) = \frac{1}{2}e^{-5/\psi_0}\left(I_0\left(\frac{4}{\psi_0}\right) - e^{4/\psi_0}\right) \quad (16)$$

$$\frac{\partial^2 f}{\partial\rho^2}(0,0) = \frac{4e^{-1/\psi_0}}{\psi_0^2}. \quad (17)$$

The maximum value of ρ will be obtained when both the CB constraint and the power constraint are met with equality. Therefore, the solution is obtained by equating (15) to zero and substituting $\gamma^2 = P-\rho^2$, so the following equality arises:

$$\frac{\rho^2}{P} = \frac{\frac{\partial^2 f}{\partial\gamma^2}(0,0)}{\frac{\partial^2 f}{\partial\gamma^2}(0,0) - \frac{\partial^2 f}{\partial\rho^2}(0,0)} = \frac{\psi_0^2\left(e^{4/\psi_0} - I_0\left(\frac{4}{\psi_0}\right)\right)}{e^{4/\psi_0}(\psi_0^2+8) - \psi_0^2 I_0\left(\frac{4}{\psi_0}\right)}. \quad (18)$$

C. $P \to \infty$

For high values of P, the high power coming from the secondary transmitter makes the primary contribution negligible. In this case it can be easily seen that the optimum filtering of the primary signal leads to $\phi = 1$, so we can write the CB constraint as

$$\eta(\gamma,\rho) = e^{-\frac{\gamma^2}{2\rho^2}} \leq \eta_0, \quad (19)$$

so the optimum value of ρ^2 will be obtained when (19) and the power constraint are met with equality, so we arrive to

$$\frac{\rho^2}{P} = \frac{1}{1-2\log(\eta_0)} = \frac{1}{1+\Upsilon_0}. \quad (20)$$

TABLE I
VALUES FOR THE DESIGN PARAMETERS ϕ, γ^2 AND ρ^2 FOR THE DIFFERENT CASES UNDER STUDY.

Case	ϕ	γ^2	ρ^2
P Moderate, $\gamma^2 < 4$	$\gamma^2/4$	$P-\rho^2$	Root of (13)
P Moderate, $\gamma^2 \geq 4$	1	$P-\rho^2$	Root of (14)
$P \to 0$, $\Upsilon_{NS} > \Upsilon_0$	N/A	0	P
$P \to 0$, $\Upsilon_{NS} = \Upsilon_0$	$\gamma^2/4$	$P-\rho^2$	(18)
$P \to \infty$	1	$\frac{P\Upsilon_0}{1+\Upsilon_0}$	$\frac{P}{1+\Upsilon_0}$

Note that this is the case when both noise and primary user power are negligible, so the constraint for the secondary user is to keep the ratio between primary and secondary messages over the limit SNR value, $\frac{\gamma^2}{\rho^2} = \Upsilon_0$.

The analytical power allocation results are summarized in Table I for the different cases.

D. Results

We will show the values of the secondary message power ρ^2 for receivers with different margins with respect to the necessary SNR for Quasi Error Free (QEF) reception, obtained with the analytical approximation $\phi = \gamma^2/4$. These results will be compared with those obtained with the optimum value $\phi = \phi_{opt}$ in order to check the accuracy of the approximation, and with those forcing $\phi = 1$, thus showing the importance of the unequal power weighting. These two latter approximation are obtained by `MATLAB fmincon` applied to the problem (10), with ϕ a degree of freedom and $\phi = 1$, respectively. The approximation $\phi = \gamma^2/4$ is obtained[8] following the *P moderate* entries in Table I.

In the simulations the selected convolutional code rate is 2/3 again, for which the bound (6) predicts a value of $\Upsilon_0 \approx 5.6dB$ for a BER of $2 \cdot 10^{-4}$, being $\eta_0 \approx 0.16$.

In Figure 4 it is shown the evolution of ρ with the total available power for moderate values of P and three different SNR values, with $\Upsilon_{NS,dB} = 10\log_{10}(\Upsilon_{NS})$. Obviously, as all the three cases have the same CB restriction, the one with the higher Υ_{NS} will require a lighter *support* from the secondary transmitter and, therefore, ρ^2 will be higher. It is also noticeable that the evolution of ρ^2 (in both the $\phi = \gamma^2/4$ and ϕ_{opt} cases) has two differentiated regimes: the *low power* regime, where all the secondary power can be allocated to the secondary message without breaking the BER constraint and, therefore, in this region $\rho^2 = P$; and the *moderate power* regime. Note also that the case of $\Upsilon_{NS} = \Upsilon_0$ does only admit the *moderate power* regime, as the BER constraint is met with equality even in absence of the secondary transmitter. The solution for $\phi = 1$ has a slightly different behavior:

- For the cases where all the power can be allocated to the secondary message without breaking the BER constraint, the solution is the same as in the other approximations. If this region does not exist (for $\Upsilon_{NS} = \Upsilon_0$) the optimum value of ρ is zero for a large range of values of P.

[8]An additional check has to be performed: If the obtained value meets $\rho^2 > P$, then all the available power can be allocated to the secondary message $\rho^2 = P$, and the CB constraint will be met with strict inequality.

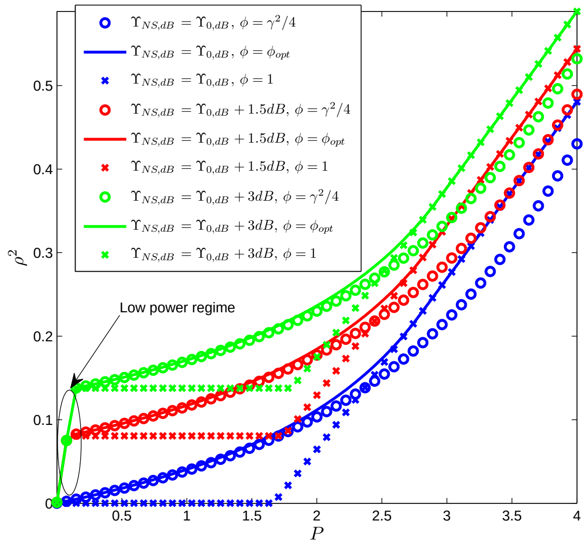

Fig. 4. Power (seen at reception) allocated to the secondary signal at the secondary transmitter as a function of the received secondary power with respect to the primary one..

- For moderate values of P, an increment on the value of P is not reflected in the value of ρ, as allocating some power to the primary message would increase the BER bound.
- For high values of P, the value of ρ increases with P. In this region, the value of ρ is obtained as the root of (14), and approximates the optimum solution as P increases.

It is also noticeable that the solution with $\phi = \gamma^2/4$ offers very little degradation with respect to the optimum value of ϕ for small values of P, while the solution for $\phi = 1$ offers a good performance for larger values. Therefore, a near-optimum solution could be obtained just by solving the $\phi = 1$ and $\phi = \gamma^2/4$ problems, and choosing the one whose performance is better, which is substantially less computationally expensive than solving the more general problem. The degradation due to the presence of echoes is transcendent for a large range of values of P, specially the lower ones. In this region, the importance of the proposed filtering is clear, as it allows the secondary transmitter to achieve a non-zero rate.

In Figure 5 the accuracy of the $P \to \infty$ and $P \to 0$ expressions for ρ^2/P is shown. For moderate values of P, it is also shown that if the target receivers have $\Upsilon_{NS} > \Upsilon_0$, then the fraction of available power used for the secondary transmission can be quite high for low values of P and then it has to decrease. In fact, in the low power regime, all the available power can be allocated to the secondary message without breaking the BER constraint, as previously stated. It can be also seen that the family of curves for $\Upsilon_{NS} > \Upsilon_0$ tend to approach the $\Upsilon_{NS} = \Upsilon_0$ curve as Υ_{NS} approaches Υ_0.

V. COVERAGE ANALYSIS

In this section we will extend the previously obtained results to the case of having several primary receivers in different reception states (i.e., different values of P and Υ_{NS}), as

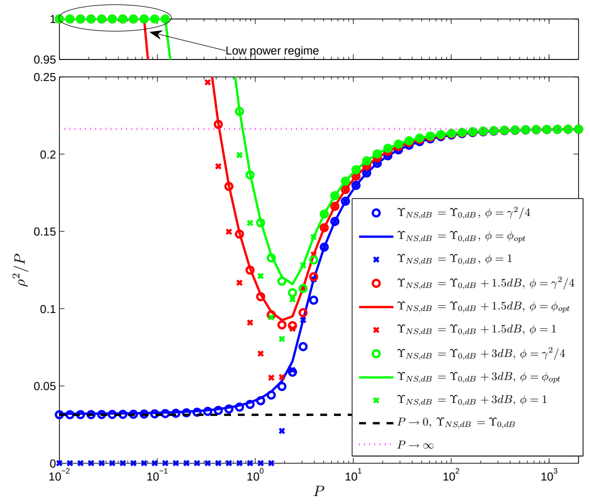

Fig. 5. Fraction of power used for the transmission of the secondary message as a function of the total received power from the secondary transmitter with respect to the primary one.

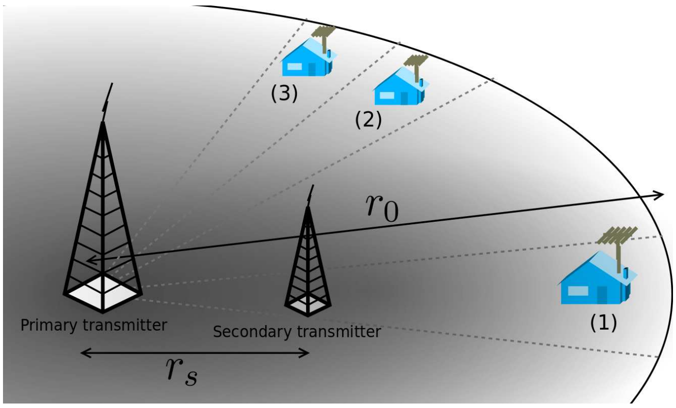

Fig. 6. Coverage diagram. Due to the assumption on the perfect directivity of the antennas, receiver (1) is affected by the secondary transmitter, but receivers (2) and (3) are not.

expected in a realistic broadcast scenario. As we will see next, obtaining a solution is more involved than just considering a *worst case* primary receiver.

Let us define the *transmit mask* $\tilde{\boldsymbol{\gamma}} = [\tilde{\gamma}_1, ..., \tilde{\gamma}_N]$, and the *secondary ratio* $\tilde{\rho}$ as the transmit parameters such that $\frac{1}{N}||\tilde{\boldsymbol{\gamma}}||_2^2 + \tilde{\rho}^2 \leq 1$, so we can write $\boldsymbol{\gamma}(\mathbf{x}) = \sqrt{P(\mathbf{x})}\tilde{\boldsymbol{\gamma}}$ and $\rho(\mathbf{x}) = \sqrt{P(\mathbf{x})}\tilde{\rho}$, where $P(\mathbf{x})$, $\boldsymbol{\gamma}(\mathbf{x})$ and $\rho(\mathbf{x})$ denote the same quantities as in previous sections with the insertion of a parameter that indicates the position $\mathbf{x}$ (in polar coordinates $\mathbf{x} = (r, \theta)$, for convenience) of a receiver located at $\mathbf{x}$. Similarly, we introduce the modified metric

$$\eta(\mathbf{x}, \tilde{\boldsymbol{\gamma}}, \tilde{\rho}) = \frac{1}{N}\sum_{k=1}^{N} e^{-\frac{1+\gamma_k^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_k(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right) \quad (21)$$

that extends (5) by adding the location parameter $\mathbf{x}$. With this extension, $\eta(\mathbf{x}, \mathbf{0}, 0)$ denotes the same metric in the absence of a secondary transmitter.

We will constrain the secondary user to keep (at least) the original coverage area of the primary system so the licensed service is not compromised. For the sake of simplicity, we will only consider those points within the coverage zone that are aligned with the primary and secondary transmitters, and have the two transmitters at the same side. This is equivalent to assuming receivers with perfectly aimed antennas with a gain of $-\infty$ dB for all angular directions (except 0º). Thus, the points that are affected by the secondary user and, therefore, the points we must take into account in the coverage constraint can be written in polar coordinates as $\mathcal{C}_0 = \{(r, \theta) \,|\, r \in [r_s, r_0],\, \theta = \theta_0\}$, where r_0 is the radius of the coverage zone, assumed to be a circle centered on the primary transmitter, and (r_s, θ_0) denotes the secondary transmitter location. This scenario is depicted in Figure 6.

Unfortunately, the problem of maximizing the secondary rate subject to a constraint on the primary coverage area is analytically intractable. However, and in order to show the effects of having several receivers under very different reception characteristics, we will study first a simplified two-user scenario, where one of the primary receivers is located near the secondary transmitter, and the other one far from it. As we will see afterwards, this two-user scenario is quite a good approximation to the solution to the complete coverage scenario, which has to be obtained numerically.

A. Two different receivers

In the proposed scenario we are likely to find two receivers that are in extremely different reception situations. For instance, if the secondary transmitter is located far from the coverage edge and its transmit power is much smaller than that of the primary transmitter, those receivers in the limit of the coverage zone will have an active CB constraint ($\eta(\mathbf{x}, \mathbf{0}, 0) = \eta_0$), and a value of $P(\mathbf{x}) \to 0$, whereas the receivers near the secondary transmitter will have a value of $P(\mathbf{x}) \to \infty$. We will study this case as a simplification of the general case covering receivers under many different values of P.

Let us denote as $\mathbf{x}_n$ the position of the receiver that is near the secondary transmitter ($P(\mathbf{x}_n) \to \infty$), and as $\mathbf{x}_f$ the position of the receiver that is far from the secondary transmitter ($P(\mathbf{x}_f) \to 0$). Even for this simple case, the optimum fraction of active carriers for the nearby receiver is $\phi = 1$, and for the far-off receiver is $\phi = \gamma^2(\mathbf{x}_f)/4 \approx 0$. For the sake of analytical tractability, we will restrict our analysis to two-level solutions for the primary power weighting, i.e., solutions of the form $\tilde{\boldsymbol{\gamma}} = [\tilde{\gamma}_1 \mathbf{1}_{N\phi}\; \tilde{\gamma}_2 \mathbf{1}_{N(1-\phi)}]$, with neither $\tilde{\gamma}_1$ nor $\tilde{\gamma}_2$ necessarily zero. In Appendix C it is shown that a fraction of power

$$\tilde{\rho}^2 = \frac{\psi_0^2\left(e^{4/\psi_0} - I_0\left(\frac{4}{\psi_0}\right)\right)}{e^{4/\psi_0}\left(\Upsilon_0(\psi_0 - 2)^2 + \psi^2 + 8\right) - (\Upsilon_0 + 1)\,\psi_0^2 I_0\left(\frac{4}{\psi_0}\right)} \quad (22)$$

can be allocated to the secondary signal in this scenario, with $\tilde{\gamma}_2 = \Upsilon_0\tilde{\rho}$, $\tilde{\gamma}_1^2 \to \infty$ and $\phi \to 0$. As we will see in the following section, this simplified scenario is a good approximation to the general one, where all the receivers in the coverage zone are taken into account.

B. Numerical approach and results

The extension of the previous analysis to the complete coverage zone implies the insertion of an infinite number of CB constraints (one for each of the infinite points in the coverage zone), so the problem can be seen to be a Semi-Infinite Program (SIP), i.e., an optimization problem with a finite number of design variables, but an infinite number of constraints. This problem is intractable due to the high dimensionality of the problem.[9]

We can reduce this dimensionality by grouping the N amplitude values $\tilde{\gamma}_1$, ..., $\tilde{\gamma}_N$ in M groups $\mathcal{G}_1$, ..., $\mathcal{G}_M$, such that the power allocation will be constant in each group, this is, $\tilde{\gamma}_j = \tilde{\gamma}_k \,\forall\, \tilde{\gamma}_j,\, \tilde{\gamma}_k \in \mathcal{G}_i$. Similarly to (9), we can rewrite the problem as

$$\begin{aligned} \text{minimize} \quad & -\tilde{\rho} \\ \text{subject to} \quad & \tilde{\eta}(\mathbf{x}, \tilde{\boldsymbol{\gamma}}, \tilde{\rho}, \boldsymbol{\phi}) \leq \eta_0 \,\forall\, \mathbf{x} \in \mathcal{C}_0, \\ & \tilde{\rho}^2 + \sum_{i=1}^{M} \tilde{\gamma}_i^2 \phi_i \leq 1 \\ & \sum_{i=1}^{M} \phi_i = 1 \\ & \phi_i \geq 0 \end{aligned} \tag{23}$$

where ϕ_i is the fraction of carriers in the i-th group, $\phi_i = \frac{|\mathcal{G}_i|}{N} \leq 1$, $|\mathcal{X}|$ denotes the cardinality of set $\mathcal{X}$, $\boldsymbol{\phi} = [\phi_1, \ldots, \phi_M]$ and

$$\tilde{\eta}(\mathbf{x}, \tilde{\boldsymbol{\gamma}}, \tilde{\rho}, \boldsymbol{\phi}) = \sum_{i=1}^{M} \phi_i e^{-\frac{1+\gamma_k^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_k(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right). \tag{24}$$

In this problem, the number of variables is $2M$: the secondary ratio $\tilde{\rho}$, the amplitudes for the different groups $\tilde{\gamma}_1$, ...,$\tilde{\gamma}_M$ and the corresponding fractions of carriers ϕ_1, ..., ϕ_{M-1}. The remaining fraction can be computed as $\phi_{M-1} = 1 - \sum_{i=1}^{M-1} \phi_i$. We will also assume that there is a large enough number of carriers in every group, so $0 \leq \phi_i \leq 1$, with $\phi_i \in \mathbb{R}$.

`MATLAB` function `fseminf` was used to obtain the solution of the optimization problem. This algorithm, of the discretization type [19], is based on a quasi-Newton Sequential Quadratic Programming (SQP) algorithm applied to a finite number of restrictions, as a result of the discretization of the semi infinite constraint. This optimization method will return a local minimum, but as the problem is not convex we cannot guarantee global optimality. In order to overcome this problem, the optimization algorithm was run 2,000 times for each pair of problem complexity and secondary position (M, r_s) with different initial random points, selecting afterwards the solution that provided the lowest value on the objective function.

Other parameters that describe the scenario (height of transmitters and receivers, transmit power...) are shown in Table II, with the Okumura-Hata propagation model equations taken from [20]. In Figure 7 the obtained results are compared with those corresponding to a single primary receiver on the border of the coverage zone (which might be thought to be a *worst case*, but as previously seen, the solution is more involved), and the two user scenario previously described by the numerical evaluation of (22).

[9] For instance, this problem for a DVB-T system operating in the 8K-Mode will have 8193 variables, although this number can be slightly lower if we take into account the guard bands, for example.

TABLE II
PARAMETERS OF THE PROPOSED SCENARIO.

Parameter	**Value**
Height of primary transmitter	324m
EIRP of primary transmitter	70dBm
Position of primary transmitter	r=0Km, $\theta = 0$
Height of secondary transmitter	40m
EIRP of secondary transmitter	36dBm
Position of secondary transmitter	$r = r_s$ (Variable), $\theta = 0$
Height of receivers	30m
Thermal Noise Power	-105dBm
Propagation model	Modified Okumura-Hata, Urban Model
Discretization step for the SIP solver	200m

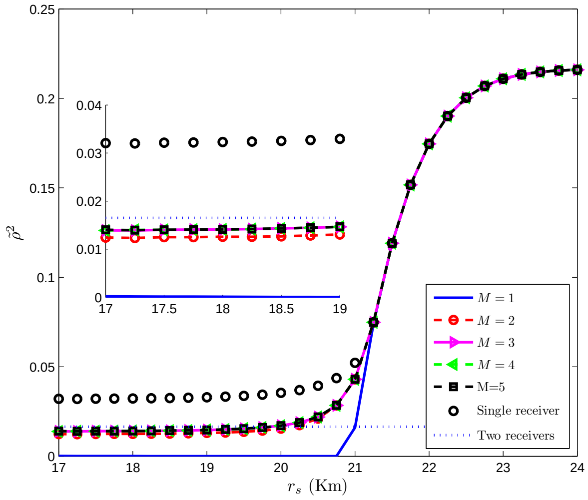

Fig. 7. Fraction of transmit power of the secondary transmitter allocated to the secondary message as a function of the secondary transmitter position.

It can be seen that the lower values of r_s suffer from quite a large degradation with respect to the single user case, while for higher values this difference does not exist. The cause of this difference resides in the variability of P among the different receivers: while for low r_s values those receivers near the secondary transmitter have $P \to \infty$ and those near the coverage limit have $P \to 0$, in the high r_s case all the receivers that are affected by the secondary transmitter experience relatively high values of P. Not surprisingly, the degradation in the low r_s zone is much more reduced if we compare the actual result with the simplified two-user scenario, as it is closer to the studied case. These results imply that the insertion of two receivers in very different situations reduces the power allocated to the secondary message, while incrementing this number of receivers (even to infinity) does not change the result too much.

For higher values of r_s, all the affected receivers have large values of P, so the optimum fraction of active carriers is one for all of them, and the obtained solution is equivalent to the *worst case* single-receiver solution. In this region the proposed approximation with two users is not realistic, as even the users on the border receive a much higher power contribution from the secondary transmitter, as previously pointed out.

With respect to the complexity of the problem (the number M of groups), for the lower values of r_s, similarly to the single receiver case, $M = 1$ results in a null power allocated to the secondary message, whereas for values of $M > 3$ no additional gain is attained. Note that the $M = 1$ case is equivalent to the transmission without the proposed power weighting in the frequency domain, which use is shown once again to be mandatory in order to achieve a nonzero rate for the secondary user. Moreover, the solution $M = 2$ (which was shown to be optimum for the single user case) suffers only a slight degradation with respect to $M = 3$. For higher values of r_s, the solution is to perform a uniform power allocation for the primary message, so the optimum number of groups is $M = 1$ and, therefore, further gain is not achieved by incrementing the order of the problem.

VI. Bound Verification: Software and Hardware Simulations

In the previous sections, the transmit parameters of the secondary system have been designed according to the BER bound (6), due to the impossibility of finding a closed form expression for the actual BER. The objective of this section is to verify the aforementioned bound, thus providing an empirical proof of the previous theoretical results.

Computer simulations and hardware measurements were conducted in order to validate the proposed power allocation for the secondary transmitter. Hardware tests were performed in order to check the potential negative effects that the proposed transmission technique could have on the synchronization and estimation stages of a real receiver. The measurement set-up is described in Figure 9[10]. In Figure 8 it can be seen that, although the bound is not remarkably tight, its use as a performance metric for the design of the proposed filtering provides a clear improvement in the primary link quality with respect to the simple transmission of the primary message, and, therefore, the achievable rate of the secondary system is going to be larger[11].

In order to show the usefulness of the proposed filtering when dealing with higher order constellations, hardware tests were run also for a 64-QAM constellation, with the corresponding results shown in Figure 10. It can be seen that the filtered approach outperforms the non-filtered transmission in all the scenarios except for the $\gamma = 0.5$ one, where some artifacts were found. These effects are expected to disappear when using higher order transmission filters or weighting directly in the DFT domain. A similar behavior was obtained for a 16-QAM constellation, although the results are omitted due to space constraints.

[10] Due to hardware constraints, the power weighting was performed in the time domain, by means of a 32-ray equivalent baseband channel, where one ray was used to emulate the primary contribution, and the remaining 31 to perform the frequency power weighting. The results were averaged in both cases for 50 different pairs (n_0, θ) of delay and phase differences between primary and secondary contributions

[11] As the CNR required for a given BER performance is going to be lower for the filtered transmission, the secondary user is allowed to allocate more power to the secondary message and, therefore, achieve a larger rate.

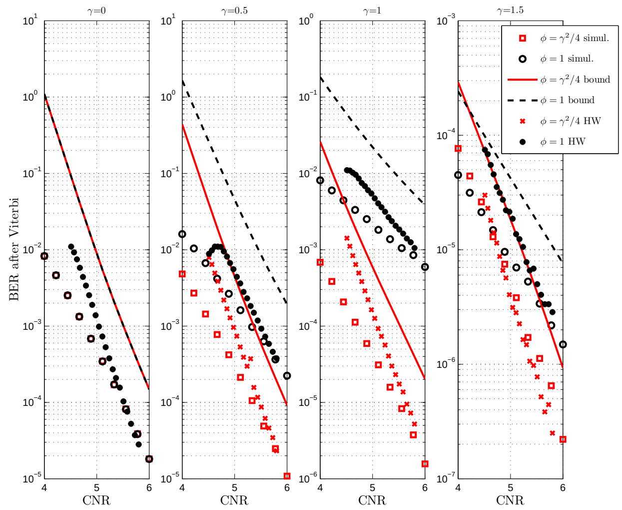

Fig. 8. Analytical bounds, simulation and hardware (HW) results for multiple CNR and γ values. DVB-T waveform with Constellation: QPSK, Code Rate: 2/3. The CNR is calculated prior to the transmission of the secondary user, i.e. $\beta = \frac{1}{2}\Upsilon = \frac{1}{2}10^{(CNR+0.33)/10}$[21]

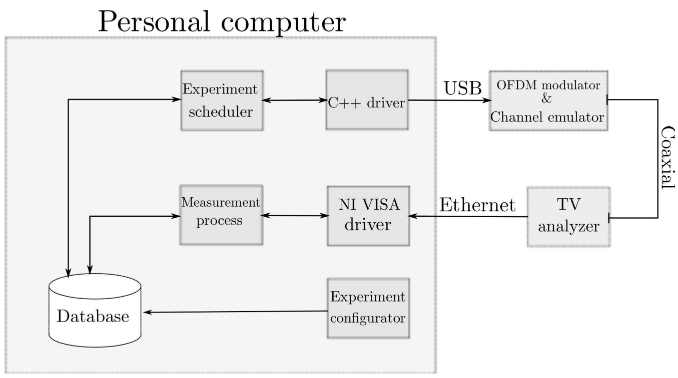

Fig. 9. Hardware measurements set-up. The OFDM signal was generated with the DekTec DTU-215 USB-2 VHF/UHF modulator [22], which allows to simulate a 32 rays baseband equivalent channel (by defining the delay, amplitude and phase of each ray), and the addition of Gaussian noise. The BER was measured with Rohde & Schwarz ETL TV Analyzer [23], and captured with MATLAB via the National Instruments (NI) VISA driver. The experiments (CNR, channel model, number of measurements...) are configured, inserted into a relational database, and finally executed by the experiment scheduler.

VII. Conclusions

In this paper we have considered the application of the overlay cognitive radio paradigm to a broadcast Single Frequency Network. Given the fact that the primary user Quality of Service is not simply a function of the Signal to Noise Ratio, our approach has taken into account the possible degradation of the primary service in strong line of sight environments due to the impossibility a of coherent combination of the primary waveforms. Optimum transmission strategies with respect to analytical BER bounds have been derived and analyzed via software simulations. The proposed approach was further verified by means of BER measurements in an actual hardware receiver. These modified transmission schemes were applied in order to maximize the transmission rate of a secondary user

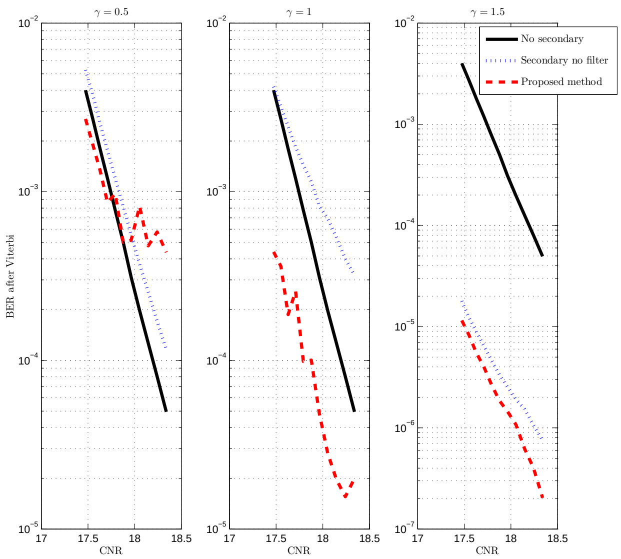

Fig. 10. Hardware tests for a 64-QAM 2/3 DVB-T waveform. The proposed method (filtering with $\phi = \gamma^2/4$) is compared with the unfiltered approach ($\phi = 1$) and with the scenario without the secondary transmitter ($\gamma = 0$).

operating at the same frequency and location as the primary user. The primary QoS is assured by means of a coverage analysis whereby the BER is restricted to be above a given threshold. Spectrum reuse is successfully achieved without requiring any modification on the primary users, and with no cooperation with the primary transmitters. Future lines on this work include the extension of the proposed scenario to Rician and Rayleigh fading channels.

Appendix A
Optimality Conditions for the Optimization Problem

The associated Karush-Kuhn-Tucker (KKT) conditions to problem (7) are

$$2e^{-\beta\gamma_k^2}\left(-\beta\gamma_k I_0(2\beta\gamma_k)+\beta I_1(2\beta\gamma_k)\right)+2\lambda\gamma_k = 0 \;\forall k, \quad (25)$$

$$\lambda\left(\sum_{k=1}^{N}\gamma_k^2 - N\gamma^2\right) = 0, \quad \lambda \geq 0. \quad (26)$$

We will distinguish two cases: *1)* when $\sum_{k=1}^{N}\gamma_k^2 - N\gamma^2 < 0$, so λ is forced to be zero in order to meet condition (26), and *2)* when $\sum_{k=1}^{N}\gamma_k^2 - N\gamma^2 = 0$, so λ is not forced to be zero (we will refer to the constraint as *active* in that case).

1) Non-active constraint: In this case, we have $\lambda = 0$, so the resulting condition is

$$2e^{-\beta\gamma_k^2}\left(-\beta\gamma_k I_0(2\beta\gamma_k)+\beta I_1(2\beta\gamma_k)\right) = 0 \;\forall k \implies \quad (27)$$
$$\implies \gamma_k I_0(2\beta\gamma_k) = I_1(2\beta\gamma_k) \;\forall k.$$

Proposition A.1: The nontrivial solutions for (27) are in the interval $\sqrt{\frac{\beta-1}{\beta}} \leq \gamma_k \leq 1$ for $\beta > 1$. For $\beta \leq 1$, the only solution is $\gamma_k = 0$.

Proof: We will assume $\gamma_k \neq 0$. Using (32), we can write $I_0(2\beta\gamma_k) \geq \frac{1}{\beta\gamma_k} I_1(2\beta\gamma_k)$. Combining this inequality with (27) we obtain $I_0(2\beta\gamma_k) \geq \frac{1}{\beta} I_0(2\beta\gamma_k)$, so $\beta \geq 1$.

Starting with (33), we have that $I_1^2(2\beta\gamma_k) > I_0(2\beta\gamma_k)I_2(2\beta\gamma_k)$, which together with (32) leads to

$$\gamma_k^2 I_0^2(2\beta\gamma_k) > I_0(2\beta\gamma_k)I_2(2\beta\gamma_k). \quad (28)$$

Finally, combining equations (32) and (27), we have that $I_2(2\beta\gamma_k) = \left(1-\frac{1}{\beta}\right) I_0(2\beta\gamma_k)$, so (28) reads as $\gamma_k^2 I_0^2(2\beta\gamma_k) > \left(1-\frac{1}{\beta}\right) I_0^2(2\beta\gamma_k)$, or, equivalently $\gamma_k > \sqrt{\frac{\beta-1}{\beta}}$.

∎

Proposition A.2: The nontrivial solutions for (27) are not local minima of the optimization problem.

Proof: In order to be a local minimum, the Hessian matrix of the objective function has to be positive definite. The Hessian is a diagonal matrix with elements

$$L(\gamma_k) = (L(\boldsymbol{\gamma}))_{k,k} = 2e^{-\beta\gamma_k^2} \times \quad (29)$$
$$\times\left(-4\beta^2\gamma_k I_1(2\beta\gamma_k) + \beta^2 I_2(2\beta\gamma_k) + \left(2\beta\gamma_k^2+\beta-1\right) I_0(2\beta\gamma_k)\right).$$

Moreover, we have that

$$\begin{aligned} &-4\beta\gamma_k I_1(2\beta\gamma_k) + \beta I_2(2\beta\gamma_k) + \quad (30)\\ &\beta\left(2\beta\gamma_k^2+\beta-1\right) I_0(2\beta\gamma_k) \overset{(i)}{=} \\ &(-2\beta\gamma_k^2 - 1 + b) I_0(2\beta\gamma_k) + \beta I_0(2\beta\gamma_k) \overset{(ii)}{=} \\ &(-2\beta\gamma^2 + 2\beta - 2) I_0(2\beta\gamma_k) \end{aligned}$$

where (i) derives from (27) and (ii) from (32) and (27).

As all the elements must be positive if $\boldsymbol{\gamma}$ is a local minimum, and since I_0 is strictly positive, the condition for the minimum is $\gamma_k < \sqrt{\frac{\beta-1}{\beta}}$, which contradicts proposition A.1. ∎

Therefore, those points with some $\gamma_k \neq 0$ and inactive power constraint are not local minimum of the optimization problem.

2) Active constraint: In this case, we have the following necessary conditions for the point $\boldsymbol{\gamma}$ to be optimal

$$-2\beta\gamma_k e^{-\beta\gamma_k^2} I_0(2\beta\gamma_k) + 2\beta I_1(2\beta\gamma_k) e^{-\beta\gamma_k^2} + 2\lambda\gamma_k = 0 \quad (31)$$
$$\forall i = 1,...,N, \lambda \geq 0.$$

The condition is met if $\gamma_k^2 = 0$, as $I_1(0) = 0$. If $\gamma_k^2 \neq 0$, we can rewrite (31) as $\lambda = \beta e^{-\beta\gamma_k^2}\left(I_0(2\beta\gamma_k) - \frac{1}{\gamma_k} I_1(2\beta\gamma_k)\right) \forall\, k$, so it can be seen that those points of the form $\boldsymbol{\gamma}_M = [\mathbf{0}_{N-M}\; k\mathbf{1}_M]$ (or their corresponding permutations) where the power constraint is active are critical points of the Lagrangian.

As the function $\lambda(\gamma_k) = \beta e^{-\beta\gamma_k^2}\left(I_0(2\beta\gamma_k) - \frac{1}{\gamma_k} I_1(2\beta\gamma_k)\right)$ is non-injective, there are some points $\gamma_1 \neq \gamma_2$ such that $\lambda(\gamma_1) = \lambda(\gamma_2)$. However, these points were found to be local maxima of the objective function by checking the second order necessary conditions for optimality.

Regarding the second order conditions, some of the points under study can be local maxima, whereas others are local minima. As we are optimizing over the whole set of points, it is expected that the solution will lead to a global optimum.

A. Properties of the Bessel functions

$$I_v(t) = I_{v-2}(t) - \frac{2(v-1)}{t} I_{v-1}(t), \quad (32)$$

$$I_1^2(t) > I_0^2(t) I_2^2(t). \quad (33)$$

APPENDIX B
ASYMPTOTIC OPTIMUM VALUE FOR ϕ

We start with $f(\phi) = (1-\phi) + \phi e^{-\beta\gamma^2/\phi} I_0(2\beta\gamma/\sqrt{\phi})$. In order to find a minimum of this function, we take its derivative

$$\frac{d}{d\phi}\left(f(\phi)\right) = e^{-\beta\gamma^2/\phi} \times \times \left(I_0(2\beta\gamma/\sqrt{\phi}) \left(1 + \frac{\beta\gamma^2}{\phi}\right) - I_1(2\beta\gamma/\sqrt{\phi}) \frac{\beta\gamma}{\sqrt{\phi}} \right) - 1. \quad (34)$$

For high SNR, if we use the asymptotic approximation for the Bessel function $I_{0,1}(x) \approx \frac{e^x}{\sqrt{2\pi x}}$, and make the variable change $\alpha = \frac{\gamma}{\sqrt{\phi}}$, after equating (34) to zero we have

$$e^{2\beta\alpha} \left(\frac{1 - \beta\alpha + \beta\alpha^2}{\sqrt{4\pi\beta\alpha}} \right) = e^{\beta\alpha^2}. \quad (35)$$

For asymptotically large β, the expression between parenthesis can be ignored, so the remaining expression is $e^{2\beta\alpha} = e^{\beta\alpha^2}$. Therefore, we have $\alpha = 2$, which leads to a value of $\phi = \frac{\gamma^2}{4}$. Note that this expression is only valid for values of $\gamma < 2$. In fact, if $\gamma > 2$, the solution of the problem is to transmit over all carriers with equal power, i.e., $\phi = 1$.

APPENDIX C
TWO DIFFERENT RECEIVERS

If we constrain the frequency power weighting to have only two different levels, we can write the Chernoff bound as

$$\eta\left(\phi, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\rho}, \mathbf{x}\right) = \phi e^{-\frac{1+\gamma_1^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_1(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right) + (1-\phi) e^{-\frac{1+\gamma_2^2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}} I_0\left(\frac{2\gamma_2(\mathbf{x})}{\psi(\mathbf{x})+2\rho^2(\mathbf{x})}\right), \quad (36)$$

where $\phi \in [0,1] \subset \mathbb{R}$ since we are assuming a large enough number of carriers, and $\gamma_i(\mathbf{x}) = \sqrt{P(\mathbf{x})}\tilde{\gamma}_i$, $\rho(\mathbf{x}) = \sqrt{P(\mathbf{x})}\tilde{\rho}$. We will try to find a solution $(\phi, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\rho})$ that fulfills the BER constraint at both receivers even with the insertion of a secondary signal, i.e., $\eta\left(\phi, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\rho}, \mathbf{x}\right) \le \eta_0$, $\forall \mathbf{x} \in \{\mathbf{x}_n, \mathbf{x}_f\}$. For the nearby receiver, the signal coming from the primary transmitter will be negligible with respect to the secondary transmission, so we have that

$$\eta\left(\phi, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\rho}, \mathbf{x}_n\right) = \phi e^{-\frac{\tilde{\gamma}_1^2}{2\tilde{\rho}^2}} + (1-\phi) e^{-\frac{\tilde{\gamma}_2^2}{2\tilde{\rho}^2}} \quad (37)$$

just by taking the limit $P(\mathbf{x}) \to \infty$ in (36).

Let us define $\tilde{\gamma}_m = \min\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$. Then $\eta\left(\phi, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\rho}, \mathbf{x}_n\right) < e^{-\frac{\tilde{\gamma}_m^2}{2\tilde{\rho}^2}}$, so if we set $e^{-\frac{\tilde{\gamma}_m^2}{2\tilde{\rho}^2}} = \eta_0$, then $\tilde{\gamma}_m^2/\tilde{\rho}^2 = \Upsilon_0$, the CB constraint will be met.

With this restriction, we can write a simplified CB constraint for the distant receiver using[12] $\phi = \gamma^2/4$ and $\gamma_2 = \sqrt{\Upsilon_0}\rho$, with $\gamma^2 = \phi\gamma_1^2$ the total power spent in the carriers with amplitude $\gamma_1 = 2$, as

$$\eta\left(\gamma, \rho, \mathbf{x}_f\right) = \frac{\gamma^2}{4} e^{-\frac{5}{\psi_0+2\rho^2}} I_0\left(\frac{4}{\psi_0+2\rho^2}\right) + \left(1 - \frac{\gamma^2}{4}\right) e^{-\frac{1+\Upsilon_0\rho^2}{\psi_0+2\rho^2}} I_0\left(\frac{2\rho\sqrt{\Upsilon_0}}{\psi_0+2\rho^2}\right). \quad (38)$$

Let us define $f(\gamma,\rho) = \eta\left(\gamma, \rho, \mathbf{x}_f\right) - \eta_0$. As $P \to 0$, we have that $\gamma \to 0$ and $\rho \to 0$, so we can write $f(\gamma,\rho) = \frac{1}{2}[\gamma\ \rho]\nabla^2_{\gamma,\rho} f(0,0)[\gamma\ \rho]^T$, being $\nabla^2_{\gamma,\rho} f(0,0)$ a diagonal matrix with entries

$$\frac{\partial^2 f}{\partial\gamma^2}(0,0) = \frac{1}{2} e^{-5/\psi_0} \left(I_0\left(\frac{4}{\psi_0}\right) - e^{4/\psi_0} \right) \quad (39)$$

$$\frac{\partial^2 f}{\partial\rho^2}(0,0) = \frac{(4 - 2\Upsilon_0(\psi_0 - 1)) e^{-1/\psi_0}}{\psi_0^2}. \quad (40)$$

The maximum value of ρ will be obtained when the power constraint is met with equality. In this case $\gamma^2 + \rho^2 + \left(1 - \frac{\gamma^2}{4}\right)\Upsilon_0\rho^2 = P$, so $\gamma^2 = \frac{4\left((\Upsilon_0+1)\rho^2 - P\right)}{\Upsilon_0\rho^2 - 4} \approx P - (\Upsilon_0 + 1)\rho^2$, where the last approximation holds provided $\Upsilon_0\rho^2$ is small enough with respect to 4. With these expressions, we get to the desired equation

$$\tilde{\rho}^2 = \frac{\rho^2}{P} = \frac{\frac{\partial^2 f}{\partial\gamma^2}(0,0)}{(\Upsilon_0+1)\frac{\partial^2 f}{\partial\gamma^2}(0,0) - \frac{\partial^2 f}{\partial\rho^2}(0,0)} = \frac{\psi_0^2\left(e^{4/\psi_0} - I_0\left(\frac{4}{\psi_0}\right)\right)}{e^{4/\psi_0}\left(\Upsilon_0(\psi_0-2)^2 + \psi_0^2 + 8\right) - (\Upsilon_0+1)\psi_0^2 I_0\left(\frac{4}{\psi_0}\right)}. \quad (41)$$

[12] With this simplification, $\gamma_1 = 2$, and as $\gamma_2 \approx 0, \gamma_m = \gamma_2$. In the following, we will omit the position indexing $(\mathbf{x})$, as we are only taking into account the far-off receiver.

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