

# ON THE ROLE OF DIFFERENTIATION FOR RESAMPLING DETECTION

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# Outline

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- 2 Theoretical Model
- 3 Minimization of self-noise power
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# Introduction

- Nowadays, a lot of powerful and intuitive editing tools allow the easy alteration of digital images.
- Image manipulations usually involve geometric transformations that introduce detectable artefacts through the resampling process.
- Two techniques for **detection of resampling traces**:
  - **Popescu & Farid**[1]: measure the correlation between pixels performing an EM algorithm and expose the periodic patterns.
  - **Gallagher**[2] and **Mahdian & Saic**[3]: apply a differentiator to the resampled signal to enhance the spectrum peaks of the covariance. The second approach introduces a Radon transform.

## Some questions arise:

- Why the derivative prefilter enhances the spectral lines in [2, 3]?
- Are there other prefilters that can yield better results?

# Analogy between a resampled signal and a PAM signal

Let us consider a **one-dimensional resampled signal** with rate  $P := L/M$  and a **baseband linearly modulated signal** (PAM):

$$y[l] = \sum_m u[m] h[lM - mL] \quad \equiv \quad x(lT_s) = \sum_m \alpha_m g(lT_s - mT)$$

$u[m]$ : pixel intensity  $\equiv \alpha_m$ : symbol amplitude

$h[\cdot]$ : interpolation kernel  $\equiv g(\cdot)$ : pulse shape

$M$ : sub-sampling factor  $\equiv T_s$ : sampling period

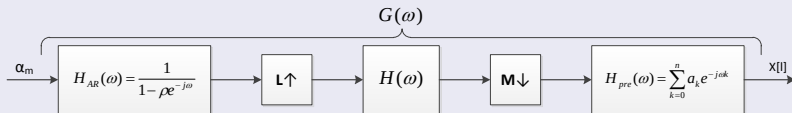
$L$ : up-sampling factor  $\equiv T$ : symbol period

Hence,  $P \equiv T/T_s$  and if WLOG, we make  $T_s = 1$ , we have:

**Resampled signal:**  $x(l) = \sum_m \alpha_m g(l - mP).$

# Resampled Image Model

One-dimensional resampled image model:



From the block diagram we have:

$$G(\omega) = H_{AR} \left( \frac{\omega L}{M} \right) H \left( \frac{\omega}{M} \right) H_{pre}(\omega) = H \left( \frac{\omega}{M} \right) \frac{\sum_{k=0}^n a_k e^{-j\omega k}}{1 - \rho e^{-j\omega L/M}} \quad (1)$$

If we denote  $D(\omega) := \frac{H(\omega/M)}{1 - \rho e^{-j\omega L/M}}$ , then we simplify (1) as:

$$G(\omega) = \sum_{k=0}^n a_k D(\omega) e^{-j\omega k}.$$

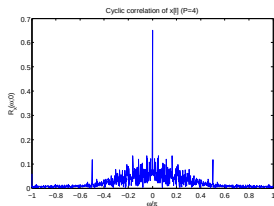
# Characterization of the resampled signal

Cyclostationary of order two in the wide sense:

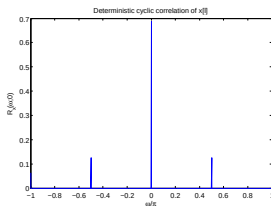
Analogy PAM signal and resampled signal  $\Rightarrow x[l]$  cyclostationary with period  $P$ :

$$E\{x[l]\} = E\{x[l + kP]\}, \quad R_x[l; \tau] = R_x[l + kP; \tau], \quad \forall k.$$

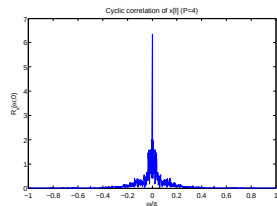
The cyclic spectrum, i.e.  $R_x[\omega; \tau] = \mathcal{FT}\{R_x[l; \tau]\}$ , is composed by two terms: **spectral lines** and **self-noise**.



(a)  $u[m] \sim \mathcal{N}(0, 1)$



(b) Theoretical ( $P = 4$ )



(c)  $u[m] \sim AR(1)$

# Phasor representation of self-noise power

- Power of the **in-phase** self-noise component:

$$E[I^2] = \frac{\pi(M-1)}{2P} \int_{-\pi}^{\pi} \left| G_0 \left( \frac{2\pi}{P} + \omega \right) + G_0^* \left( \frac{2\pi}{P} - \omega \right) \right|^2 d\omega \\ + \frac{2\pi}{P} \sum_{q=1}^{\infty} \int_{-\pi}^{\pi} \left| G_q \left( \frac{2\pi}{P} + \omega \right) + G_q^* \left( \frac{2\pi}{P} - \omega \right) \right|^2 d\omega.$$

- Power of the **quadrature** self-noise component:

$$E[Q^2] = \frac{\pi(M-1)}{2P} \int_{-\pi}^{\pi} \left| G_0 \left( \frac{2\pi}{P} + \omega \right) - G_0^* \left( \frac{2\pi}{P} - \omega \right) \right|^2 d\omega \\ + \frac{2\pi}{P} \sum_{q=1}^{\infty} \int_{-\pi}^{\pi} \left| G_q \left( \frac{2\pi}{P} + \omega \right) - G_q^* \left( \frac{2\pi}{P} - \omega \right) \right|^2 d\omega,$$

where  $G_q(\omega) = \int_{-\pi}^{\pi} G \left( \frac{\omega}{2} + \theta \right) G \left( \frac{\omega}{2} - \theta \right) e^{\pm j2\pi\theta Pq} d\theta$  and  $M = E[\alpha_m^4]$ .

# Is there a closed-form solution for the minimization?

## Objective:

Find the prefilter coefficients  $a_k$  which minimize  $E[I^2]$  and  $E[Q^2]$ .

Given the resampled image model:  $G_q(\omega) = \sum_{k=0}^n \sum_{r=0}^n a_k a_r \beta_{k,r}^q(\omega)$ ,  
with:

$$\beta_{k,r}^q(\omega) = \int_{-\pi}^{\pi} D\left(\frac{\omega}{2} + \theta\right) D\left(\frac{\omega}{2} - \theta\right) e^{-j[(\frac{\omega}{2} + \theta)k + (\frac{\omega}{2} - \theta)r \mp 2\pi q\theta]} d\theta.$$

For instance, express  $E[I^2]$  as a function of the prefilter coefficients:

$$E[I^2] = \frac{\pi(M-1)}{2P} \sum_{k,r=0}^n \sum_{i,s=0}^n a_k a_r a_i a_s \xi_{k,r,i,s}^0 + \frac{2\pi}{P} \sum_{q=1}^{\infty} \sum_{k,r=0}^n \sum_{i,s=0}^n a_k a_r a_i a_s \xi_{k,r,i,s}^q$$

where  $\xi_{k,r,i,s}^q =$

$$\int_{-\pi}^{\pi} \left[ \beta_{k,r}^q\left(\frac{2\pi}{P} + \omega\right) + \beta_{k,r}^q\left(\frac{2\pi}{P} - \omega\right)^* \right] \left[ \beta_{i,s}^q\left(\frac{2\pi}{P} + \omega\right) + \beta_{i,s}^q\left(\frac{2\pi}{P} - \omega\right)^* \right]^* d\omega.$$

# Nearly optimum solution

① How can we get  $E[I^2] \rightarrow 0$ ?

- If  $G_q \left( \frac{2\pi}{P} + \omega \right) = -G_q^* \left( \frac{2\pi}{P} - \omega \right)$  then  $E[I^2] = 0$ .
- **This condition cannot be achieved for real signals.**

Minimize the energy of  $G(\omega)$ :

**Cost function:**  $\epsilon_I = \int_{-\pi}^{\pi} |G(\omega)|^2 d\omega.$

② How can we get  $E[Q^2] \rightarrow 0$ ?

- If  $G_q \left( \frac{2\pi}{P} + \omega \right) = G_q^* \left( \frac{2\pi}{P} - \omega \right)$  then  $E[Q^2] = 0$ .
- Equivalently  $G \left( \frac{\pi}{P} + \omega \right) = G^* \left( \frac{\pi}{P} - \omega \right)$ , so  $G(\omega)$  has to be symmetric with respect to  $\frac{\pi}{P}$ .

Minimize the *non-spectral symmetry* of  $G(\omega)$ :

**Cost function:**  $\epsilon_Q = \int_0^{\frac{\pi}{P}} \left| G \left( \frac{\pi}{P} + \omega \right) - G^* \left( \frac{\pi}{P} - \omega \right) \right|^2 d\omega.$

# Spectral symmetry (I)

$\epsilon_Q$  as a function of the prefilter coefficients:

$$\epsilon_Q = \sum_{k=0}^n \sum_{r=0}^n a_k a_r \int_0^{\frac{\pi}{P}} C_k(\omega) C_r^*(\omega) d\omega,$$

$$\text{with } C_k(\omega) = \left[ D \left( \frac{\pi}{P} - \omega \right) e^{-j\frac{\pi}{P}k} - D \left( \frac{\pi}{P} + \omega \right)^* e^{j\frac{\pi}{P}k} \right] e^{j\omega k}$$

- Considering  $Y_{k,r} = \int_0^{\frac{\pi}{P}} C_k(\omega) C_r^*(\omega) d\omega$  and noting that  $Y_{k,r} = Y_{r,k}^*$ :

$$\begin{aligned} \epsilon_Q = & Y_{0,0} + 2a_i \operatorname{Re}\{Y_{i,0}\} + 2 \sum_{k=1, k \neq i}^n a_k \operatorname{Re}\{Y_{0,k}\} + a_i^2 Y_{i,i} \\ & + 2 \sum_{k=1, k \neq i}^n a_i a_k \operatorname{Re}\{Y_{i,k}\} + \sum_{k=1, k \neq i}^n \sum_{r=1, r \neq i}^n a_k a_r Y_{k,r} \end{aligned}$$

## Spectral symmetry (II)

Minimization of  $\epsilon_Q$ , given by:

$$\begin{aligned}\bar{a}_i &= \arg \min_{a_i} \epsilon_Q = -\frac{\operatorname{Re}\{Y_{i,0}\}}{Y_{i,i}} - \sum_{k=1, k \neq i}^n a_k \frac{\operatorname{Re}\{Y_{i,k}\}}{Y_{i,i}} \\ &= -A_{i,0} - \sum_{k=1, k \neq i}^n a_k A_{i,k}\end{aligned}$$

### Property

Let  $A_{i,k} = \frac{\operatorname{Re}\{Y_{i,k}\}}{Y_{i,i}}$ . Then  $\lim_{P \rightarrow \infty} A_{i,k} = 1, \quad \forall i, k$ .

► Proof

**Asymptotic Result:**  $\bar{a}_i = -1 - \sum_{k=1, k \neq i}^n \bar{a}_k, \quad \forall i = 1, \dots, n.$

# Energy conditions

$\epsilon_I$  as a function of the prefilter coefficients:

$$\epsilon_I = \sum_{k=0}^n \sum_{r=0}^n a_k a_r Z_{k,r}, \quad \text{with } Z_{k,r} = \int_{-\pi}^{\pi} |D(\omega)|^2 e^{j\omega(r-k)} d\omega$$

Minimization of  $\epsilon_I$ , given by:

$$\bar{a}_i = \arg \min_{a_i} \epsilon_I = -\frac{\operatorname{Re}(Z_{i,0})}{Z_{i,i}} - \sum_{k=1, k \neq i}^n a_k \frac{\operatorname{Re}(Z_{i,k})}{Z_{i,i}} = -B_{i,0} - \sum_{k=1, k \neq i}^n a_k B_{i,k}$$

Two different approximate criteria lead to the same result:

**Property:** If  $B_{i,k} = \frac{\operatorname{Re}\{Z_{i,k}\}}{Z_{i,i}} \Rightarrow \lim_{P \rightarrow \infty} B_{i,k} = 1, \forall i, k.$

**Asymptotic Result:**  $\bar{a}_i = -1 - \sum_{k=1, k \neq i}^n \bar{a}_k, \quad \forall i = 1, \dots, n.$

# Prefilter design

Set of equations that must be satisfied:

$$\bar{a}_i = -1 - \sum_{k=1, k \neq i}^n \bar{a}_k, \quad \forall i = 1, \dots, n.$$

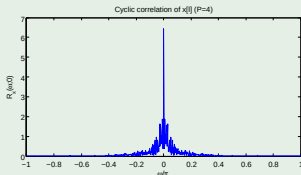
## Example

$$\mathbf{n} = 1 \quad \bar{a}_0 = 1 \quad \bar{a}_1 = -1$$

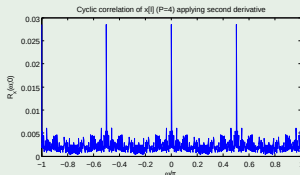
$$\mathbf{n} = 2 \quad \bar{a}_0 = 1 \quad \bar{a}_1 = -2$$

$$\bar{a}_2 = 1 \quad H_{pre}(z) = 1 - z^{-1}$$

$$H_{pre}(z) = 1 - 2z^{-1} + z^{-2}$$



(a) AR(1),  $\rho = 0.95$ .



(b) AR(1),  $\rho = 0.95$  and  $n = 2$ .

# Results

- Asymptotic results for a first-order filter considering  $\bar{a}_0 = 1$  and so:  
 $\bar{a}_1 = -A_{1,0}$  and  $\bar{a}_1 = -B_{1,0}$ .

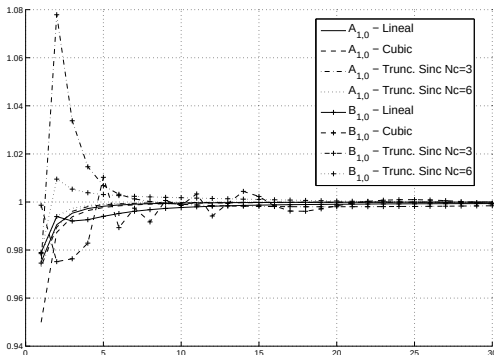


Figure: Factors  $A_{1,0}$  and  $B_{1,0}$

## Results (II)

- Monte Carlo simulations with different interpolation kernels and averaging results for  $P = 3, 7, 12$ .

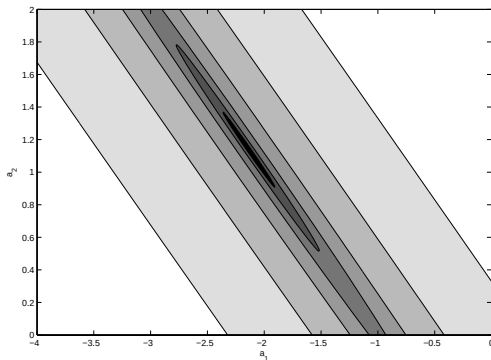


Figure: Self-noise power ( $E[I^2] + E[Q^2]$ ) for a three coefficient prefilter.

# Conclusions

Theoretical analysis of the prefiltering as a way of boosting resampling traces:

- Resampled image model based on the Digital Communications framework.
- Derivative prefilters are a nearly optimum solution for asymptotically large  $P$ .
- A set of possible solutions for the prefilter coefficients is provided.

Future Work:

- Extension of this analysis to the 2-D case.
- Analysis of the prefiltering in more realistic scenarios, for example when  $1 < P < 2$ .

# References

-  A.C. Popescu and H. Farid, "Exposing digital forgeries by detecting traces of resampling," in *IEEE Transactions on Signal Processing*, vol. 53, no. 2, pp. 758-767, Feb. 2005.
-  A.C. Gallagher, "Detection of linear and cubic interpolation in jpeg compressed images," in *Proceedings of The 2nd Canadian Conference on Computer and Robot Vision*, p. 6572, 2005.
-  B. Mahdian and S. Saic, "Blind authentication using periodic properties of interpolation," in *IEEE Transactions on Information Forensics and Security*, vol. 3, no. 3, pp. 529-538, Sept. 2008.

Thanks for your attention.

Questions?

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## Proof

Since  $A_{i,k} = \frac{\text{Re}\{Y_{i,k}\}}{Y_{i,i}}$ , then

$$A_{i,k} = \lim_{P \rightarrow \infty} \frac{\int_0^1 \text{Re}\{C_i(v \frac{\pi}{P}) C_k^*(v \frac{\pi}{P})\} dv \frac{\pi}{P}}{\int_0^1 |C_k(v \frac{\pi}{P})|^2 dv \frac{\pi}{P}},$$

where,

$$\begin{aligned} \mathcal{R}e \{ C_i \left( v \frac{\pi}{\bar{P}} \right) C_k^* \left( v \frac{\pi}{\bar{P}} \right) \} = \\ \mathcal{R}e \{ [D \left( \frac{\pi}{\bar{P}} (1 - v) \right) e^{-j \frac{\pi}{\bar{P}} i} - D^* \left( \frac{\pi}{\bar{P}} (1 + v) \right) e^{j \frac{\pi}{\bar{P}} i}] \\ \times [D \left( \frac{\pi}{\bar{P}} (1 - v) \right) e^{-j \frac{\pi}{\bar{P}} k} - D^* \left( \frac{\pi}{\bar{P}} (1 + v) \right) e^{j \frac{\pi}{\bar{P}} k}]^* e^{j(i-k)v \frac{\pi}{\bar{P}}} \} \end{aligned}$$

and

$$|C_k(v\frac{\pi}{P})|^2 = |D(\frac{\pi}{P}(1-v))e^{-j\frac{\pi}{P}k} - D^*(\frac{\pi}{P}(1+v))e^{j\frac{\pi}{P}k}|^2.$$

Given that  $|C(0)|^2 \neq 0$  and as  $P \rightarrow \infty$ :  $e^{j\gamma \frac{\pi}{P}} \rightarrow 1$ .

So,  $\operatorname{Re} \left\{ C_i \left( \nu \frac{\pi}{\rho} \right) C_k^* \left( \nu \frac{\pi}{\rho} \right) \right\} \rightarrow \left| C_k \left( \nu \frac{\pi}{\rho} \right) \right|^2$  and, hence,  $A_{i,k} \rightarrow 1, \forall i, k$ .