TWO-DIMENSIONAL STATISTICAL TEST FOR THE PRESENCE OF ALMOST CYCLOSTATIONARITY ON IMAGES

David Vázquez-Padín*, Carlos Mosquera*,
Fernando Pérez-González*,†

*. Signal Theory and Communications Department, University of Vigo, Vigo 36310
†. Electrical and Computer Engineering Department, University of New Mexico, Albuquerque, NM

Hong Kong. September 28, 2010.

Outline

1. Introduction
2. Preliminaries and Problem Statement
3. Test for the presence of cyclostationarity
4. Experimental Results
5. Conclusions

Introduction

Problem

- A lot of poweful and intuitive image editing tools facilitate the manipulation and alteration of digital images.

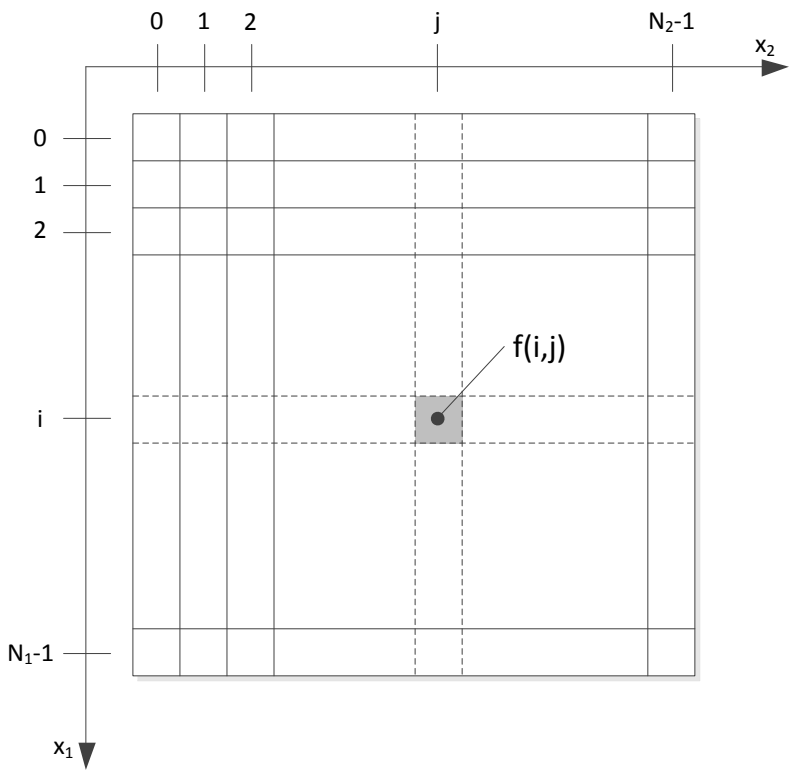
Previous work

- **Popescu & Farid**[1]: measure the correlation between pixels performing an EM algorithm and expose the periodic patterns.
- **Mahdian & Saic**[2]: apply a differentiator to the image, use a Radon transform and then process the unidimensional covariance.

Contributions

- Theoretical framework based on a cyclostationary signal analysis.
- Detection/Estimation of spatial transformations applied to an image.

Notation



Coordinate convention used to represent digital images.

Matrix and vector representation

- **Matrices** are denoted by bold capital letters, e.g. $\mathbf{F}$.
- **Vectors** are denoted by bold lowercase letters, e.g. $\mathbf{x}$.

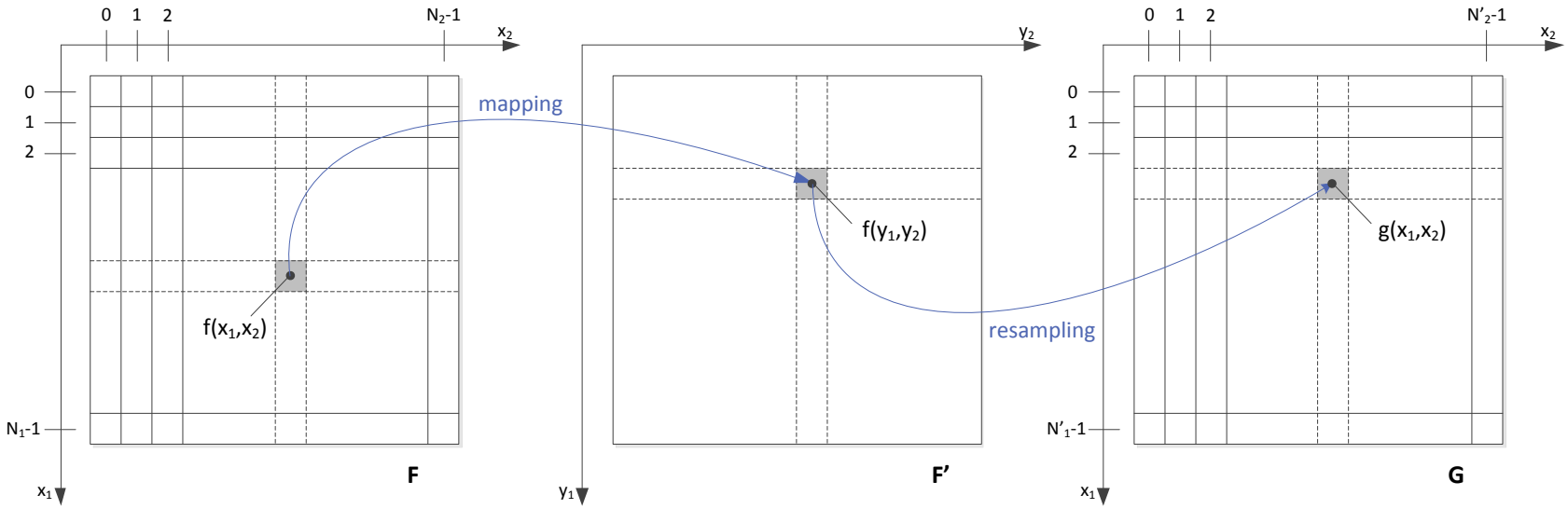
Digital image representation:

$$\begin{aligned} \mathbf{F}: &\quad \text{matrix, size } N_1 \times N_2. \\ \mathbf{x} = (x_1, x_2): &\quad \text{pixel location.} \\ f(x_1, x_2): &\quad \text{pixel intensity.} \end{aligned}$$

Properties:

$$(x_1, x_2) \in \mathcal{Z}^2, \quad f(x_1, x_2) \in \{0, \ldots, 255\}$$

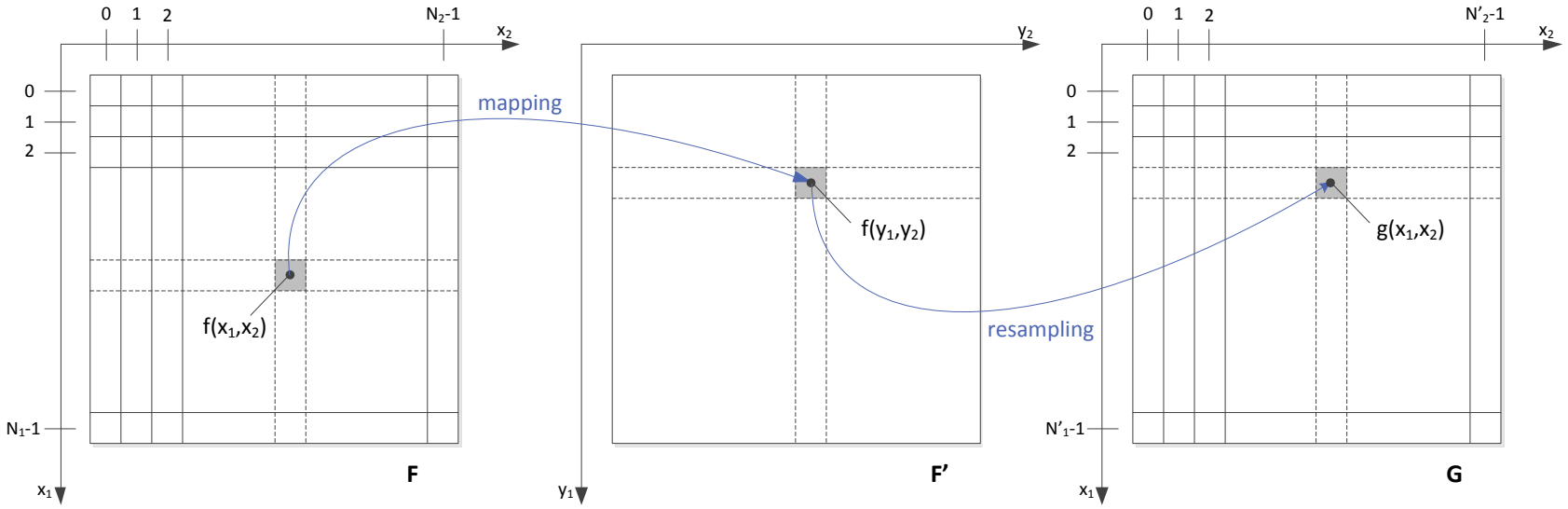
Spatial Transformations



Mapping for a spatial transformation:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{b}, \text{ where } \begin{cases} \text{Rotation} & \mathbf{A} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \\ \text{Scaling} & \mathbf{A} = \begin{bmatrix} N_{s_1} & 0 \\ 0 & N_{s_2} \end{bmatrix} \end{cases}$$

Spatial Transformations (II)



Resampling by a factor $\mathbf{n}_s = (L_1/M_1, L_2/M_2)$

$$g(x_1, x_2) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} f(i,j) h(x_1 M_1 - i L_1, x_2 M_2 - j L_2)$$

Question:

- $f(x_1, x_2)$ is periodically filtered with the same kernel $h(x_1, x_2)$. Will $g(x_1, x_2)$ exhibit periodically correlated fields?

Cyclostationary Approach

Definition

A random field $x(m_1, m_2)$ is **almost cyclostationary** with period (T_1, T_2) iff:

$$\mu_x(m_1, m_2) = \mu_x(m_1 + kT_1, m_2 + lT_2)$$
$$c_{xx}((m_1, m_2); \boldsymbol{\tau}) = c_{xx}((m_1 + kT_1, m_2 + lT_2); \boldsymbol{\tau}),$$

for all integers $m_1, m_2, \tau_1, \tau_2, k, l$ and rational numbers T_1, T_2.

Is a block $x(m_1, m_2)$ of $g(x_1, x_2)$, almost cyclostationary?

Considering $g(x_1, x_2) \sim \mathcal{N}(0, 1)$, then: $\mu_x(m_1, m_2) = 0$ and

$$c_{xx}((m_1, m_2); (\tau_1, \tau_2)) = E\{x(m_1, m_2)x(m_1 + \tau_1, m_2 + \tau_2)\}$$

$$= \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h(m_1 M_1 - iL_1, m_2 M_2 - jL_2) h((m_1 + \tau_1)M_1 - iL_1, (m_2 + \tau_2)M_2 - jL_2)$$

$$= c_{xx}((m_1 + kL_1/M_1, m_2 + lL_2/M_2); (\tau_1, \tau_2))$$

Cyclostationary Approach (II)

Unless the kernel used $h(x_1, x_2)$ is ideal,

$x(m_1, m_2)$ is almost cyclostationary with period $\mathbf{n}_s = (L_1/M_1, L_2/M_2)$.

How do we know $\mathbf{n}_s$?

From the estimation of the cyclic correlation, i.e. the Fourier Transform of $c_{xx}(\mathbf{m}; \boldsymbol{\tau})$:

$$\hat{C}_{xx}((\alpha_1, \alpha_2); (\tau_1, \tau_2)) = \frac{1}{N^2} \sum_{m_1=0}^{N-1} \sum_{m_2=0}^{N-1} x(m_1, m_2) x(m_1 + \tau_1, m_2 + \tau_2) e^{-j(\alpha_1 m_1 + \alpha_2 m_2)}$$

where the cyclic frequencies $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ are related with $\mathbf{n}_s$ as follows $\alpha_i = \frac{2\pi}{N_{s_i}}$.

Problem

The use of the estimator $\hat{C}_{xx}(\boldsymbol{\alpha}; \boldsymbol{\tau})$ works well for Gaussian fields, but not for **real images**.

Test for the presence of cyclostationarity

Dandawaté & Giannakis[3]: test for the presence of cyclostationarity.

Two-dimensional extension for images:

- $\hat{C}_{xx}(\boldsymbol{\alpha};\boldsymbol{\tau}) = C_{xx}(\boldsymbol{\alpha};\boldsymbol{\tau}) + e_{xx}(\boldsymbol{\alpha};\boldsymbol{\tau})$
- For different K lags $\{\boldsymbol{\tau_k}\}_{k=1}^{K}$, i.e.

$$\hat{\mathbf{c}}_{xx} = \frac{1}{\sqrt{2}}\left[\hat{C}_{xx}\left(\boldsymbol{\alpha};\boldsymbol{\tau}_1\right),\ldots,\hat{C}_{xx}\left(\boldsymbol{\alpha};\boldsymbol{\tau}_K\right),\hat{C}^{*}_{xx}\left(\boldsymbol{\alpha};\boldsymbol{\tau}_1\right),\ldots,\hat{C}^{*}_{xx}\left(\boldsymbol{\alpha};\boldsymbol{\tau}_K\right)\right]^{T}.$$

- **Hypothesis testing problem**:

$$\begin{aligned}
&\mathcal{H}_0 : \boldsymbol{\alpha} \notin \mathcal{A}_{xx}, \forall\{\boldsymbol{\tau}_k\}_{k=1}^{K} && \Rightarrow && \hat{\mathbf{c}}_{xx} = \mathbf{e}_{xx}\\
&\mathcal{H}_1 : \boldsymbol{\alpha} \in \mathcal{A}_{xx}, \text{for some } \{\boldsymbol{\tau}_k\}_{k=1}^{K} && \Rightarrow && \hat{\mathbf{c}}_{xx} = \mathbf{c}_{xx} + \mathbf{e}_{xx},
\end{aligned}$$

where $\mathcal{A}_{xx} \doteq \{\boldsymbol{\alpha} \ : \ C_{xx}(\boldsymbol{\alpha};\boldsymbol{\tau}) \neq 0, -\pi < \alpha_1, \alpha_2 \leq \pi\}$.

Test for the presence of cyclostationarity (II)

Characterization of the estimation error $\mathbf{e}_{xx}$:

- $N \to \infty \quad \Rightarrow \quad \mathbf{e}_{xx} \to \mathbf{0}$,
- $\lim_{N\to\infty} N\mathbf{e}_{xx} \stackrel{\mathcal{D}}{=} \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_{xx})$,
- $\mathbf{\Sigma}_{xx} = \lim_{N\to\infty} N^2 \mathrm{cov}\{\hat{\mathbf{c}}_{xx}, \hat{\mathbf{c}}_{xx}^H\}$.

Likelihood ratio test with a threshold Γ:

$$\mathcal{T}_{xx} = N^2 \hat{\mathbf{c}}_{xx}^H \hat{\mathbf{\Sigma}}_{xx}^{-1} \hat{\mathbf{c}}_{xx} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \Gamma, \begin{cases} \mathcal{H}_1: & \mathcal{T}_{xx} \sim \mathcal{N}(N^2 \hat{\mathbf{c}}_{xx}^H \hat{\mathbf{\Sigma}}_{xx}^{-1} \hat{\mathbf{c}}_{xx}, 4N^2 \hat{\mathbf{c}}_{xx}^H \hat{\mathbf{\Sigma}}_{xx}^{-1} \hat{\mathbf{c}}_{xx}) \\ \mathcal{H}_0: & \lim_{N\to\infty} \mathcal{T}_{xx} \stackrel{\mathcal{D}}{=} \chi^2_{2K} \end{cases}$$

Set the threshold Γ:

Probability of false alarm: $P_F = \Pr(\mathcal{T}_{xx} \geq \Gamma | \mathcal{H}_0) \quad \Leftrightarrow \quad P_F = \Pr(\chi^2_{2K} \geq \Gamma)$

Test for the presence of cyclostationarity (III)

For each frequency pair (α_1, α_2):

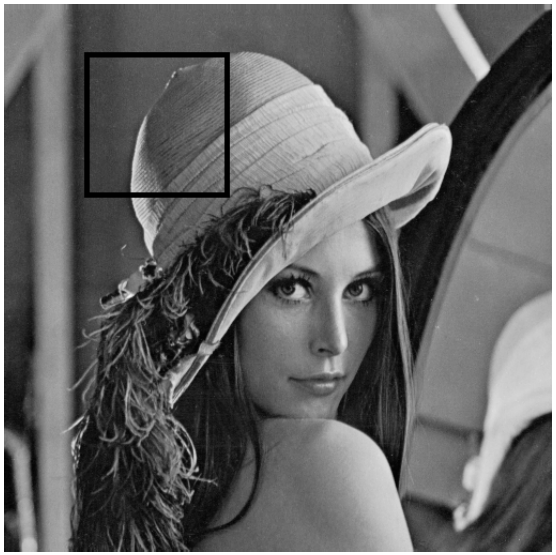
1. Compute $\hat{\mathbf{c}}_{xx}$ for a fixed set of K lags.
2. Estimate the asymptotic covariance matrix $\mathbf{\Sigma}_{xx}$.
   - Smoothed periodogram with a frequency window of size $P \times P$.
3. Calculate the test statistic $\mathcal{T}_{xx} = N^2 \hat{\mathbf{c}}_{xx}^H \hat{\mathbf{\Sigma}}_{xx}^{-1} \hat{\mathbf{c}}_{xx}$.
4. For a given probability of false alarm P_F, set Γ.
5. Declare the frequency pair (α_1, α_2) as cyclic if $\mathcal{T}_{xx} \geq \Gamma$

How do we apply this method to an image?

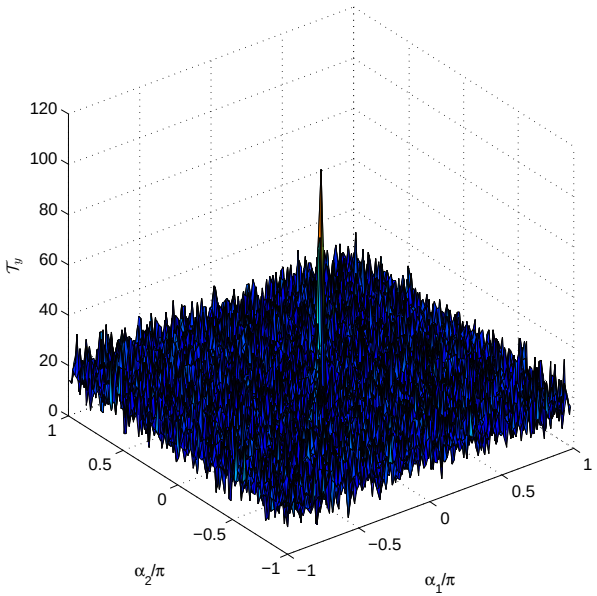
- Selection of the image block of size 128×128 for the analysis.
- Application of the previous algorithm $\Rightarrow$ (Detection)
- Identification of the spatial transformation $\Rightarrow$ (Estimation)

Experimental Results

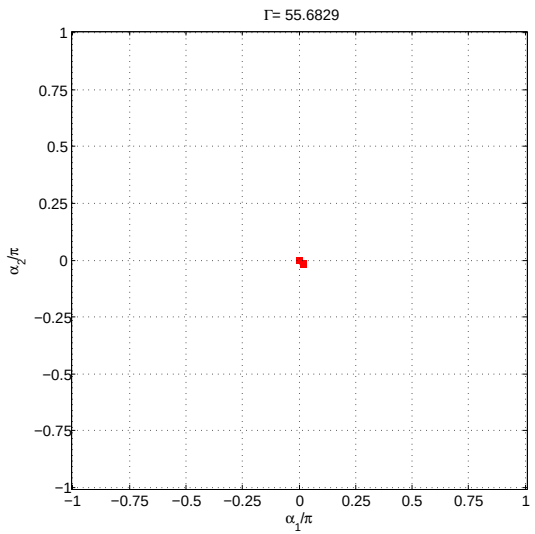
Example (I): original image.



(a) Original Image



(b) Analysis (128×128)

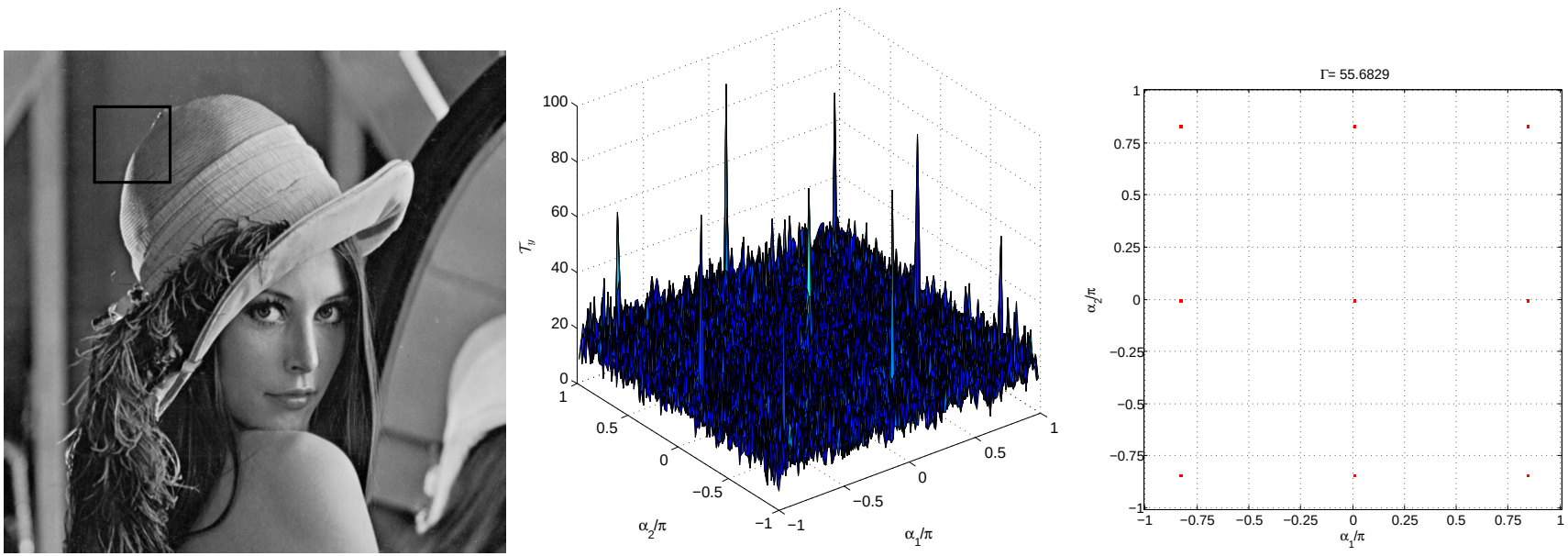


(c) $P_F = 10^{-5}$

Figure: Graphical results obtained with our method for an original image.

Experimental Results (II)

Example (II): scaled image by $\mathbf{n}_s = (1.7, 1.7)$.

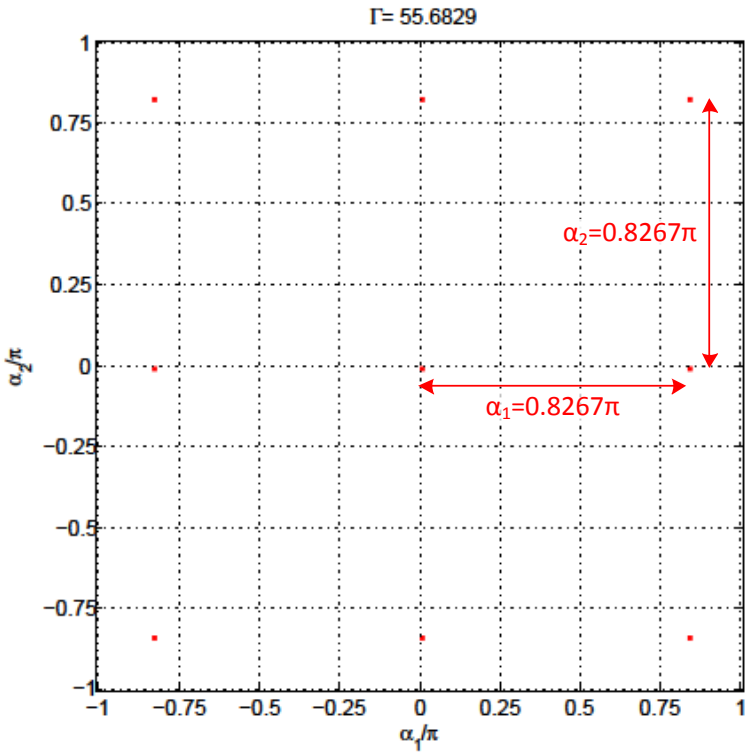


(a) Scaled Image (b) Analysis (128×128) (c) $P_F = 10^{-5}$

Figure: Graphical results obtained with our method for a scaled image.

Experimental Results (II)

Example (II): scaled image by $\mathbf{n}_s = (1.7, 1.7)$.



Cyclic frequencies detected.

Estimation of $\mathbf{n}_s = (N_{s_1}, N_{s_2})$:

- $\alpha_i = \frac{2\pi}{N_{s_i}}$, with $i = \{1, 2\}$.
- Aliasing: same set $\mathcal{A}_{xx}$ for N_{s_i} and $\frac{N_{s_i}}{N_{s_i}-1}$.

$$\hat{N}_{s_i} = \begin{cases} \dfrac{2\pi}{2\pi - |\alpha_i|}, & -\pi \leq \alpha_i \leq \pi \quad (\hat{N}_{s_i} \leq 2) \\ \dfrac{2\pi}{|\alpha_i|}, & -\pi \leq \alpha_i \leq \pi \quad (\hat{N}_{s_i} \geq 2) \end{cases}.$$

Example

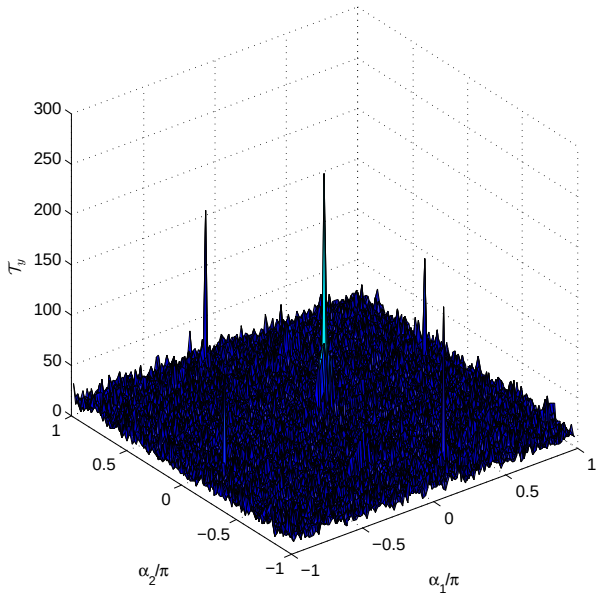
- $N_{s_i} = 1.7 < 2$, for $i = \{1, 2\}$.
- $\hat{N}_{s_1} = 2\pi/(2\pi - 0.8267\pi) = 1.7046$.
- $\hat{N}_{s_2} = 2\pi/(2\pi - 0.8267\pi) = 1.7046$.

Experimental Results (III)

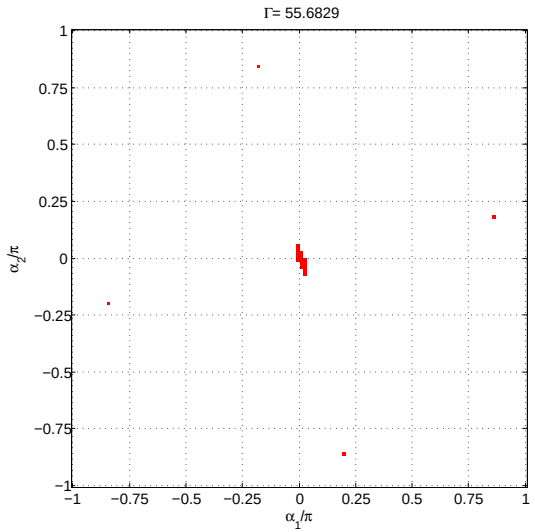
Example (III): rotated image by $\theta = -25^\circ$.



(a) Rotated Image



(b) Analysis (128×128)

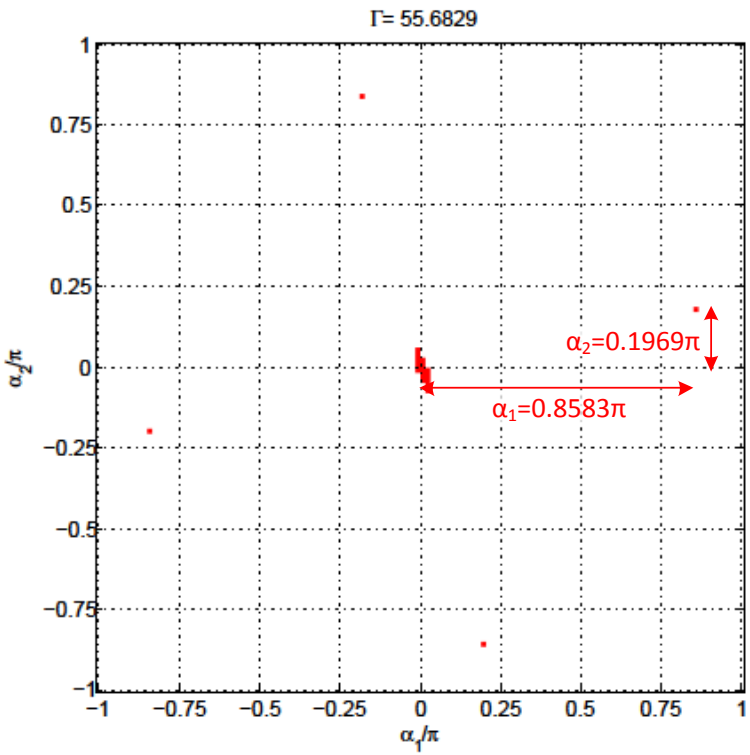


(c) $P_F = 10^{-5}$

Figure: Graphical results obtained with our method for a rotated image.

Experimental Results (III)

Example (III): rotated image by $\theta = -25^\circ$.



Cyclic frequencies detected.

Estimation of θ:

- Ambiguities for $\theta = -30^\circ$ and $\theta = -60^\circ$.
- $\beta = \arctan\left(\frac{\alpha_2}{\alpha_1}\right) \mod \frac{\pi}{2}$

$$\hat{\theta} = \begin{cases} -2\beta, & \text{if} \quad 0 \leq \beta \leq \frac{\pi}{12} \\ -\arccos(\kappa), & \text{if} \quad \frac{\pi}{12} < \beta \leq \frac{5\pi}{12} \\ \frac{\pi}{2} - 2\beta, & \text{if} \quad \frac{5\pi}{12} < \beta \leq \frac{\pi}{2} \end{cases},$$

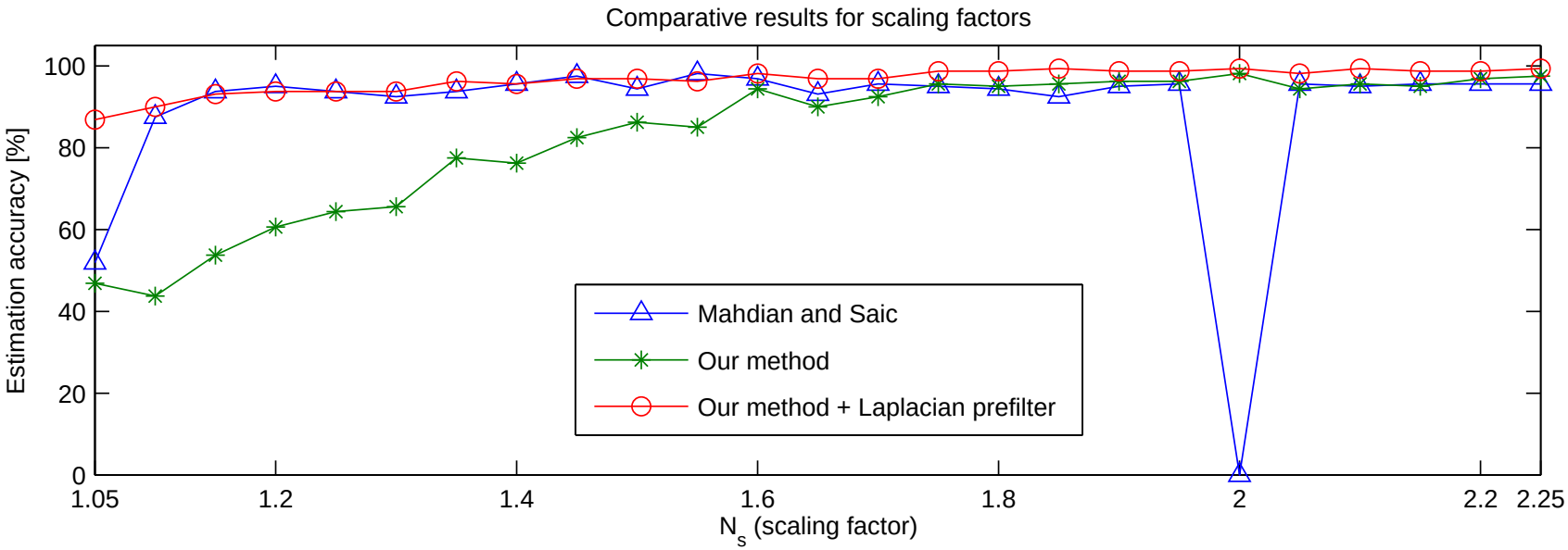
where $\kappa \doteq \cos^2(\beta)(\sqrt{2\tan(\beta)} - \tan(\beta) + \tan^2(\beta))$.

Example

- $\theta = -25^\circ$.
- $\beta = \arctan\left(\frac{\alpha_2}{\alpha_1}\right) \mod \frac{\pi}{2} = 0.2255 < \frac{\pi}{12}$.
- $\hat{\theta} = -2\beta = -0.4510 \quad \Leftrightarrow \quad \hat{\theta} = -25.841^\circ$.

Experimental Results (IV)

Comparative results (I):

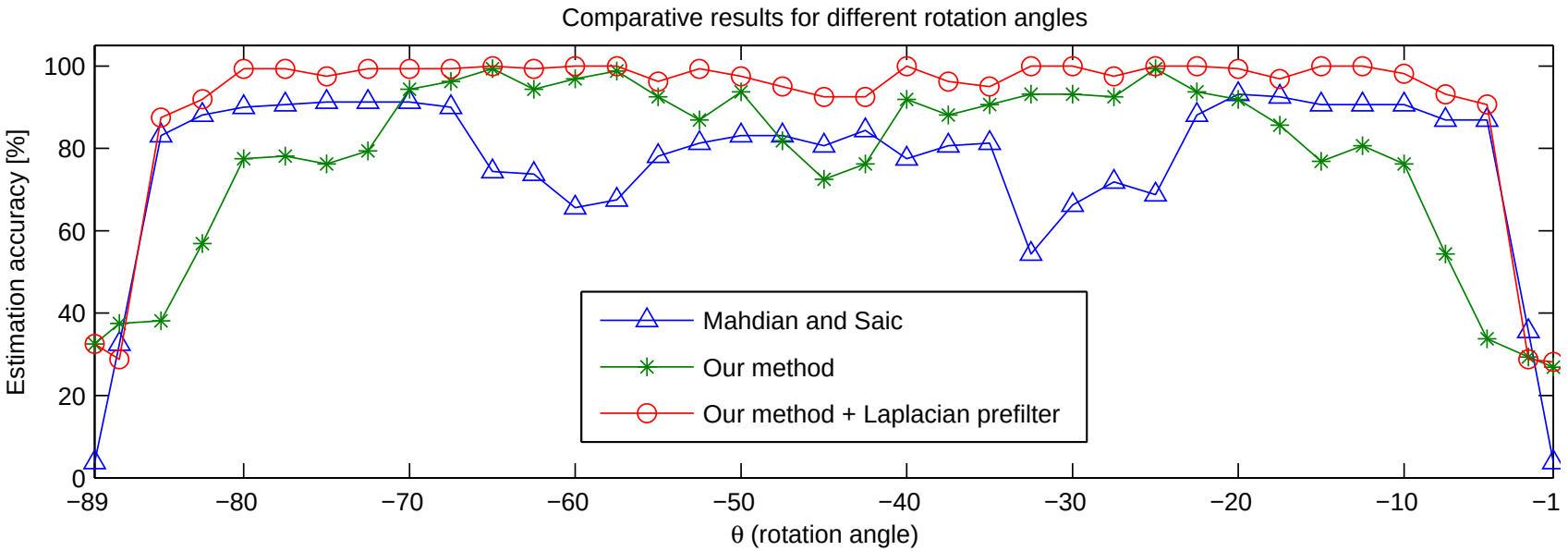


Parameters: $P = 11$, $K = 9$, $P_F = 10^{-4}$ and $\Gamma = 49.18$.

Figure: Comparative results for different scaling factors.

Experimental Results (V)

Comparative results (II):



Parameters: $P = 21$, $K = 9$, $P_F = 10^{-5}$ y $\Gamma = 55.68$.

Figure: Comparative results for different rotation angles.

Conclusions

Method to detect and estimate spatial transformations applied to digital images:

- Theoretical framework based on a cyclostationary analysis.
- Detection based on a fixed probability of false alarm.
- Two-dimensional processing $\Rightarrow$ less ambiguities.
- Two-dimensional processing $\Rightarrow$ more time consuming.

Future work:

- Explanation of the improvement aforded by the Laplacian prefilter.
- Is there an optimum prefilter for this method?
- Is there an optimum weighting matrix for the detection?

References


A.C. Popescu and H. Farid, "Exposing digital forgeries by detecting traces of resampling," in *IEEE Transactions on Signal Processing*, vol. 53, no. 2, pp. 758-767, Feb. 2005.


B. Mahdian and S. Saic, "Blind authentication using periodic properties of interpolation," in *IEEE Transactions on Information Forensics and Security*, vol. 3, no. 3, pp. 529-538, Sept. 2008.


A.V. Dandawaté and G.B. Giannakis, "Statistical tests for presence of cyclostationarity," in *IEEE Transactions on Signal Processing*, vol. 42, no. 9, pp. 2355-2369, Sept. 1994.

Thanks for your attention.

Questions?

David Vázquez-Padín
dvazquez@gts.tsc.uvigo.es