ON THE ROLE OF DIFFERENTIATION FOR RESAMPLING DETECTION

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ABSTRACT

Detection of resampling traces for digital image blind authentication has been addressed recently by A. C. Gallagher and later extended by B. Mahdian and S. Saic. On the other side, it is well known from the synchronization area in communications that prefiltering is an appropriate tool to improve the performance of those schemes exploiting the underlying cyclostationarity of communication signals. Thus, the detection of resampling manipulations improves significantly when the derivative of the interpolated signal is used for covariance computation. This work focuses on the role of prefiltering as a way of boosting resampling traces and, in particular, on the use of derivation.

***Index Terms*—** Image forensics, resampling detection, prefiltering, cyclostationarity, interpolation.

1. INTRODUCTION

Blind authentication of digital images has become an important research topic since the advent of inexpensive digital cameras and powerful editing software that eases the creation of fake images with real appearance. When a digital image is altered, certain transformations such as scaling, skewing or rotation (i.e., geometrical transformations) are very likely to be used. In each of these transformations a resampling process is applied [1]. Two main effective techniques have been developed aiming at detecting the resampling. A. C. Popescu and H. Farid [2] have proposed a method in which image tampering can be detected by finding a set of pixels that are correlated with their neighbors in the same way. This relation between neighbors is given by the interpolation process, that forms part of resampling along with decimation. Then, an expectation/maximization algorithm is used in order to generate a probability map which exposes periodicities depending on the mentioned correlations, allowing to detect traces of resampling and, with some ambiguities, the resampling factor. This method depends on certain initialization parameters that may produce different results for the same input; furthermore, the algorithm is computationally expensive. For this reason, we focus on the approach first introduced by A. C. Gallagher [3] and further developed by B. Mahdian and S. Saic [1], which is simple and computationally efficient. It is based on the fact that interpolated signals and its derivatives exhibit periodicities in their second order statistics. Such signals are called cyclostationary of order two in the wide sense [4]. Mentioned periodicities appear as lines in the spectrum of the covariance of the interpolated signals. The position of those lines in the spectrum is related to the resampling factor thus allowing for its estimation. A key element of the Mahdian-Saic method is the role of derivatives in the detection of the spectral lines, i.e., when interpolated signals are differentiated, spectral lines are enhanced. The importance of this enhancement becomes apparent when one considers that in many cases, the absence of a differentiation prefilter leads to a wrong detection. Although in [1] it is graphically shown that the application of a differentiator to the resampled signal enhances the amplitude of the spectral lines, no analytical explanation of this feature is provided. Moreover, the question of whether other prefilters may yield better results remains open.

Our contribution starts by writing the resampled signal as a baseband linearly modulated signal, such as PAM (pulse-amplitude-modulated). On this basis, it is possible to notice that blind feedforward synchronization schemes in communications, in particular those used for symbol rate estimation, also related to timing offset estimation [4] [5] [6], closely resemble the Mahdian-Saic method. In blind feedforward synchronization schemes, a nonlinearity is applied to the modulated signal in order to generate a spectral line whose position and phase are related to the symbol rate and timing offset, respectively. The nonlinearity is often a squaring operation; analogously, in the Mahdian-Saic method the variance of the signal is computed. The cyclic spectrum of a squared linearly modulated signal (resampled signal in our case) is composed by two terms: the spectral lines (due to resampling) and a so-called self-noise term [4]. This self-noise is due to the random nature of the signal and can seriously degrade the spectral line of interest. In the context of symbol timing recovery, the quadrature (i.e., imaginary) component of self-noise causes jitter in the estimated spectral peak phase. This jitter can be eliminated by cancelling the quadrature component of self-noise; this in turn can be achieved if certain conditions on the transmission pulse shape (interpolation kernel in our case) are fulfilled. In order to modify the pulse shape [7] [8] suggest the use of a prefilter, i.e., a filter applied to the signal before squaring. Knowing this, we can think that differentiation, interpreted as a prefiltering operation, tend to modify the interpolation kernel in such a way that self-noise is reduced. In this paper we prove that differentiators used as prefilters are nearly optimal with respect to the minimization of the power of the in-phase and quadrature components of the self-noise.

A linearly modulated signal can be expressed as follows:

$$x(lT_s) = \sum_m \alpha_m g(lT_s - mT - eT)$$

where x, T_s, T, l, α_m, $g(.)$, and e, are respectively, the linearly modulated signal sampled, the sampling period, the symbol period, the sample index, the symbols amplitude, the pulse shape, and the timing symbol delay. The oversampling factor is defined as $P := T/T_s$; then, an analogy with a resampled signal can be done if we consider

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that the symbols amplitude α_m correspond to the pixel intensities, $g(.)$ is regarded as the interpolation kernel, and x the resampled signal with rate $P := L/M$, where, in the context of resampling,L is the up-sampling factor and M the down-sampling factor. Then, if without loss of generality we make $T_s = 1$ the resampled signal can be written as a function of P as follows:

$$x(l) = \sum_m \alpha_m g(l - mP - eP).$$

In the next section we introduce a resampled image model. In Sect. 3 we discuss the direct minimization of the self-noise power of in-phase and quadrature components. In Sect. 4 we provide alternative cost functions to achieve the nearly self-noise power optimization. In Sect. 5 we verify the obtained results. Finally, the conclusions are presented in Sect. 6.

2. RESAMPLED IMAGE MODEL

In order to simplify the following discussion, a one-dimensional resampled image model is presented in the frequency domain. We remark that our approach can be straightforwardly extended to two dimensions. Fig. 1 shows the model block diagram. The first block corresponds to a natural image model without compression whose input is a white Gaussian process with zero mean and unit variance. For this case, a simple and convenient approximation is a first-order autoregressive model (AR(1)) with correlation coefficient ρ=0.95, whose transfer function is denoted as $H_{AR}(\omega)$ [9] [10]. The resampling block comprises an expander by a factor L, an interpolation filter $H(\omega)$ and a decimator by a factor M [11]. The third block represents the transfer function of the FIR prefilter $H_{pre}(\omega)$, with coefficients a_k, $k = 0, ..., n$. For the purpose of the design of this prefilter, and without loss of generality, we assume $a_0 = 1$. The overall system comprising the AR filter, the sampling rate conversion and the prefilter is denoted as $G(\omega)$, which is the Fourier Transform of the modified interpolation kernel $g(l)$ that includes the effect of the AR(1) model and the prefilter, in this way we can use expressions of communication framework directly:

$$G(\omega) = H_{AR}\left(\frac{\omega L}{M}\right) H\left(\frac{\omega}{M}\right) H_{pre}(\omega) = H\left(\frac{\omega}{M}\right) \frac{\sum_{k=0}^{n} a_k e^{-j\omega k}}{1 - \rho e^{-j\omega L/M}}. \quad (1)$$

If we denote $D(\omega) := \frac{H(\omega/M)}{1-\rho e^{-j\omega L/M}}$, then (1) can be written as

$$G(\omega) = \sum_{k=0}^{n} a_k D(\omega) e^{-j\omega k}. \quad (2)$$

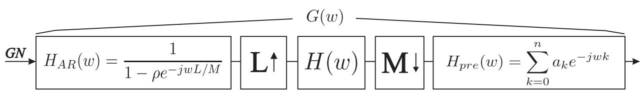

Fig. 1. Resampled image model.

3. MINIMIZATION OF IN-PHASE AND QUADRATURE SELF-NOISE POWER

The use of prefilters and postfilters to enhance synchronization performance has been common among communication practitioners for years. Thus, classical analog timing recovery schemes employed a bandpass postfilter centered at the symbol rate frequency, $1/T$, after squaring the signal, thus filtering out unimportant components for timing extraction. Prefiltering can be also beneficial for timing recovery, as illustrated in [8]. However, symbol rate estimation or the corresponding resampling estimation under study cannot benefit from the same filters, at least as a first approach, since the center frequency $2\pi/P$ depends on the parameter to be estimated, and hence it is unknown. Thus, we assume that no postfilter exists, and focus on the design of an appropriate prefilter. Based on [4], the power of the in-phase and quadrature self-noise components derived from its phasor representation with respect to the spectral line at $2\pi/P$ at the output of $G(w)$ are, respectively,

$$E[I^2] = \tfrac{\pi(M-1)}{2P} \int_{-\pi}^{\pi} \left| G_0\left(\tfrac{2\pi}{P} + \omega\right) + G_0^*\left(\tfrac{2\pi}{P} - \omega\right)\right|^2 d\omega \quad (3)$$

$$+ \tfrac{2\pi}{P} \sum_{q=1}^{\infty} \int_{-\pi}^{\pi} \left| G_q\left(\tfrac{2\pi}{P} + \omega\right) + G_q^*\left(\tfrac{2\pi}{P} - \omega\right)\right|^2 d\omega,$$

$$E[Q^2] = \tfrac{\pi(M-1)}{2P} \int_{-\pi}^{\pi} \left| G_0\left(\tfrac{2\pi}{P} + \omega\right) - G_0^*\left(\tfrac{2\pi}{P} - \omega\right)\right|^2 d\omega \quad (4)$$

$$+ \tfrac{2\pi}{P} \sum_{q=1}^{\infty} \int_{-\pi}^{\pi} \left| G_q\left(\tfrac{2\pi}{P} + \omega\right) - G_q^*\left(\tfrac{2\pi}{P} - \omega\right)\right|^2 d\omega,$$

where $G_q(\omega) \doteq \int_{-\pi}^{\pi} G(\frac{\omega}{2} + \theta) G(\frac{\omega}{2} - \theta) e^{\pm 2\pi j\theta P q} d\theta$, $E[\cdot]$ stands for the expectation operator, $M \doteq E[\alpha_m^4]$.

Now, if we want to find the prefilter coefficients a_k which minimize $E[I^2]$, we can use (2) to rewrite $G_q(\omega)$ as

$$G_q(\omega) = \sum_{k=0}^{n} \sum_{r=0}^{n} a_k a_r \beta_{k,r}^q(\omega) \quad (5)$$

where

$$\beta_{k,r}^q(\omega) \doteq \int_{-\pi}^{\pi} D\left(\tfrac{\omega}{2} + \theta\right) D\left(\tfrac{\omega}{2} - \theta\right) e^{-j[(\frac{\omega}{2}+\theta)k + (\frac{\omega}{2}-\theta)r \mp 2q\pi\theta]} d\theta.$$

Thus, $E[I^2]$ in (3) can be expressed as an explicit function of the prefilter coefficients a_k:

$$E[I^2] = \frac{2\pi(M-1)}{4P} \sum_{k=0}^{n} \sum_{r=0}^{n} \sum_{i=0}^{n} \sum_{s=0}^{n} a_k a_r a_i a_s \xi_{k,r,i,s}^0 \quad (6)$$

$$+ \frac{2\pi}{P} \sum_{q=0}^{\infty} \sum_{k=0}^{n} \sum_{r=0}^{n} \sum_{i=0}^{n} \sum_{s=0}^{n} a_k a_r a_i a_s \xi_{k,r,i,s}^q$$

with:

$$\xi_{k,r,i,s}^q = \int_{-\pi}^{\pi} \beta_{k,r}^q\left(\tfrac{2\pi}{P} + \omega\right)^* \beta_{i,s}^q\left(\tfrac{2\pi}{P} + \omega\right) + 2Re[\beta_{k,r}^q\left(\tfrac{2\pi}{P} - \omega\right) \beta_{i,s}^q\left(\tfrac{2\pi}{P} + \omega\right)] + \beta_{k,r}^q\left(\tfrac{2\pi}{P} - \omega\right)^* \beta_{i,s}^q\left(\tfrac{2\pi}{P} - \omega\right)\} d\omega. \quad (7)$$

A closed-form solution for the minimization of (6) cannot be obtained, with similar considerations for the quadrature self-noise component. However, nearly optimum coefficients can be obtained as detailed in the following section.

4. SPECTRAL SYMMETRY AND ENERGY CONDITIONS

Quadrature self-noise power in (4) can be made equal to zero provided that $G_q(2\pi/P+\omega)=G_q^*(2\pi/P-\omega)$, that is, $G_q(\omega)$ must be Hermitian with respect to the frequency $2\pi/P$. Equivalently, $G(\pi/P+\omega)=G^*(\pi/P-\omega)$ [4, pp. 363-389], that is, Hermitian with respect to π/P. This conjugate symmetry condition around π/P allows to eliminate the quadrature component of the self-noise as was pointed out in [7]. Analogously, $G_q(2\pi/P+\omega)=-G_q^*(2\pi/P-\omega)$ nulls out the in-phase self-noise component in (3). Unfortunately, this latter condition cannot be achieved for real signals, since both real and imaginary parts would need to be equal. In consequence, given that it is not possible to cancel both components (3) and (4), we propose the minimization of the following cost function to get a solution with a high degree of symmetry:

$$\varepsilon_Q=\int_0^{\pi/P}\left|G\left(\frac{\pi}{P}+\omega\right)-G^*\left(\frac{\pi}{P}-\omega\right)\right|^2 d\omega \tag{8}$$

We restrict the integration upper limit given the low-pass character of interpolation filters. Now, we insert (2) into (8), which can be rewritten as

$$\varepsilon_Q=\sum_{k=0}^{n}\sum_{r=0}^{n}a_k.a_r\int_0^{\pi/P}C_k(\omega)C_r(\omega)^*d\omega \tag{9}$$

where

$$C_k(\omega)\doteq\left[D\left(\tfrac{\pi}{P}-\omega\right).e^{-j\frac{\pi}{P}k}-D\left(\tfrac{\pi}{P}+\omega\right)^*.e^{j\frac{\pi}{P}k}\right]e^{j\omega k} \tag{10}$$

If we make $Y_{k,r}\doteq\int_0^{\pi/P}C_k(w)C_r^*(w)dw$ and note that $Y_{k,r}=Y_{r,k}^*$, then ε_Q can be put as an explicit function of the prefilter coefficients:

$$\varepsilon_Q=Y_{0,0}+2a_iRe\{Y_{i,0}\}+2\sum_{k=1,k\neq i}^{n}a_kRe\{Y_{0,k}\}+a_i^2Y_{i,i}$$
$$+2\sum_{k=1,k\neq i}^{n}a_ia_kRe\{Y_{i,k}\}+\sum_{k=1,k\neq i}^{n}\sum_{r=1,r\neq i}^{n}a_ka_rY_{k,r} \tag{11}$$

from which

$$\bar{a}_i=\arg\min_{a_i}\varepsilon_Q=-\frac{Re(Y_{i,0})}{Y_{i,i}}-\sum_{k=1,k\neq i}^{n}a_k\frac{Re(Y_{i,k})}{Y_{i,i}}$$
$$\doteq-A_{i,0}-\sum_{k=1,k\neq i}^{n}a_kA_{i,k}. \tag{12}$$

We could also devise the corresponding cost function to (8) to try to approach the condition $G_q(2\pi/P+\omega)=-G_q^*(2\pi/P-\omega)$ which would eliminate the in-phase component. This is not possible with real coefficients, so we propose to minimize the energy of $G(\omega)$:

$$\varepsilon_I=\textstyle\int_{-\pi}^{\pi}|G(\omega)|^2\,d\omega. \tag{13}$$

Following similar steps as for the case of ε_Q, (13) can be expressed as

$$\varepsilon_I=\sum_{k=0}^{n}\sum_{r=0}^{n}a_ka_rZ_{k,r} \tag{14}$$

being $Z_{k,r}=\int_{-\pi}^{\pi}|D(\omega)|^2e^{j\omega(r-k)}d\omega$. Then, the minimization of ε_I is given by

$$\bar{a}_i=\arg\min_{a_i}\varepsilon_I=-\frac{Re(Z_{i,0})}{Z_{i,i}}-\sum_{k=1,k\neq i}^{n}\frac{a_kRe(Z_{i,k})}{Z_{i,i}}$$
$$\doteq-B_{i,0}-\sum_{k=1,k\neq i}^{n}a_kB_{i,k}. \tag{15}$$

As detailed next, the terms $A_{i,k}$ and $B_{i,k}$ get asymptotically to one as the resampling factor P goes to infinity.

4.1. Asymptotic Results

Property 1 *Let* $A_{i,k}=Re(Y_{i,k})/Y_{i,i}$. *Then* $\lim_{P\to\infty}A_{i,k}=1,\ \forall\ i,k$.

Proof: We have that $A_{i,k}$ is given by

$$A_{i,k}=\lim_{P\to\infty}\frac{\int_0^1 Re\{C_i\left(v\frac{\pi}{P}\right)\cdot C_k^*\left(v\frac{\pi}{P}\right)\}dv\frac{\pi}{P}}{\int_0^1\left|C_k\left(v\frac{\pi}{P}\right)\right|^2dv\frac{\pi}{P}}$$

where

$$Re\{C_i\left(\tfrac{v\pi}{P}\right)C_k^*\left(\tfrac{v\pi}{P}\right)\}$$
$$=Re\{\left[D\left(\tfrac{\pi}{P}(1-v)\right)e^{-j\frac{\pi}{P}i}-D^*\left(\tfrac{\pi}{P}(1+v)\right)e^{j\frac{\pi}{P}i}\right]$$
$$\times\left[D\left(\tfrac{\pi}{P}(1-v)\right)e^{-j\frac{\pi}{P}k}-D^*\left(\tfrac{\pi}{P}(1+v)\right)e^{j\frac{\pi}{P}k}\right]^*e^{j(i-k)v\frac{\pi}{P}}\}$$

and

$$\left|C_k\left(\tfrac{v\pi}{P}\right)\right|^2=\left|D\left(\tfrac{\pi}{P}(1-v)\right)e^{-j\frac{\pi}{P}k}-D\left(\tfrac{\pi}{P}(1+v)\right)^*e^{j\frac{\pi}{P}k}\right|^2$$

Since $|C(0)|\neq 0$, and as $P\to\infty$ the exponentials in the integrands of both the numerator and denominator approach 1, then $Re\{C_i(v\pi/P)C_k^*(v\pi/P)\}$ becomes equal to $|C_k(v\pi/P)|^2$ and, hence, $A_{i,k}\to 1$ for all i,k.

Therefore, for $P\to\infty$, the prefilter coefficients must satisfy the equations

$$\bar{a}_i=-1-\sum_{k=1,k\neq i}^{n}\bar{a}_k,\forall\ i=1,...,n. \tag{16}$$

For the case $n=1$ we have $\bar{a}_0=1$ and $\bar{a}_1=-1$, that is, the differentiation filter $H_{pre}(z)=1-z^{-1}$.

Analogously, $B_{i,k}$ goes also asymptotically to one, leading in turn to (16). Thus, we have seen how different approximate criteria end up asymptotically with the same set of coefficients, which for a first-order prefilter boils down to a derivative.

5. RESULTS

We show in Figure 2 the prefilter coefficients with respect to the resampling factor P for a first-order filter, which according to the above results are given by $\bar{a}_1=-A_{1,0}$ or $\bar{a}_1=-B_{1,0}$, with $\bar{a}_0=1$. As expected, both cases approach asymptotically the differentiation filter, thus supporting the use of derivatives to enhance spectral lines due to resampling operations. Although not shown, similar

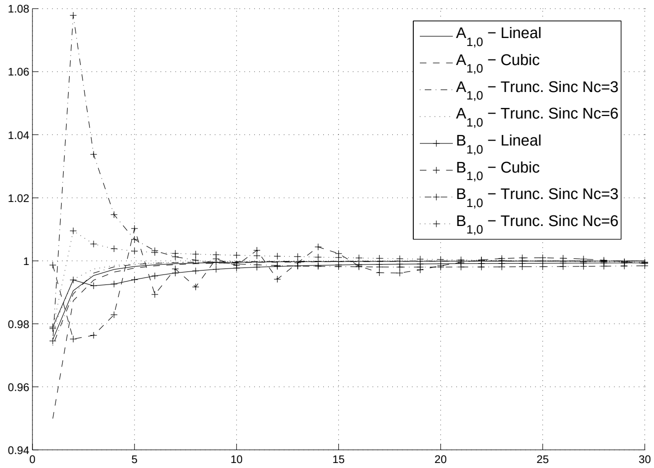

Fig. 2. Factors $A_{1,0}$ and $B_{1,0}$.

conclusions can be drawn from higher order cases. Additionally we have carried out some Monte Carlo simulations to characterize in-phase and quadrature self-noise powers with respect to prefilter coefficients. Figure 3 shows $E[I^2] + E[Q^2]$ for a three coefficient prefilter $[1, a_1, a_2]$ after averaging the corresponding results for $P = 3, 7, 12$ and different interpolation kernels (Truncated Sinc lobes, Cubic and linear). The curves correspond to filter coefficients with the same self-noise level, with a minimum close to the second-order differentiation filter $[1, -2, 1]$. In fact, as depicted in Figure 3, self-noise is almost at its minimum level all along the line $a_1 + a_2 = -1$; interestingly, this is the set of solutions arising from (16), thus opening the set of prefilters to be used before resampling detection.

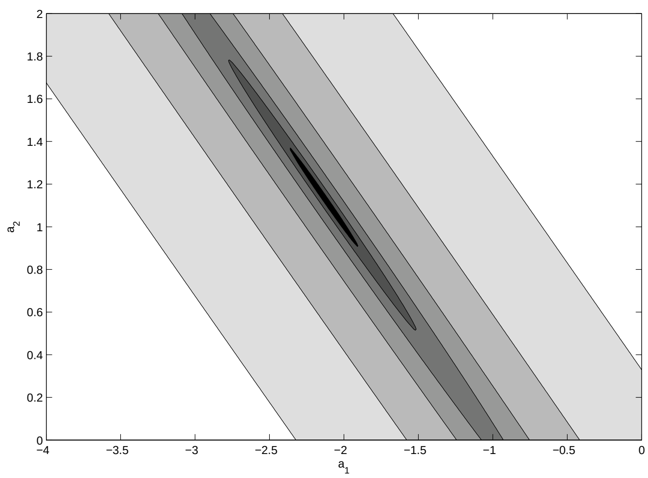

Fig. 3. Self-noise power for a three coefficient prefilter. Lower error values are displayed with darker gray.

6. CONCLUSIONS

Previous works on estimation of resampling factor P resorted to the use of differentiation prefilters to enhance performance by improving spectral lines visibility, although not clear reasons were given. In this paper the use of derivatives has been analytically supported, at least for asymptotically large P, and practical rules for the design of prefilters for practical values of P have been obtained, thus extending the range of possible filters which can be used.

7. REFERENCES

[1] M. Babak and S. Stanislav, "Blind authentication using periodic properties of interpolation," in *IEEE Transactions on Information Forensics and Security*, 2008, vol. 3, pp. 529–538.

[2] A. Popescu and H. Farid, "Exposing digital forgeries by detecting traces of re-sampling," in *IEEE Transactions on Signal Processing*, 2005, vol. 53, pp. 758–767.

[3] A. C. Gallagher, "Detection of linear and cubic interpolation in jpeg compressed images," in *Proceedings of the The 2nd Canadian Conference on Computer and Robot Vision*, 2005, p. 6572.

[4] W. A. Gardner, "Cyclostationarity in communications and signal processing," in *Ed. New York: IEEE Press*, 1994.

[5] Kai Shi, Yan Wang, and Erchin Serpedin, "On the design of digital blind feedforward, nearly jitter-free timing recovery scheme for linear modulations," in *IEEE Transactions on Communications*, 2004, vol. 52.

[6] C. Mosquera, S. Scalise, and R. López-Valcarce, "Non-data-aided symbol rate estimation of linearly modulated signals," in *IEEE Transactions on Signal Processing*, 2008, vol. 56.

[7] L. E. Franks and J. P. Bubrouski, "Statistical properties of timing jitter in a PAM timing recovery scheme," in *IEEE Trans on Comm.*, 1974, vol. COM-22, pp. 913–920.

[8] A. N. D′Andrea, U. Mengali, and Moro M., "Nearly optimum prefiltering in clock recovery," in *IEEE Trans. on Comm*, 1986, vol. COM-34, pp. 1081–88.

[9] O. Koval, S. Voloshynovskiy, F. Pérez-González, F. Deguillaume, and T. Pun, "Spread spectrum watermarking for real images: Is everything so hopeless?," in *XII European Signal Processing Conference*, 2004.

[10] J. Liang and T. D. Tran, "Fast multiplierless approximations of the dct with the lifting scheme," in *IEEE Transactions on Signal Processing*, 2002, vol. 49, pp. 3032–3044.

[11] Ramesh A. Gopinath and C. Sidney Burrus., "On upsampling, downsampling, and rational sampling rate filter banks," in *IEEE Transactions on Signal Processing*, 2008, vol. 56.