DISTRIBUTED AOA-BASED SOURCE POSITIONING IN NLOS WITH SENSOR NETWORKS

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***Abstract*—This paper focuses on the problem of positioning a source using angle-of-arrival measurements taken by a wireless sensor network in which some of the nodes experience non line-of-sight (LOS) propagation conditions. In order to mitigate the errors induced by the nodes in NLOS, we derive an algorithm that combines the expectation-maximization algorithm with a weighted least-squares estimation of the source position so that the nodes in NLOS are eventually identified and discarded. Moreover, a distributed version of this algorithm based on a diffusion strategy that iteratively refines the position estimate while driving the network to a consensus is presented.**

***Index Terms*—positioning, AOA, NLOS, distributed estimation, sensor networks.**

I. INTRODUCTION

Wireless sensor networks (WSNs) consist of a large number of inexpensive sensor nodes deployed over an area to monitor the environment. Each sensor has limited processing capability and communicates with other sensors, which allows for the implementation of distributed processing techniques with the potential to reduce the overall power consumption and offer an increased robustness against node failures. WSNs are especially suited for the positioning of a transmitting source, which is usually needed in surveillance and military applications [1], thanks to their ability to cover large areas at a minimum cost.

Source positioning is a well-studied topic [2] with many applications in today's wireless networks [3]. Most positioning techniques rely on the measurement of the time of arrival (TOA), received signal strength (RSS) or angle of arrival (AOA) to locate the source. Although AOA-based positioning is usually not considered for WSNs due to the need to equip each node with an antenna array, the usage of these antennae is both feasible and desirable [4]. In the positioning of a source, the AOA at each node restricts the position of the source to a line in the direction of the measured angle. This line is called line of bearing (LOB), and the point at which the LOB from two or more nodes cross indicates the position of the source. In [5], a closed-form least squares (LS) solution for the positioning of a source using AOA measurements is proposed. In [6], a similar method is used to locate the nodes in a network with the AOA measurements taken by several anchor nodes.

One of the main sources of error in AOA-based positioning is the lack of direct visibility between the transmitter and the receiver, which is known as NLOS. NLOS identification and mitigation techniques have been widely studied for TOA-based positioning [7]. Other hybrid approaches combine TOA, AOA and RSS measurements to identify the nodes in NLOS [8] and mitigate the NLOS error [9]. In [10], a simple outlier detection problem is proposed to identify and discard AOA measurements taken in NLOS conditions based on the fact that these exhibit a completely different statistical behavior. Another option is the use of machine learning algorithms for the identification of outliers in WSNs, as proposed in [11]. In general, it is well-known that NLOS mitigation techniques require a large number of samples in order to effectively reduce the estimation error [9].

In this paper we present an algorithm to estimate the position of a source based on AOA measurements taken by a WSN, with some nodes experiencing NLOS propagation conditions. Further, we propose a distributed implementation of the algorithm suited for WSNs in which each node can only communicate with the neighbors located within a given range. The proposed algorithm is based on the LS estimator from [5] embedded in a difussion-based distributed algorithm [12]. As it will be seen, each node is able to estimate its own LOS probability and reach a final consensus with the other nodes on the source position estimate.

This paper is organized as follows. Section II introduces the signal model. The centralized estimator is derived in section III, and its distributed version is presented in Section IV. Section V includes the simulation results and, finally, the conclusions are drawn in Section VI.

II. SIGNAL MODEL

Consider a set of N nodes with known positions $\{\boldsymbol{u}_i = [x_i, y_i]^T \ \forall i = 1, \dots, N\}$ deployed to estimate the position of a source located at $\boldsymbol{r} = [x, y]^T$. Each node measures the AOA of a signal transmitted by the source. Most nodes are in LOS propagation conditions, where their measured AOA is assumed to be corrupted by zero-mean additive Gaussian noise with variance σ^2. We also assume that a fraction of the nodes experience an NLOS propagation channel and their estimated AOA follows a uniform distribution within the range of $[-\pi, \pi)$ [13]. Thus, the estimated AOA at node i can be expressed as

$$z_i = a_i \cdot \psi_i + (1 - a_i) \cdot \mu_i, \qquad i = 1, \dots, N, \tag{1}$$

where $\{a_i, \forall i\} = \{0, 1\}$ are independent identically distributed (iid) Bernoulli random variables (rv's) with probability of LOS $p \triangleq \Pr\{a_i = 1\}$, $\{\mu_i, \forall i\}$ are iid rv's uniformly

This work is supported by the "Ministerio de Economía y Competitividad" of the Spanish Government and ERDF funds (TEC2013-41315-R DISNET, TEC2013-47020-C2-1-R COMPASS and CONSOLIDER INGENIO 2010 CSD2008 00010 COMONSENS), the Catalan Government (2014 SGR 60 AGAUR) and the Galician Government (CN 2012/260 AtlantTIC, TACTICA, GRC2013/009).

distributed in $[-\pi, \pi)$, and $\{\psi_i, \forall i\}$ are iid Gaussian rv's with mean α_i, variance σ^2 and probability density function (PDF)

$$f(\psi_i) = \begin{cases} \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z_i-\alpha_i-2\pi n)^2}{2\sigma^2}} & \text{if } -\pi \leq \psi_i < \pi; \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $\{\alpha_i, \forall i\}$ are the true AOAs, equal to

$$\alpha_i = \arctan\left(\frac{x_i - x}{y_i - y}\right). \tag{3}$$

We further assume that the sets $\{a_i, \forall i\}$, $\{\mu_i, \forall i\}$ and $\{\psi_i, \forall i\}$ are mutually independent. Thus, a value of $a_i = 1$ indicates that node i is in LOS and, conversely, in NLOS if $a_i = 0$. Denoting $\boldsymbol{z} = [z_1, \ldots, z_N]^T$ and approximating $f(\psi)$ for its value at $n = 0$, the PDF of $\boldsymbol{z}$ is

$$f(\boldsymbol{z}|\boldsymbol{\theta}) = \prod_{i=1}^{N} \left[\frac{p}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z_i-\alpha_i)^2}{2\sigma^2}} + (1-p)g(z_i)\right], \tag{4}$$

where $\boldsymbol{\theta} = [x, y, p, \sigma^2]^T$ is the vector of parameters and $g(z_i)$ is the PDF of a uniform rv in $[-\pi, \pi)$, i.e.

$$g(z_i) = \frac{1}{2\pi} \prod \left(\frac{z_i}{2\pi}\right). \tag{5}$$

The parameter to be estimated is the position of the source $\boldsymbol{r}$, which is embedded in (4) through (3), whereas σ^2 and p are regarded as unknown *nuisance* parameters.

III. CENTRALIZED ALGORITHM

In this section we derive a centralized algorithm that estimates the source position and nuisance parameters assuming the AOA measurements taken by all the nodes, i.e. $\boldsymbol{z}$, are available at a central entity. The algorithm estimates the position of the source iteratively by solving a weighted least squares (WLS) problem whose parameters are estimated using the expectation-maximization (EM) algorithm, an iterative algorithm to compute the maximum likelihood (ML) estimate in the presence of unobserved data [14], [15]. In our case, the unobserved data correspond to the LOS/NLOS state of the nodes. We regard the observation vector $\boldsymbol{z}$ as the *incomplete* observation and $\{\boldsymbol{z}, \boldsymbol{a}\}$ as the *complete* one, where $\boldsymbol{a} = [a_1, \cdots, a_N]^T$. At iteration t, the EM algorithm performs the following:

1) *E-step:* given an estimate $\hat{\boldsymbol{\theta}}_t = [\hat{x}_t, \hat{y}_t, \hat{\sigma}_t^2, \hat{p}_t]^T$, compute the conditional expectation

$$Q(\tilde{\boldsymbol{\theta}}\,;\, \hat{\boldsymbol{\theta}}_t) = \mathbb{E}_{\boldsymbol{a}}\{\log f(\boldsymbol{z}, \boldsymbol{a} \,|\, \tilde{\boldsymbol{\theta}}) \mid \hat{\boldsymbol{\theta}}_t, \boldsymbol{z}\}, \tag{6}$$

where $\tilde{\boldsymbol{\theta}} = [\tilde{x}, \tilde{y}, \tilde{\sigma}^2, \tilde{p}]^T$ denotes a trial value of $\boldsymbol{\theta}$.

2) *M-step:* obtain the estimate for the next iteration as

$$\hat{\boldsymbol{\theta}}_{t+1} = \arg\max_{\tilde{\boldsymbol{\theta}}} Q(\tilde{\boldsymbol{\theta}}\,;\, \hat{\boldsymbol{\theta}}_t). \tag{7}$$

Let $\hat{a}_{i,t} \triangleq \Pr\left\{a_i = 1 \,|\, \hat{\boldsymbol{\theta}}_t, z_i\right\}$ denote the a posteriori probability of a_i being equal to one at iteration t. After some algebra (6) becomes

$$\begin{aligned} Q(\tilde{\boldsymbol{\theta}}\,;\, \hat{\boldsymbol{\theta}}_t) &= -\frac{1}{2}\log 2\pi\tilde{\sigma}^2 \sum_{i=1}^{N} \hat{a}_{i,t} \\ &\quad - \frac{1}{2\tilde{\sigma}^2} \sum_{i=1}^{N} \hat{a}_{i,t}(z_i - \tilde{\alpha}_i)^2 + \log\tilde{p}\sum_{i=1}^{N}\hat{a}_{i,t} \\ &\quad + \log(1-\tilde{p})\sum_{i=1}^{N}(1-\hat{a}_{i,t}) + \sum_{i=1}^{N}(1-\hat{a}_{i,t})\log g(z_i), \end{aligned} \tag{8}$$

where

$$\tilde{\alpha}_i = \arctan\left(\frac{x_i - \tilde{x}}{y_i - \tilde{y}}\right). \tag{9}$$

In order to complete the E-step, $\hat{a}_{i,t}$ is computed using Bayes' theorem as follows

$$\begin{aligned} \hat{a}_{i,t} &= \frac{f(z_i \,|\, \hat{\boldsymbol{\theta}}_t, a_i = 1) \cdot \Pr\left\{a_i = 1 \,|\, \hat{\boldsymbol{\theta}}_t\right\}}{f(z_i \,|\, \hat{\boldsymbol{\theta}}_t)} \\ &= \frac{\rho_{i,t} \cdot \hat{p}_t}{\rho_{i,t} \cdot \hat{p}_t + g(z_i) \cdot (1 - \hat{p}_t)}, \end{aligned} \tag{10}$$

where

$$\rho_{i,t} \quad = \quad f(z_i|a_i = 1; \hat{\boldsymbol{\theta}}_t) = \tfrac{1}{\sqrt{2\pi\hat{\sigma}_t^2}} e^{-\frac{(z_i-\hat{\alpha}_{i,t})^2}{2\hat{\sigma}_t^2}}, \tag{11}$$

The M-step is accomplished maximizing (8) with respect to $\tilde{\boldsymbol{\theta}}$, and it yields to

$$\hat{p}_{t+1} = \frac{1}{N}\sum_{i=1}^{N}\hat{a}_{i,t}, \tag{12}$$

$$\hat{\sigma}_{t+1}^2 = \frac{\sum_{i=1}^{N}\hat{a}_{i,t}(z_i - \hat{\alpha}_{i,t+1})^2}{\sum_{i=1}^{N}\hat{a}_{i,t}}, \tag{13}$$

where

$$\hat{\alpha}_{i,t+1} = \arctan\left(\frac{x_i - \hat{x}_{t+1}}{y_i - \hat{y}_{t+1}}\right). \tag{14}$$

The maximization of (8) with respect to $\tilde{x}$ and $\tilde{y}$ leads to two nonlinear equations that cannot be solved analytically. In order to compute the pair $\hat{\boldsymbol{r}}_{t+1} = [\hat{x}_{t+1}, \hat{y}_{t+1}]^T$ we resort to a weighted version of the LS method proposed in [5], which gives the position of the source as

$$\hat{\boldsymbol{r}}_{t+1} = \left(\boldsymbol{H}^T\hat{\boldsymbol{A}}_t\hat{\boldsymbol{D}}_t^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^T\hat{\boldsymbol{A}}_t\hat{\boldsymbol{D}}_t^{-1}\boldsymbol{b}, \tag{15}$$

where $\boldsymbol{b} = [b_1, \ldots, b_N]$ and $b_i = -x_i \cdot \sin z_i + y_i \cdot \cos z_i$; matrix $\boldsymbol{H}$ is equal to

$$\boldsymbol{H} = \begin{bmatrix} -\sin z_1 & \cos z_1 \\ \vdots & \vdots \\ -\sin z_N & \cos z_N \end{bmatrix}; \tag{16}$$

and the weight matrices are $\hat{\boldsymbol{A}}_t = \text{diag}[\hat{a}_{1,t}, \ldots, \hat{a}_{N,t}]$ and $\hat{\boldsymbol{D}}_t = \text{diag}[\hat{d}_{1,t}, \ldots, \hat{d}_{N,t}]$, where $\hat{d}_{i,k} = \|\hat{\boldsymbol{r}}_{i,t} - \boldsymbol{u}_i\|_2$. Since each $\hat{a}_{i,t}$ is a soft decision on the LOS/NLOS state of the node, the LS equations are weighted by matrix $\hat{\boldsymbol{A}}_t$ to discard the nodes in NLOS. Moreover, given that the errors in the AOA measurements induce a larger positioning error as the distance between the source and the nodes increases, the equations are also weighted by the inverse of matrix $\hat{\boldsymbol{D}}_t$.

Although the use of the WLS method to find the estimate $\hat{\boldsymbol{r}}_{t+1}$ that maximizes (8) is suboptimal in general, it is less computational intensive and allows for a distributed implementation of the algorithm as explained hereafter.

IV. A DIFFUSION-BASED DISTRIBUTED ALGORITHM

In order to obtain a distributed implementation of the algorithm described in Section III, each node must be able to compute the parameters (12), (13), and (15) locally, which require global information. The distributed implementation is based on the DB-DEM algorithm proposed in [12], where it is shown that it enables a distributed implementation of the EM algorithm whenever the parameters are expressed as a summation over i of variables available at the nodes. For this purpose, note that the elements involved in (15) have the following expressions

$$\boldsymbol{H}^T\hat{\boldsymbol{A}}_t\hat{\boldsymbol{D}}_t^{-1}\boldsymbol{H} = \begin{bmatrix} \sum \frac{\hat{a}_{i,t}}{\hat{d}_{i,t}}\sin^2 z_i & -\sum \frac{\hat{a}_{i,t}}{\hat{d}_{i,t}}\sin z_i \cos z_i \\ -\sum \frac{\hat{a}_{i,t}}{\hat{d}_{i,t}}\sin z_i \cos z_i & \sum \frac{\hat{a}_{i,t}}{\hat{d}_{i,t}}\cos^2 z_i \end{bmatrix} \tag{17}$$

$$\boldsymbol{H}^T\hat{\boldsymbol{A}}_t\hat{\boldsymbol{D}}_t^{-1}\boldsymbol{b} = \begin{bmatrix} -\sum \frac{\hat{a}_{i,t}}{\hat{d}_{i,t}}\sin z_i(-x_i\sin z_i + y_i\cos z_i) \\ \sum \frac{\hat{a}_{i,t}}{\hat{d}_{i,t}}\cos z_i(-x_i\sin z_i + y_i\cos z_i) \end{bmatrix}, \tag{18}$$

and that the nuisance parameters in (12) and (13) are calculated as summations as well.

Consider then a WSN with N nodes in which each node can only communicate with the neighbors located within a certain radius. Let us define a weight matrix $\boldsymbol{W} \in R^{N\times N}$ with nonzero w_{ij} entries only if there is a direct connection between node i and node j. Let us assume that this matrix also satisfies

$$\boldsymbol{W}\mathbf{1} = \mathbf{1}, \quad \mathbf{1}^T\boldsymbol{W} = \mathbf{1}^T, \quad \rho(\boldsymbol{W} - \frac{\mathbf{11}}{N}) < 1. \tag{19}$$

The distributed algorithm is run iteratively after the initial angle measurements. At each time instant k[1], node i keeps local variables $\hat{\boldsymbol{\theta}}_{i,k} = [\hat{x}_{i,k}, \hat{y}_{i,k}, \hat{\sigma}^2_{i,k}, \hat{p}_{i,k}]^T$ and auxiliary variables

$$\begin{aligned}
f_a(i,k) &= \xi_{i,k}; & f_c(i,k) &= 1 \\
f_1(i,k) &= \frac{\xi_{i,k}}{\hat{d}_{i,k}}\sin^2 z_i; & f_2(i,k) &= -\frac{\xi_{i,k}}{\hat{d}_{i,k}}\sin z_i\cos z_i \\
f_3(i,k) &= \frac{\xi_{i,k}}{\hat{d}_{i,k}}\cos^2 z_i; & f_4(i,k) &= -\frac{\xi_{i,k}}{\hat{d}_{i,k}}\sin z_i \\
& & &\quad (-x_i\sin z_i + y_i\cos z_i) \\
f_5(i,k) &= -f_4\frac{\cos z_i}{\sin z_i}; & f_6(i,k) &= \xi_{i,k}(z_i - \hat{\alpha}_{i,k})^2
\end{aligned}$$

[1]We use the index k for the distributed implementation to avoid confusion with the centralized approach of Sec. III, for which we use the index t.

For $i = 1, \cdots, N$

1) *Initialization:*
   - Set $\hat{a}_{i,0}$ to an initial guess and $\hat{d}_{i,0} = 1$.
   - Compute the auxiliary variables $f_\nu(i,0)$ for $\nu \in \{a, c, 1, \ldots, 5\}$ as in (**??**).
   - Compute the intermediate variables $\phi_\nu(i,0) = \sum_{j=1}^{N} W_{ij} f_\nu(j,0)$ for $\nu \in \{a, c, 1, \ldots, 5\}$.
   - Compute $\hat{\boldsymbol{r}}_{i,1}$ and $\hat{p}_{i,1}$ as

$$\begin{aligned}
\hat{\boldsymbol{r}}_{i,1} &= \begin{bmatrix} \phi_1(i,0) & \phi_2(i,0) \\ \phi_2(i,0) & \phi_3(i,0) \end{bmatrix}^{-1} \begin{bmatrix} \phi_4(i,0) \\ \phi_5(i,0) \end{bmatrix} \\
\hat{p}_{i,1} &= \frac{\phi_a(i,0)}{\phi_c(i,0)}
\end{aligned} \tag{21}$$

   - Compute $\hat{\alpha}_{i,0}$, $f_6(i,0)$, $\phi_6(i,0)$ and then $\hat{\sigma}^2_{i,1}$ as

$$\hat{\sigma}^2_{i,1} = \frac{\phi_6(i,0)}{\phi_a(i,0)} \tag{22}$$

For $k \geq 1$,

2) *E-Step*: given $\hat{\boldsymbol{\theta}}_{i,k}$, compute $\xi_{i,k}$, $\hat{d}_{i,k}$ and $\hat{\alpha}_{i,k}$.
3) *M-Step*: for every subindex $\nu \in \{a, c, 1, \ldots, 6\}$, update the auxiliary variables f_ν with the new $\xi_{i,k}$ and compute the intermediate variables

$$\phi_\nu(i,k) = \sum_{j=1}^{N} W_{ij}((1-\beta(k))\phi_\nu(j,k-1) + \alpha(k)f_\nu(j,k)),$$

where

$$\alpha(k) = \frac{1}{k}, \quad \beta(k) = \frac{1}{k^\delta}, \quad 0 < \delta < 1, \quad k = 1, 2, \ldots \tag{23}$$

and then update

$$\begin{aligned}
\hat{\boldsymbol{r}}_{i,k+1} &= \begin{bmatrix} \phi_1(i,k) & \phi_2(i,k) \\ \phi_2(i,k) & \phi_3(i,k) \end{bmatrix}^{-1} \begin{bmatrix} \phi_4(i,k) \\ \phi_5(i,k) \end{bmatrix} \\
\hat{p}_{i,k+1} &= \frac{\phi_a(i,k)}{\phi_c(i,k)} \\
\hat{\sigma}^2_{i,k+1} &= \frac{\phi_6(i,k)}{\phi_a(i,k)}
\end{aligned} \tag{24}$$

4) *Repeat* steps 2 and 3 until convergence.

TABLE I
DIFFUSION-BASED DISTRIBUTED ALGORITHM

where $\xi_{i,k} \triangleq \Pr\left\{a_i = 1 \,|\, \hat{\boldsymbol{\theta}}_{i,k}, z_i\right\}$ is a local estimation at the i^{th} node of the LOS/NLOS state a_i, given by

$$\xi_{i,k} = \frac{\hat{p}_{i,k}\cdot\frac{1}{\sqrt{2\pi}\hat{\sigma}_{i,k}}e^{-\frac{(z_i-\hat{\alpha}_{i,k})^2}{2\hat{\sigma}^2_{i,k}}}}{\hat{p}_{i,k}\cdot\frac{1}{\sqrt{2\pi}\hat{\sigma}_{i,k}}e^{-\frac{(z_i-\hat{\alpha}_{i,k})^2}{2\hat{\sigma}^2_{i,k}}} + (1-\hat{p}_{i,k})\cdot\frac{1}{2\pi}}, \tag{20}$$

where $\hat{d}_{i,k} = \|\hat{\boldsymbol{r}}_{i,k} - \boldsymbol{u}_i\|_2$ and $\hat{\alpha}_{i,k} = \arctan\left(\frac{\hat{x}_{i,k}-x}{\hat{y}_{i,k}-y}\right)$.

Table I summarizes the execution of the distributed algorithm, which can be described as follows. In the expectation step, each node updates the value of $\xi_{i,k}$, the estimated angle $\hat{\alpha}_{i,k}$ and the distance $\hat{d}_{i,k}$ using the estimates $\hat{\boldsymbol{\theta}}_{i,k}$ obtained in the previous iteration. In the maximization step, information is exchanged between the neighboring nodes to update the auxiliary variables $f_\nu(i,k)$ and the intermediate variables $\phi_\nu(i,k)$. Finally, the estimates $\hat{\boldsymbol{\theta}}_{i,k+1}$ are calculated using the intermediate variables. These steps are repeated until convergence.

The sum of the two terms weighted by $(1-\beta(k))$ and $\alpha(k)$ in (23) is responsible for propagating the information over the network. The weight of the first term, which drives the network towards a consensus, increases with k whereas the second term, which diffuses the updated variables, vanishes. The relationship between the two terms can be tuned with the parameter δ, with lower values of δ delaying the consensus in favour of more refined estimates of the unobserved data ξ_i.

V. SIMULATION RESULTS

We have simulated several configurations of a graph network with N nodes that estimates the position of a source randomly positioned in a 200×200 m^2 square. The nodes have a connectivity radius of 40 m and experience a LOS propagation channel with probability $p = 0.7$. The range of the noise variance σ^2 is $[10^{-4.5}, 1]$ rad. We compare the centralized estimator, a clairvoyant (CV) WLS estimator that obtains the source position using (15) with $\hat{\boldsymbol{A}} = \text{diag}[a_1, \ldots, a_N]$, and the distributed estimator. The CV estimator is run twice; first with $\hat{\boldsymbol{D}} = \boldsymbol{I}$ to obtain an estimate of the distances $\hat{d}_i$, and a second time with the weight matrix $\hat{\boldsymbol{D}} = \text{diag}[\hat{d}_1, \ldots, \hat{d}_N]$. The centralized algorithm is run $N_{it} = 300$ iterations and the distributed algorithm is run $N_{it}^D = 1000$ iterations with $\delta = \{0.6, 0.7, 0.8\}$ and a Metropolis weight matrix $\boldsymbol{W}$ [16]. All three algorithms are run a total of $N_{rea} = 1000$ realizations for each value of σ^2. A new random position of the source source is used in each realization, as well as new random LOS or NLOS channels for the nodes. As a performance metric we use the logarithm of the MSE, defined as

$$10 \log_{10} \text{MSE} = 10 \log_{10} \left[\mathbb{E} \left(\frac{1}{N} \sum_{i=1}^{N} \| \hat{\boldsymbol{r}}_{i,N_{it}} - \boldsymbol{r} \|_2^2 \right) \right], \tag{25}$$

Fig. 1 shows the MSE vs. σ^2 for $N = 100$, with a random deployment of the nodes in each realization, and $\delta = \{0.6, 0.7, 0.8\}$. We observe that the MSE of the centralized algorithm is slightly above the MSE of the CV estimator and that at the lower σ^2 the two curves merge. We also observed in the simulations that the centralized algorithm converges in less than 10 iterations for noise variances lower than -5 dBradians. The distributed algorithm performs as well as the centralized version and also converges to the CV estimator for $\delta = 0.7$, whereas for $\delta = \{0.6, 08\}$ it does not. With $\delta = 0.6$ the consensus is slower than with $\delta = 0.7$, so each node relies longer on the information from its neighbors. On the other hand, with $\delta = 0.8$ the consensus is reached too soon, which hinders the propagation of the updated estimates $\xi_{i,k}$ over the network. Moreover, we have observed that in a percentage of the realizations (4% for $\delta = 0.6$, 2% for $\delta = 0.7$ and 11% for $\delta = 0.7$) the algorithm is unable to identify all the nodes in NLOS conditions independently of the noise variance.

Fig. 2 shows the MSE vs. σ^2 for $N = \{50, 100\}$ nodes deployed in a rectangular grid and $\delta = 0.7$. We observe that the centralized algorithm converges to the CV estimator for both N, whereas the distributed algorithm only converges for $N = 100$. The high percentage of nodes in NLOS, i.e. 30% on average, the reduced connectivity and number of samples of the network for $N = 50$ prevent the distributed algorithm from being able to discard the nodes in NLOS in some realizations and, consequently, the MSE saturates.

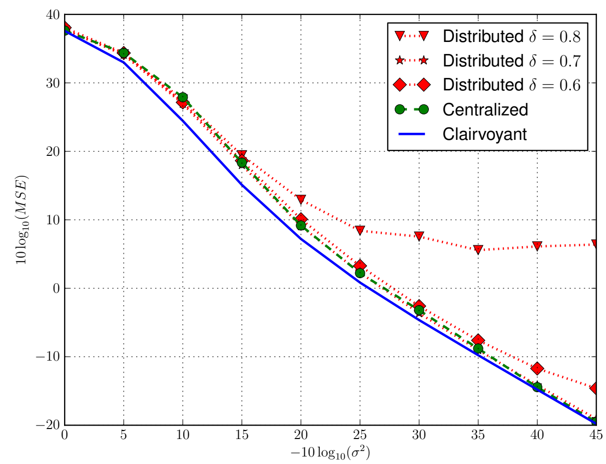

Fig. 1. MSE (dB) vs. $-10\log_{10}\sigma^2$ for $N = 100$ and $\delta = \{0.6, 0.7, 0.8\}$.

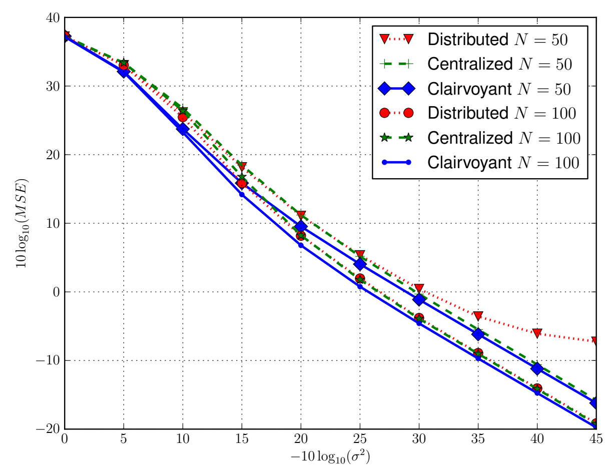

Fig. 2. MSE (dB) vs. $-10\log_{10}\sigma^2$ for $N = \{50, 100\}$ and $\delta = 0.7$.

VI. CONCLUSIONS

An algorithm for the mitigation of the NLOS error in AOA-based positioning in WSNs has been proposed. The scheme relies on the EM algorithm to identify the nodes in NLOS and obtain the ML estimate of several nuisance parameters, whereas a WLS estimator is used to calculate the position of the source. This approach allows for a distributed implementation in which each node iteratively refines its estimates and diffuses the new information over the network, while a consensus process is gradually switched on so that all nodes reach a final agreement on the estimated values. Simulation results show that the centralized algorithm offers performance close to that of a CV estimator and it only needs a few iterations to converge to a solution, whereas the distributed version, given a correct balance between consensus speed and propagation of new information, is able to attain the same level of performance.

References

[1] C.-Y. Chong and S. P. Kumar, "Sensor networks: evolution, opportunities, and challenges," *Proceedings of the IEEE*, vol. 91, no. 8, pp. 1247–1256, 2003.

[2] D. Torrieri, "Statistical Theory of Passive Location Systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-20, no. 2, pp. 183–198, Mar. 1984.

[3] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *Signal Processing Magazine, IEEE*, vol. 22, no. 4, pp. 24–40, 2005.

[4] R. Huang and Y. Manoli, "Phased array and adaptive antenna transceivers in wireless sensor networks," in *Digital System Design, 2004. DSD 2004. Euromicro Symposium on*. IEEE, 2004, pp. 587–592.

[5] J. Vidal, D. Brooks *et al.*, "Closed-form solution for positioning based on angle of arrival measurements," in *Personal, Indoor and Mobile Radio Communications, 2002. The 13th IEEE International Symposium on*, vol. 4. IEEE, 2002, pp. 1522–1526.

[6] P. Kułakowski, J. Vales-Alonso, E. Egea-López, W. Ludwin, and J. García-Haro, "Angle-of-arrival localization based on antenna arrays for wireless sensor networks," *Computers & Electrical Engineering*, vol. 36, no. 6, pp. 1181–1186, 2010.

[7] I. Guvenc and C.-C. Chong, "A survey on toa based wireless localization and nlos mitigation techniques," *Communications Surveys & Tutorials, IEEE*, vol. 11, no. 3, pp. 107–124, 2009.

[8] K. Yu and Y. J. Guo, "Statistical nlos identification based on aoa, toa, and signal strength," *Vehicular Technology, IEEE Transactions on*, vol. 58, no. 1, pp. 274–286, 2009.

[9] L. Cong and W. Zhuang, "Non-line-of-sight error mitigation in tdoa mobile location," in *Global Telecommunications Conference, 2001. GLOBECOM'01. IEEE*, vol. 1. IEEE, 2001, pp. 680–684.

[10] L. Xiong, "A selective model to suppress nlos signals in angle-of-arrival (aoa) location estimation," in *Personal, Indoor and Mobile Radio Communications, 1998. The Ninth IEEE International Symposium on*, vol. 1. IEEE, 1998, pp. 461–465.

[11] Y. Zhang, N. Meratnia, and P. Havinga, "Outlier detection techniques for wireless sensor networks: A survey," *Communications Surveys & Tutorials, IEEE*, vol. 12, no. 2, pp. 159–170, 2010.

[12] S. S. Pereira, R. Lopez-Valcarce *et al.*, "A diffusion-based em algorithm for distributed estimation in unreliable sensor networks," *Signal Processing Letters, IEEE*, vol. 20, no. 6, pp. 595–598, 2013.

[13] A. Goldsmith, *Wireless communications*. Cambridge University Press, 2005.

[14] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the em algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 1–38, 1977.

[15] T. K. Moon, "The expectation-maximization algorithm," *Signal processing magazine, IEEE*, vol. 13, no. 6, pp. 47–60, 1996.

[16] L. Xiao, S. Boyd, and S. Lall, "A scheme for robust distributed sensor fusion based on average consensus," in *Information Processing in Sensor Networks, 2005. IPSN 2005. Fourth International Symposium on*. IEEE, 2005, pp. 63–70.