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Multipath Compensation in Acoustic Local Positioning Systems

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***Abstract*—This work proposes a novel method to estimate the Line-of-Sight Time-of-Flights (LOS-TOFs) in a broadband acoustic local positioning system (ALPS) with strong multipath interference. The proposed method is based on the Matching Pursuit channel estimation algorithm that provides a low complexity approximation to the Maximum Likelihood solution for sparse channels. A multichannel version of this algorithm has been implemented to estimate a minimum of three coefficients in the channel responses of a particular ALPS, composed of four beacons that perform the simultaneous emission of BPSK modulated 255-bit Kasami codes. A statistical analysis of performance has been carried out by using a set of test signals synthetically generated to simulate different positions and reflection coefficients of the environment. The results of this analysis show the enhanced capability of the proposed method to estimate the LOS-TOFs under strong multipath interference, with respect to that of a classical system based on correlation + thresholding.**

***Index Terms*—Acoustic Local Positioning Systems, Multipath propagation, Matching Pursuit Channel Estimation**

I. Introduction

Man has always felt the need to position himself, using the sun and the stars as the only reliable reference systems for many centuries. Today, with the advent of global navigation satellite systems (GNSS) global positioning in the earth's surface is a successfully overcome problem. Nonetheless, local positioning in indoor environments is still a matter of active research, since GNSS signals get severely degraded in this type of environments due to multipath and attenuation losses, and thus they cannot be used to track people or objects with acceptable accuracy [1]. Many local positioning systems (LPS) have been developed during the past two decades based on different technologies that include acoustic [2]–[4], radio-frequency [5], [6], optic [7], [8] and magnetic [9], [10], among others. Today, there is a general agreement in considering the relatively low cost and high resolution of acoustic systems as their main advantages, the latter being a direct consequence of the low speed of sound in air.

During the first years of the former decade some acoustic LPS (ALPS) were proposed that achieved centimetric precision through the emission of short ultrasonic pulses, both centralized, where the object to be located acts as the emitter [11], and privacy-oriented, where this object is in charge of computing its own position using the signals received from different beacons [12]. The use of narrowband signals made these systems very susceptible to noise though, and also, the update rate was very limited due to the necessity to avoid signal collisions.

Shortly after, signal coding and matched filtering detection were incorporated into these systems, choosing for this purpose families of binary codes with good correlation properties. The use of Direct Sequence Spread Spectrum (DSSS) techniques brought important advantages such as robustness to noise, multiple access capability (allowing increased update rates) and higher resolution. One of the first broadband systems was proposed by Hazas and Ward, who used Gold sequences in the design of both centralized [2] and privacy-oriented [13] ALPS. Since then, several works based on DSSS have arisen that employ more efficient encoding schemes, using Kasami [3] or Loosely Synchronous (LS) [14] codes. Also, Frequency Hopping Spread Spectrum (FHSS) techniques are being recently used as an alternative to DSSS that seems to improve the accuracy of these systems under conditions of noise and reverberation [4].

The use of longer and simultaneous emissions by these evolved systems comes hand in hand with new problems, though. These problems, such as Multiple Access Interference (MAI), the Near-Far problem, and the effect of multipath propagation, are the subject of current interest for researchers in the field. However, this is, to the authors knowledge, the first work that addresses and proposes a solution to the effects that multipath propagation can have on the performance of an ALPS. This solution is based on the Matching Pursuit (MP) channel estimation algorithm [15], [16] which has been proven to accurately detect direct transmission path signals which are highly attenuated compared with reflected path signals in DS-CDMA systems.

The rest of the work is organized as follows. Section II describes the fundamentals of ALPS and the proposed LPS architecture. In Section III, the effects of multipath propagation on the performance of an ALPS are exposed, and a solution based on the MP channel estimation algorithm is proposed. Section IV first introduces some preliminay results obtained to tune the parameters of the proposed method, and then it presents the results of the performance analysis. The main conclusions drawn from this work are finally outlined in

Section V.

II. ALPS MODEL DESCRIPTION

A. Broadband Acoustic Local Positioning

The most extensively developed acoustic positioning systems are those based on the measurement of the time that the signal takes to travel from the emitter to the receiver. In these systems, several beacons distributed in the environment emit acoustic signals that are detected by the receiver placed in the object or person to be located. This receiver estimates the Time-of-Flight (TOF) of these signals in order to obtain from them the desired position $\mathbf{r}$ by solving the following system of sphere equations

$$\|\mathbf{r_k} - \mathbf{r}\| = c \cdot TOF_k \tag{1}$$

where $\mathbf{r_k} = (x_k, y_k, z_k)$ with $k = 1, 2, \ldots, N$ are the beacons known positions, and c is the speed of sound in air. This process of determining the target location is known as trilateration, and it requires a minimum of $N = 3$ non-aligned beacons in order to get the 3D position of the target. However, the set of nonlinear equations (1) is rarely solved directly, and it is usually linearized by introducing an additional beacon, expanding the equations, and grouping the nonlinear terms in an addicional variable that can be used to verify the estimated position [17]. Trilateration positioning requires the target and the beacons to be perfectly synchronized, which is usually achieved by using a triggering RF signal whose propagation time is negligible compared to that of the acoustic signal.

In broadband ALPS, all beacons are wired synchronized and perform the simultaneous and periodic emission of pseudo-orthogonal signal patterns. These signals are detected in the receiver through of a bank of correlators, in whose outputs a maximum peak is obtained every time the matched emission is received. The time of ocurrence of this peak, known as Time-of-Arrival (TOA), is then computed by simple thresholding, and from it the TOF of the correspoding emission is obtained by subtracting the emission duration [3], [18].

B. Proposed architecture

In this work we have analysed the behavior of a broadband ALPS composed of four beacons, forming a 40 cm side square in the ceiling of a $3 \times 3 \times 3$ m^3 room, as represented in Fig. 1. The pseudo-orthogonal signals emitted by these beacons are BPSK modulated 255-bit Kasami codes [19]. These binary codes have better aperiodic correlation properties than maximal and Gold sequences, at the expense of reducing the number of members in a family with these desirable properties. In the proposed ALPS these codes are BPSK modulated with a 20 kHz carrier, in order to adapt the emissions spectra to the frequency response of the acoustic transducers.

III. MULTIPATH PROPAGATION

A. The effect of multipath on the performance of the ALPS

As stated in the introduction, multipath propagation is one of the problems that must be faced by the designers of broadband ALPS. The effect of this phenomenon can be disruptive

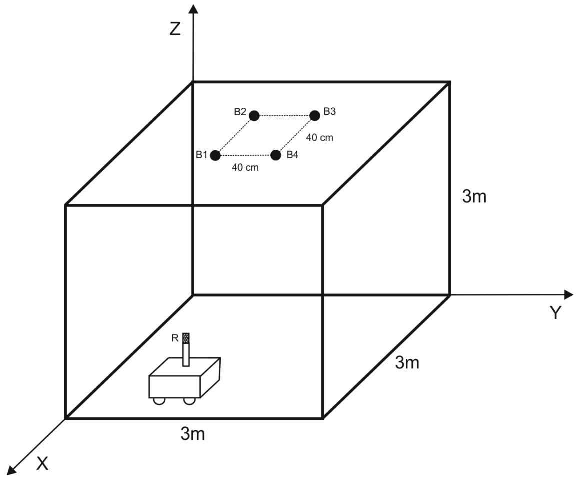

Fig. 1. Proposed ALPS architecture.

in the proximity of the room limits, where the reflected signals interfere with the Line-of-Sight (LOS) emissions, thus altering the ideal correlation properties of these emissions. Out of these limits, or when the reflection coefficients of the room's walls are low, the interference is weak and the LOS emissions can still be detected by locating the maximum value at the outputs of the matched filter bank. However, when the receiver is close to the walls of a room whose reflection coefficients are high, the loss of orthogonality due to interference has as a consequence that the maximum values at the outputs of the matched filter bank no longer represent the instant of arrival of the LOS emissions. This is the situation depicted in Fig. 2, where these outputs have been represented when the receiver is placed at coordinates (0.05, 0.08, 0.03) m, assuming a reflection coefficient $\gamma = 0.8$ for all the walls. As can be seen, the estimated TOAs for the emissions coming from beacons 3 and 4 have an error of 0.14 and 0.13 ms respectively, what would translate into an error of several decimeters in the estimation of the receiver position by means of the trilateration algorithm.

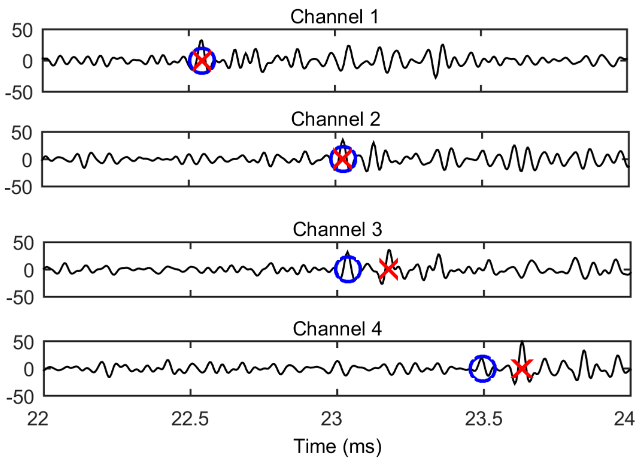

Fig. 2. Estimated (red crosses) and theorical (blue circles) TOAs for a receiver placed at coordinates (0.05, 0.08, 0.03) m when $\gamma = 0.8$.

B. Sparse multichannel estimation

The erroneous calculation of the LOS TOAs described above can be resolved if a precise estimation of the channel impulse response is performed. Since in an acoustic LPS a Line-of-Sight path between the beacons and the receiver must always be assured, the time of ocurrence of the first non-null coeficient in the estimated impulse response represents the desired TOF. Let us consider first the case of a single channel composed by a beacon-receiver pair. After A/D conversion at the sampling rate $f_s = 1/T_s$, the received signal $\mathbf{y} = [y[N_s],\, y[N_s{+}1],\, \cdots, y[N_s{+}p{-}1]]^T \in R^{p\times 1}$ is denoted by

$$\mathbf{y} = \mathbf{S}\mathbf{f} + \mathbf{n} \tag{2}$$

where $\mathbf{f} = [f_1,\, f_2,\, \cdots,\, f_{N_s}]^T \in R^{N_s\times 1}$ is the channel coefficient vector, $\mathbf{n} = [n[1],\, n[2],\, \cdots,\, n[p]]^T \in R^{p\times 1}$ is a vector of white Gaussian noise samples and

$$\mathbf{S} = \begin{bmatrix} s[N_s] & s[N_s-1] & \cdots & s[1] \\ s[N_s+1] & s[N_s] & \cdots & s[2] \\ s[N_s+2] & s[N_s+1] & \cdots & s[3] \\ \vdots & \vdots & \cdots & \vdots \\ s[N_s+p-1] & s[N_s+p-2] & \cdots & s[p] \end{bmatrix} \in R^{p\times N_s}$$

is the characteristic signal matrix collecting samples $s[n]$ of the signal transmitted by the beacon. Note that the multipath spread has been assumed to be $N_s \times T_s$. As we know, the maximum likelihood (ML) estimate for the channel coefficients is the solution to the following minimization problem

$$\hat{\mathbf{f}} = \underset{\mathbf{f}}{\operatorname{argmin}} \left\{ \|\mathbf{y} - \mathbf{S}\mathbf{f}\|^2 \right\} \tag{3}$$

whose optimal solution is given by the Least Squares (LS) estimate

$$\hat{\mathbf{f}} = (\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H\mathbf{y} \tag{4}$$

The characteristic signal matrix is known *a priori* and thus, its pseudo-inverse $(\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H$ can be precomputed to facilitate the ML channel estimation defined by (4). However, taking into account that the number of samples in a typical ALPS channel impulse response is around several thousands, it should be clear that the ML solution is not a feasible implementation for a real-time operating system. This issue is even more problematic in practice with multiple beacon-receiver channels to be estimated jointly.

In this work we have explored the Matching Pursuit (MP) channel estimation algorithm as an alternative to the ML solution. This algorithm provides a low complexity approximation to the ML solution for sparse channels [16], i.e., channels where the number of coefficients with non-negligible magnitude is much lower than the total number of coefficients, as represented in Fig. (2). The MP algorithm minimizes (3) iteratively, one estimated channel coefficient $\hat{f}_{q_j}$ at a time, using a greedy approach in wich the detected path index q_j and $\hat{f}_{q_j}$ are selected such that the decrease in (3) at each stage j is the largest possible [15]. That is, the multipath signal components are estimated via successive interference cancellation. For $j = 1, 2, \ldots, N_f$,

$$q_j = \underset{i \neq q_1,\ldots,q_{j-1}}{\operatorname{argmax}} \left\{ \frac{\left|\mathbf{S}_i^H \mathbf{y}^j\right|^2}{\|\mathbf{S}_i\|^2} \right\} \tag{5}$$

and

$$\hat{f}_{q_j} = \frac{\mathbf{S}_{q_j}^H \mathbf{y}^j}{\left\|\mathbf{S}_{q_j}\right\|^2} \tag{6}$$

with

$$\mathbf{y}^{j+1} = \mathbf{y}^j - \frac{\mathbf{S}_{q_j}^H \mathbf{y}^j \mathbf{S}_{q_j}}{\left\|\mathbf{S}_{q_j}\right\|^2} \tag{7}$$

where $\mathbf{S}_i$ represents the $i-th$ column vector of matrix $\mathbf{S}$ and $\mathbf{y}^1 = \mathbf{y}$. The algorithm terminates after stage $j = N_f$.

For K simultaneous channels, the received signal model (2) can be rewritten as

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{S}^k \mathbf{f}^k + \mathbf{n} \tag{8}$$

where $\mathbf{f}^k$ is the $k-th$ channel coefficient vector and $\mathbf{S}^k$ is the characteristic signal matrix of the $k-th$ beacon. Now, in every new iteration of the MP algorithm, Eqs. (5) and (6) are computed K times, one for each channel, and only the largest coefficent $\hat{f}^k_{q_j}$ is stored. Then, the corresponding estimated signal $\hat{f}^k_{q_j}\mathbf{S}^k_{q_j}$, which lies closest to the current cancelled signal $\mathbf{y}^l$, is substracted from this signal to obtain the updated received signal $\mathbf{y}^{l+1}$ as indicated by (7).

IV. Results

A. LOS TOFs calculation

It has been shown in the previous section that the TOF of the Line-of-Sight emission can be derived from a precise estimation of the channel impulse response. Next issue to be addressed is to determine the required level of precision for this estimation, which is directly related to the number of coefficients to be calculated by the MP algorithm. It is obvious that this number should be kept as low as possible to minimize the computational load of the MP algorithm but, on the other hand, it must be large enough as to be useful in a wide variety of different situations. Preliminary results confirm that a total number of four coefficients (one per channel) would be enough to estimate the LOS TOFs in the trivial case of very weak multipath interference. However, this is not possible with very strong multipath interference that may cause the appearance of larger coefficients in the impulse response than those associated with the Line-of-Sight paths. In this case, the results show that a minimun of three coefficients estimated per channel (for a total minimum of 12 coefficients) can assure a precise estimation of the desired TOFs. This situation is depicted in Fig. 3.

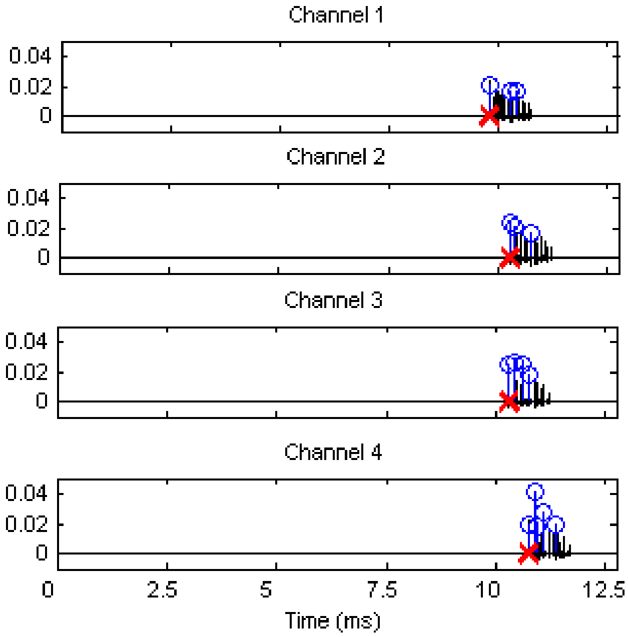

Fig. 3. Impulsive response (black line), estimated coefficients (blue circles) and estimated LOS TOF (red crosses) under strong multipath interference conditions. The MP algorithm was programmed to calculate a minimum of three coefficients per channel.

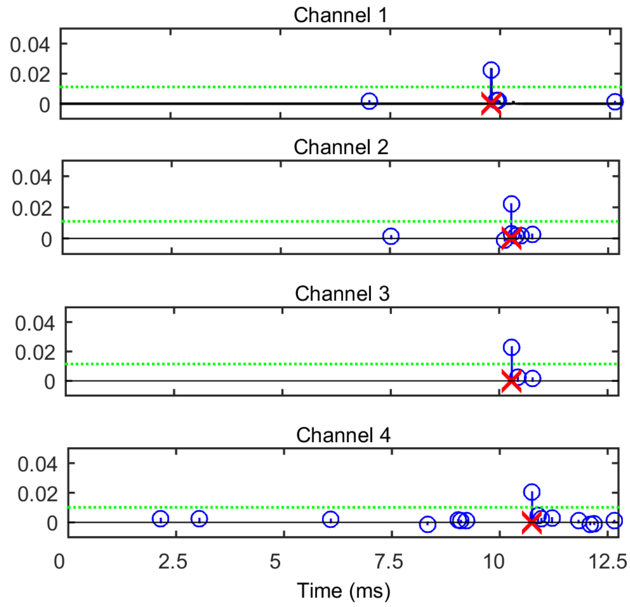

Fig. 4. Impulsive response (black solid line), estimated coefficients (blue circles), detection threshold (green dotted line) and estimated LOS TOF (red crosses) under weak multipath interference conditions. The MP algorithm was programmed to calculate a minimum of three coefficients per channel.

The problem now is that, if a large number of coefficients is calculated in the case of weak multipath interference or not interference at all, some spurious coefficients may appear before the Line-of-Sight path coefficient. This problem can be easily solved by establishing a detection threshold below which all calculated coefficients are discarded as LOS TOF candidates. Fig. 4 represents a very weak multipath situation when the MP algorithm is forced to calculate a minimum of three coefficients per channel. As can be seen, in this case, a detection threshold chosen as half the value of the largest coefficient estimated in each channel (dotted green line), is above all spurious coefficients thus allowing the correct estimation of the LOS TOFs.

Hence, the method proposed in this work makes use of the mutichannel MP algorithm to first calculate at least three coefficients of every channel impulse response, and then estimates the TOF of the Line-of-Sight path by detecting the time of occurence of the first coefficient whose value is above half the value of the largest coefficent estimated in each channel. The performance of this algorithm is evaluated next.

B. Perfomance analysis

The performance of the LOS-TOFs detection algorithm proposed in this work has been statistically analysed by modelling the signals that would reach the receiver in three different positions with increasing multipath interference $\{\mathbf{r}_1 = [0.05,\ 1,\ 1.2],\ \mathbf{r}_2 = [0.05,\ 1,\ 0.02],\ \mathbf{r}_3 = [0.05,\ 0.03,\ 0.02]\}$ close to 1, 2 and 3 walls respectively, when two different values for the reflection coefficient $\{\gamma_1 = 0.5,\ \gamma_2 = 0.9\}$ are considered. For each one of the six $(\mathbf{r}_i, \gamma_j)$ with $i \in \{1, 2, 3\}$ and $j \in \{1, 2\}$ possible combinations, four SNR levels have been assumed ranging from 4 dB to 16 dB in increments of 4 dBs. Every situation has been simulated 10^3 times to obtain $4 \cdot 10^3$ TOFs whose error distribution is represented in terms of cumulative distribution functions (CDF). First, for comparison purposes, the performance of a classical broadband ALPS where the signals are detected by correlation + thresholding has been represented in Fig. 5. As can be seen in this figure, 100% of the TOF measurements exhibit an error below 0.004 ms in the first position with a reflection coefficient $\gamma_1 = 0.5$ (first column in the first row). With this reflection coefficient but in the second position (close to two walls), we can see a slight degradation in the TOF measurements, especially for low SNR values. This degradation becomes evident in the third position (close to three walls), where only 75% of the measurements have an error below 0.07 ms even for the highest SNR value of 16 dB considered, although this number increases up to 100% for errors below 0.1 ms in all cases. The problems caused by multipath in a classical broadband ALPS, already discussed in section III-A, are clearly manifested in the second row of Fig. 5, where the highest reflection coefficient $\gamma_2 = 0.9$ is represented. Even in the less problematic $\mathbf{r}_1$ position, the total number of measurements with error below 0.1 ms is around 28% for the highest SNR level. This is also true in the other two positions, although in these cases the minimum error below which there are practically no measurements increases up to 0.05 and 0.08 ms for positions $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively.

The performance of the proposed MP algorithm-based detection method has been represented in Fig. 6. First thing to notice is that this method seems to perform slightly worse than the classical correlation + thresholding-based detection method under weak multipath interference for low SNR conditions, as represented in the first column of the first row ($\mathbf{r}_1$,γ_1). The performance of both systems is very similar in the second position with a reflection coefficient $\gamma_1 = 0.5$. From thereon, the improvement introduced by the proposed method becomes evident. Hence, in the third position with $\gamma_1 = 0.5$, 100% of the TOF measurements has an error below 0.004 ms with $SNR = 16$ dB, whereas this percentage is only achieved for errors below 0.08 ms in the classical system. Under strong multipath interference ($\gamma_2 = 0.9$), maximum values of 84%, 80% and 82% of the measurements with errors below 0.1 ms are obtained in positions $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ respectively for $SNR = 16$ dB. Minimum values of 75%, 75% and 69% are obtained in the respective positions for the lowest value of $SNR = 4$ dB. These values are, in all cases, far above the 28% obtained in the classical system, and the differences are even higher if lower errors are considered.

V. Conclusions and Future Work

In this work a novel LOS-TOFs estimation method for broadband ALPS with strong multipath interference has been proposed. This method uses the Matching Pursuit algorithm to estimate a minimum of three coefficients of the channel impulse responses, and obtains the desired LOS-TOFs as the time of ocurrence of the first coefficient whose value is above half the value of the largest coefficient estimated in each channel. The performance of this method has been analysed for a particular ALPS composed of four beacons installed in the ceiling of a $3 \times 3 \times 3$ m^3 room, that perform the simultaneous emission of BPSK modulated 255-bit Kasami codes. A set of test signals has been generated that models the signal reaching a receiver placed in three different positions, for three different wall reflection coefficients and under four different SNR values. The results show that the proposed method has a slightly worse performance than the classical correlation + thresholding-based detection method under weak multipath interference and with low SNR, but it clearly outperforms this method under strong multipath interference, multiplying by a factor between 2 and 3 the total number of TOF measurements obtained with an admisible error in this case.

In spite of this remarkable improvement, the results obtained with the new method are far from being ideal though, what leaves a wide margin for future actions. These actions should focus on the development of more evolved algorithms that could, for example, make use of more complex detection thresholds or exploit some properties inherent to the beacons architecture, as the maximum possible delay among LOS TOFs.

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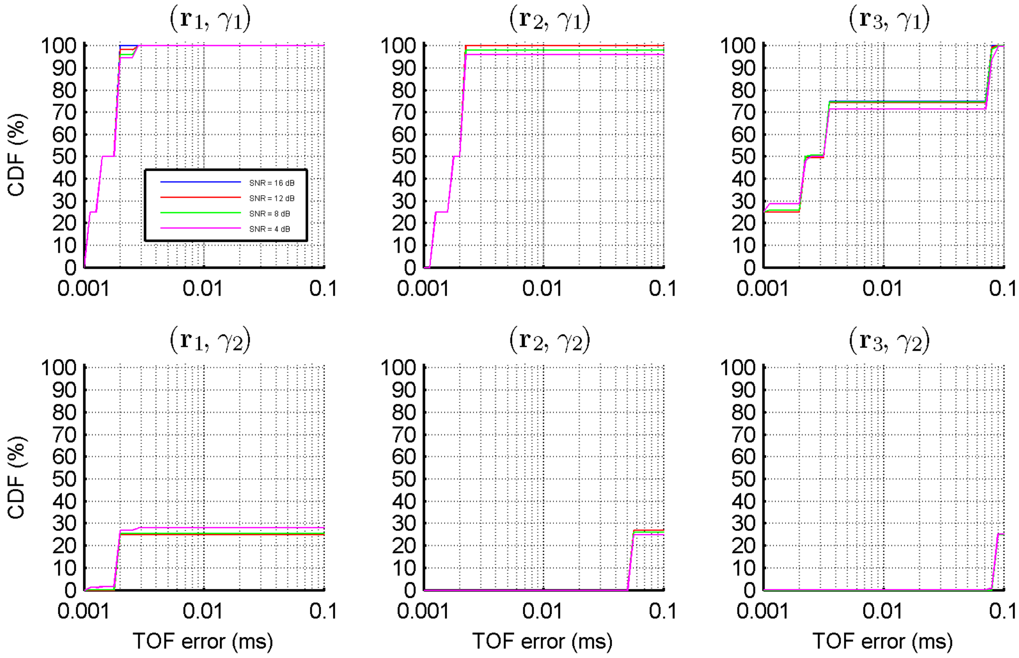

Fig. 5. Cumulative error in the TOF estimates of a classical broadband ALPS (correlation + thresholding-based detection). Three different positions ($\mathbf{r}_1 = [0.05,\ 1,\ 1.2]$, $\mathbf{r}_2 = [0.05,\ 1,\ 0.02]$, $\mathbf{r}_3 = [0.05,\ 0.03,\ 0.02]$), and two values for the reflection coefficient ($\gamma_1 = 0.5$, $\gamma_2 = 0.9$) are considered in columns and rows respectively. In each situation, four SNR values from 4 dB to 16 dB in increments of 4 dBs have been represented.

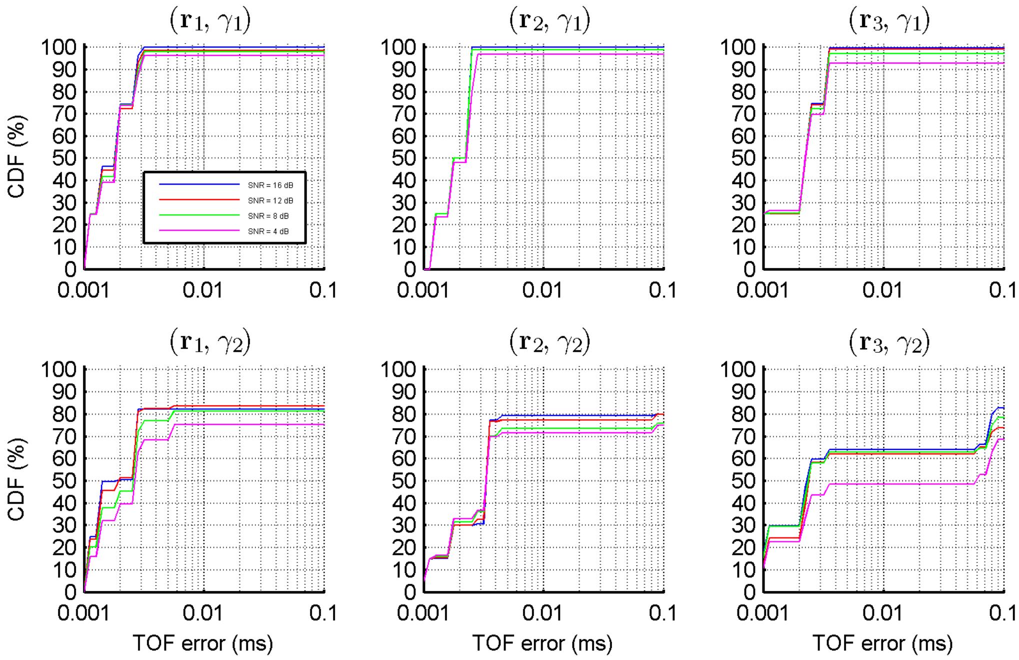

Fig. 6. Cumulative error in the TOF estimates of the proposed system (MP algorithm-based detection). Three different positions ($\mathbf{r}_1 = [0.05,\ 1,\ 1.2]$, $\mathbf{r}_2 = [0.05,\ 1,\ 0.02]$, $\mathbf{r}_3 = [0.05,\ 0.03,\ 0.02]$), and two values for the reflection coefficient ($\gamma_1 = 0.5$, $\gamma_2 = 0.9$) are considered in columns and rows respectively. In each situation, four SNR values from 4 dB to 16 dB in increments of 4 dBs have been represented.